ASSET-LIABILITY MANAGEMENT OF LIFE INSURERS IN THE NEGATIVE INTEREST RATE ENVIRONMENT

Yijia Lin

University of Nebraska - Lincoln

Sheen Liu

Washington State University

Ken Seng Tan

Nanyang Technological University

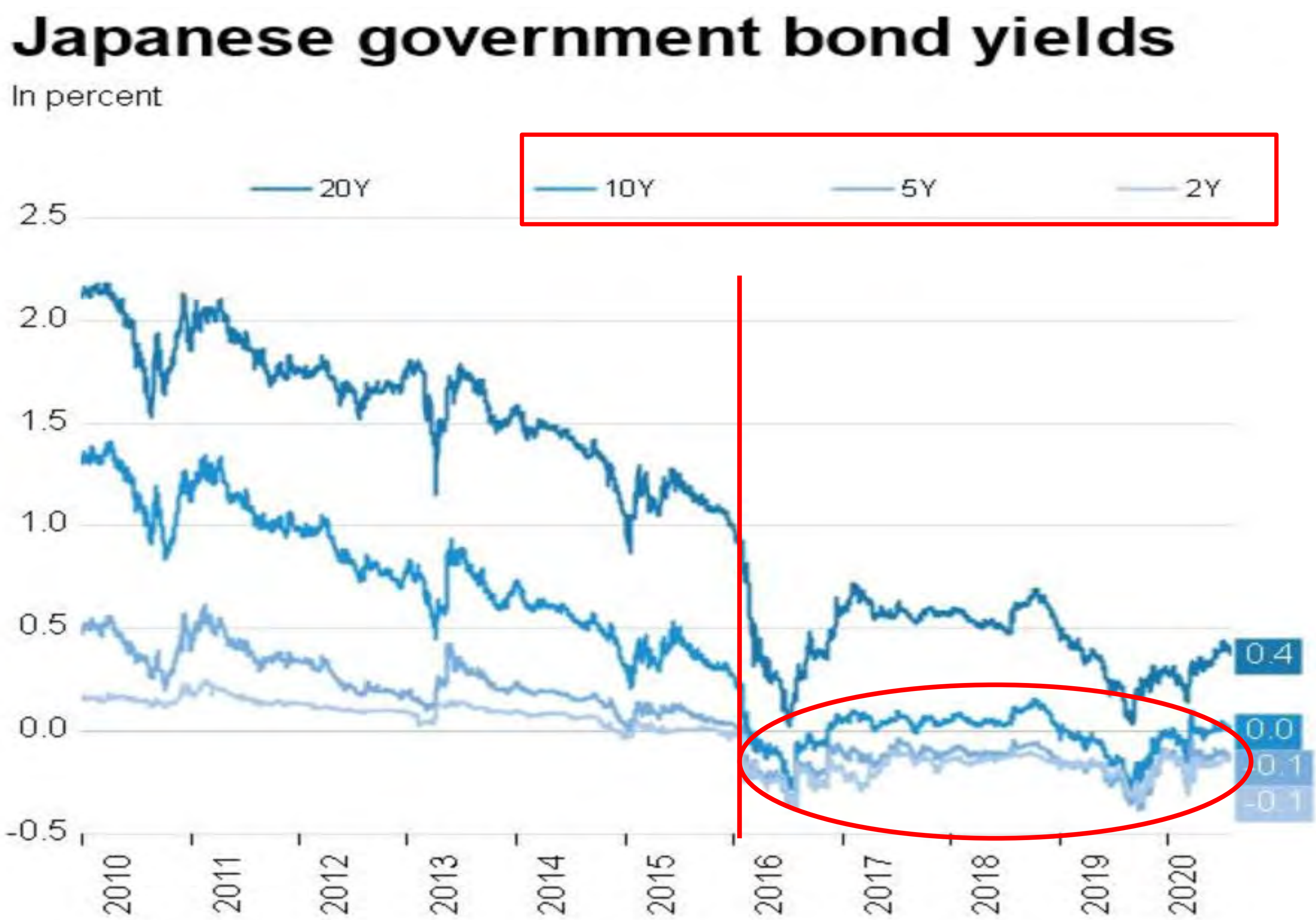
Xun Zhang*

Central University of Finance and Economics

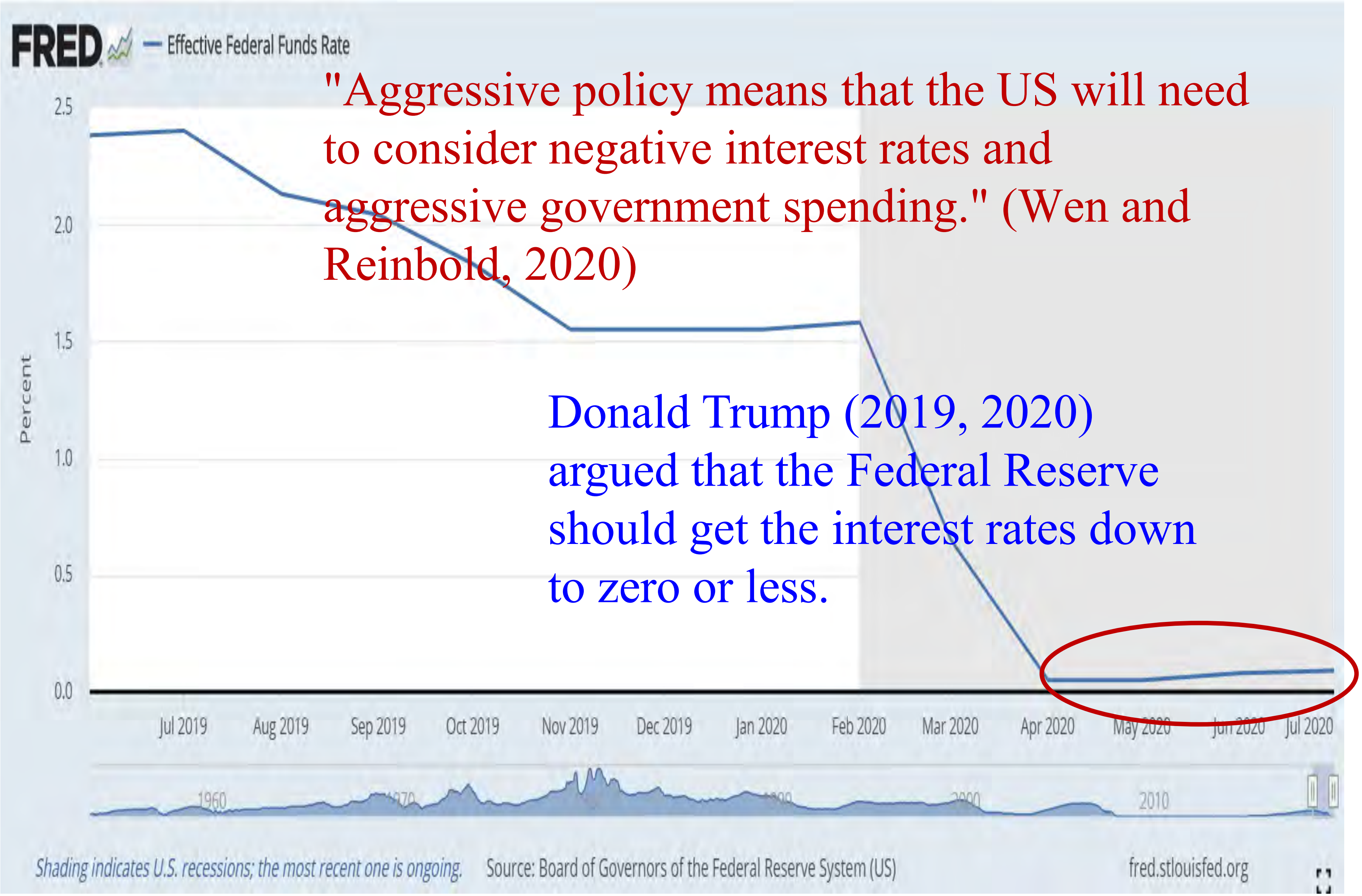
2020 Actuarial Research Virtual Conference

August 12, 2020

Negative Interest Rates



Trend of U.S. Interest Rates

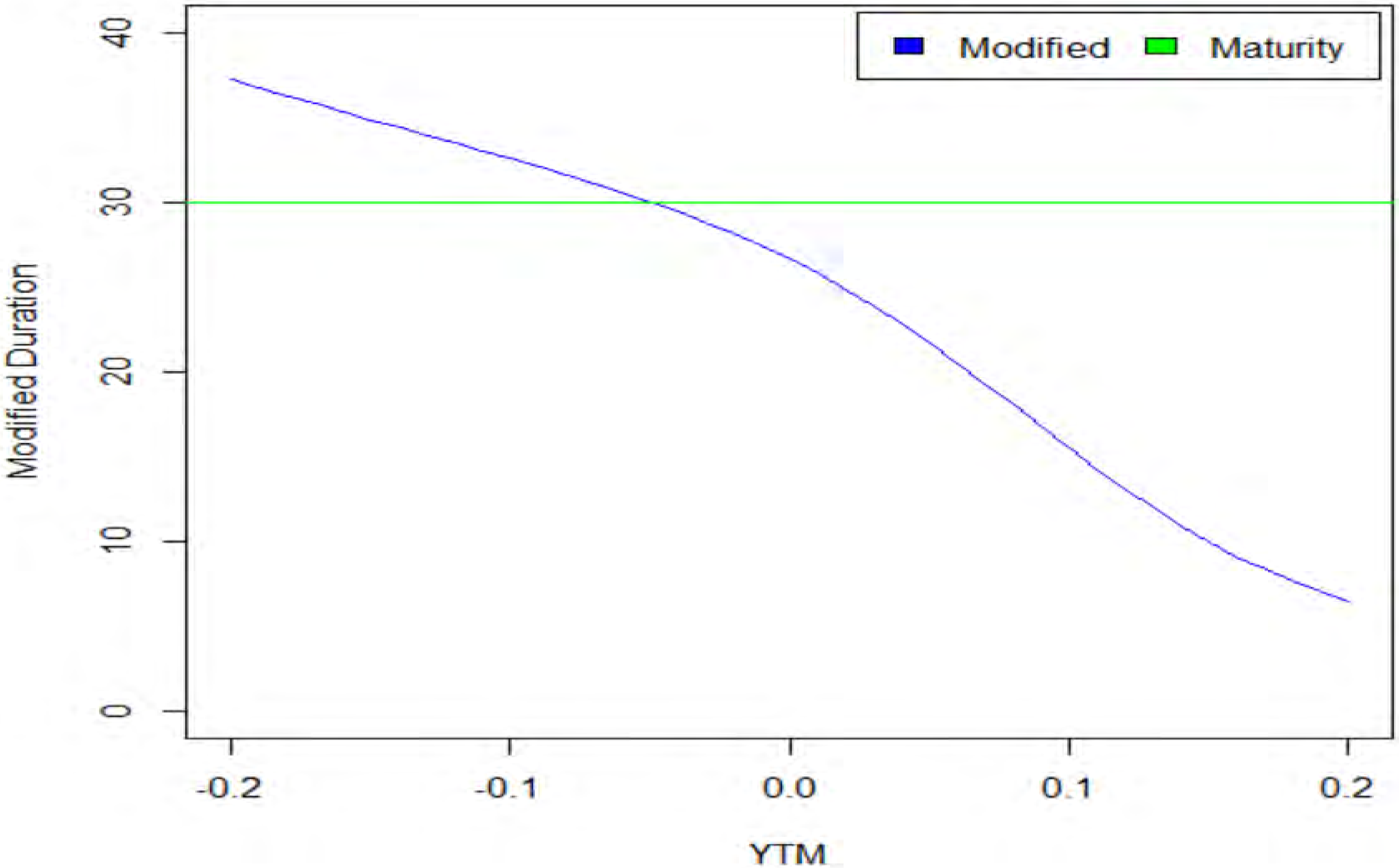


Motivation

- Research on the effect of a negative interest rate policy (NIRP) on the insurance industry is limited (Alberts, 2020).

- Whether some widely adopted duration measures (e.g. the Macauley and modified durations) can be used in the negative interest rate environment remains poorly understood.

Modified Duration (D^M)



$$D^M = \frac{1}{P(1+YTM)}\left[\sum_{t=1}^{T} \frac{t\,c}{(1+YTM)^t} + \frac{T}{(1+YTM)^T}\right] \quad (1)$$

Contributions

- We provide initial evidence that a decline in interest rates in the negative interest rate environment produces a much more serious consequence on life insurers than that in the positive interest rate environment.

- We add to the asset-liability management (ALM) literature by studying a life insurer's optimal decision with a duration constraint based on the Vasicek interest rate model (1977).

Presentation Outline

- Empirical analyses
- Duration based on Vasicek model
- Basic ALM framework for life insurers
- Numerical illustration
- Conclusion

Empirical Analyses

- Data: 38 Japanese life insurers and 2-,3-,5-,7- year Japanese government coupon bonds' yearly YTM from 1999 to 2018.
- OLS Regressions

$$ROA_{i,t} = \beta_0 + \beta_1 \; Interest\ rate\ variable_{n,t} + \Gamma' \; Control\ variable_{i,t} + Firm\ Effect_i + \epsilon_{i,t} \qquad (2)$$

Variable	(1)	(2)	(3)	(4)	(5)
$YTM_{n,t}$	√				
Indicator variable of $YTM_{n,t}$		√			
Positive $YTM_{n,t}$			√		√
Negative $YTM_{n,t}$				√	√
Premium Growth $Rate_{i,t}$	√	√	√	√	√
$\log(Total\ Asset)_{i,t}$	√	√	√	√	√
$Firm\ Effect_i$	√	√	√	√	√

OLS Model (1)

- Interest rate-return association analysis

Variable	(1)	(2)	(3)	(4)	(5)
$YTM_{n,t}$	√				
Indicator variable of $YTM_{n,t}$		√			
Positive $YTM_{n,t}$			√		√
Negative $YTM_{n,t}$				√	√
Premium Growth $Rate_{i,t}$	√	√	√	√	√
$\log(Total\ Asset)_{i,t}$	√	√	√	√	√
$Firm\ Effect_i$	√	√	√	√	√

OLS Model (1) Result

Table 3: The impact of interest rate on Japanese life insurers' performance

Variable	(1)		(2)		(3)		(4)		(5)	
Intercept	-0.2555 (-7.66)	***	-0.2438 (-7.96)	***	-0.2544 (-7.63)	***	-0.2437 (-7.97)	***	-0.2535 (-7.65)	***
5-year Japanese government bond YTM (%)	0.0077 (3.51)	***								
=1 if 5-year Japanese government bond YTM<0; 0 otherwise			-0.0052 (-3.24)	***						
Positive 5-year Japanese government bond YTM (%)					0.0077 (3.25)	***			0.0053 (1.75)	*
Negative 5-year Japanese government bond YTM (%)							0.0432 (3.03)	***	0.0313 (2.22)	**
Premium growth rate	0.0045 (0.82)		0.0029 (0.52)		0.0046 (0.86)		0.0029 (0.54)		0.0038 (0.69)	
Log(total assets)	0.0163 (7.94)	***	0.0158 (8.18)	***	0.0162 (7.92)	***	0.0158 (8.18)	***	0.0163 (7.95)	***
R^2	0.6989		0.6983		0.6976		0.6981		0.7007	
Firm-fixed Effect	Yes		Yes		Yes		Yes		Yes	
No. of Obs	282		282		282		282		282	

Note: t-statistics based on standard errors clustered at the year level are reported in parentheses.*, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels.

OLS Model (2)

- Interest rate-return association analysis

Variable	(1)	(2)	(3)	(4)	(5)
$YTM_{n,t}$	√				
Indicator variable of $YTM_{n,t}$		√			
Positive $YTM_{n,t}$			√		√
Negative $YTM_{n,t}$				√	√
Premium Growth $Rate_{i,t}$	√	√	√	√	√
$\log(Total\ Asset)_{i,t}$	√	√	√	√	√
$Firm\ Effect_i$	√	√	√	√	√

Model (2) Result

Table 3: The impact of interest rate on Japanese life insurers' performance

Variable	(1)		(2)		(3)		(4)		(5)	
Intercept	-0.2555 (-7.66)	***	-0.2438 (-7.96)	***	-0.2544 (-7.63)	***	-0.2437 (-7.97)	***	-0.2535 (-7.65)	***
5-year Japanese government bond YTM (%)	0.0077 (3.51)	***								
=1 if 5-year Japanese government bond YTM<0; 0 otherwise			-0.0052 (-3.24)	***						
Positive 5-year Japanese government bond YTM (%)					0.0077 (3.25)	***			0.0053 (1.75)	*
Negative 5-year Japanese government bond YTM (%)							0.0432 (3.03)	***	0.0313 (2.22)	**
Premium growth rate	0.0045 (0.82)		0.0029 (0.52)		0.0046 (0.86)		0.0029 (0.54)		0.0038 (0.69)	
Log(total assets)	0.0163 (7.94)	***	0.0158 (8.18)	***	0.0162 (7.92)	***	0.0158 (8.18)	***	0.0163 (7.95)	***
R^2	0.6989		0.6983		0.6976		0.6981		0.7007	
Firm-fixed Effect	Yes		Yes		Yes		Yes		Yes	
No. of Obs	282		282		282		282		282	

Note: t-statistics based on standard errors clustered at the year level are reported in parentheses.*, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels.

OLS Model (5)

- Interest rate-return association analysis

Variable	(1)	(2)	(3)	(4)	(5)
$YTM_{n,t}$	√				
Indicator variable of $YTM_{n,t}$		√			
Positive $YTM_{n,t}$			√		√
Negative $YTM_{n,t}$				√	√
Premium Growth $Rate_{i,t}$	√	√	√	√	√
$\log(Total\ Asset)_{i,t}$	√	√	√	√	√
$Firm\ Effect_i$	√	√	√	√	√

Model (5) Result

Table 3: The impact of interest rate on Japanese life insurers' performance

Variable	(1)	(2)	(3)	(4)	(5)
Intercept	-0.2555 *** (-7.66)	-0.2438 *** (-7.96)	-0.2544 *** (-7.63)	-0.2437 *** (-7.97)	-0.2535 *** (-7.65)
5-year Japanese government bond YTM (%)	0.0077 *** (3.51)				
=1 if 5-year Japanese government bond YTM<0; 0 otherwise		-0.0052 *** (-3.24)			
Positive 5-year Japanese government bond YTM (%) → $= \begin{cases} YTM, YTM > 0 \\ 0, otherwise \end{cases}$			0.0077 *** (3.25)		0.0053 * (1.75)
Negative 5-year Japanese government bond YTM (%) → $= \begin{cases} YTM, YTM < 0 \\ 0, otherwise \end{cases}$				0.0432 *** (3.03)	0.0313 ** (2.22)
Premium growth rate	0.0045 (0.82)	0.0029 (0.52)	0.0046 (0.86)	0.0029 (0.54)	0.0038 (0.69)
Log(total assets)	0.0163 *** (7.94)	0.0158 *** (8.18)	0.0162 *** (7.92)	0.0158 *** (8.18)	0.0163 *** (7.95)
R^2	0.6989	0.6983	0.6976	0.6981	0.7007
Firm-fixed Effect	Yes	Yes	Yes	Yes	Yes
No. of Obs	282	282	282	282	282

Note: t-statistics based on standard errors clustered at the year level are reported in parentheses.*, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels.

Implications

- A decrease in interest rates in the negative interest rate environment imposes a **much more serious consequence** on life insurers than that in the positive interest rate environment.

- We need to investigate a new ALM model for life insurers in the negative interest rate environment.

Duration Based on Vasicek model (D^V)

$$dr_t = a(b - r_t) + \sigma_r dW_r(t) \qquad (3)$$

$$P(0,t) = A(0,t)\exp(-B(0,t)r_0) \qquad (4)$$

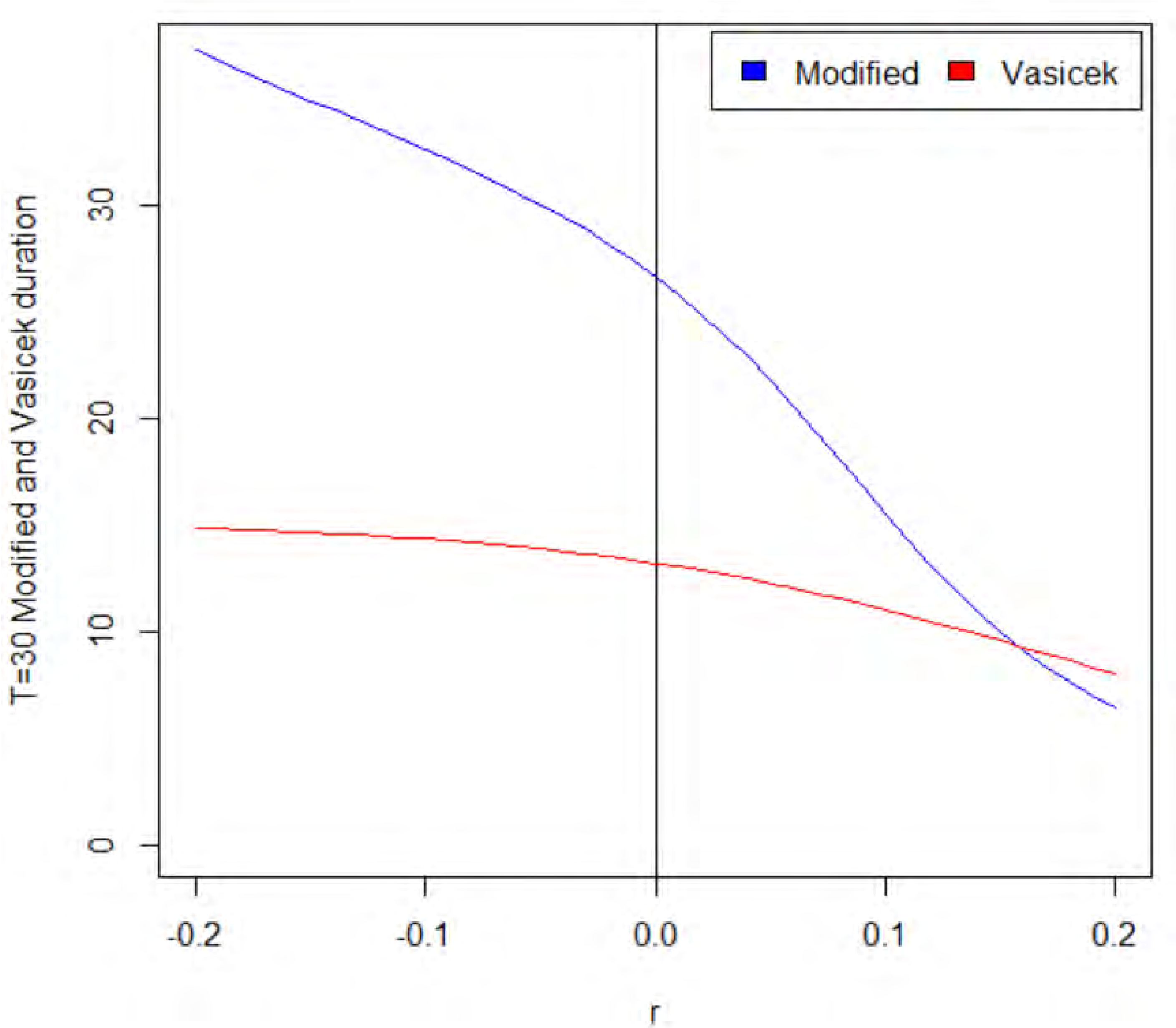
$$P_c(0,T) = c\sum_{t=1}^{T} P(0,t) + P(0,T) \qquad (5)$$

$$D^V = -\frac{dP_c(0,T)}{dr_0}\frac{1}{P_c(0,T)}$$

$$= \frac{c\sum_{t=1}^{T} B(0,t)P(0,t) + B(0,T)P(0,T)}{P_c(0,T)} \qquad (6)$$

Modified Duration vs. Duration Based on Vasicek Model

c	0.0100
a	0.0506
b	0.0692
σ_r	0.0100
T	30



Basic ALM Framework for Life Insurer

- Mortality model
  - Lee and Carter (1992)'s model:

$$\ln q_{x,t} = \alpha_x + b_x k_t + \varepsilon_{x,t} \tag{7}$$

- Annuity contracts
  - Consider a life insurer only has the annuity business.
  - Total liability at time t :

$$L(t) = N_0(x_0 + t) \cdot E \cdot a_{x_0+t} \qquad t = 0,1,2,\ldots \tag{8}$$

Total Assets

- 30-year Japanese government bond

$$dr_t = a(b - r_t) + \sigma_r dW_r(t)$$

- Asia-Pacific corporate bond index

$$dB(t) = B\,(t)[\mu dt + \sigma_B dW_B(t)] \qquad (9)$$

- Brownian motions of these two assets are correlated

$$dW_B(t)dW_r(t) = \rho dt \qquad (10)$$

- Total assets at time t:

$$\mathrm{A}(t) = \begin{cases} C_0 + N_0(x_0) \cdot E \cdot a_{x_0} \cdot (1 + l_p) & t = 0 \\ A\,(t^-) - E \cdot N_0(x_0 + t) & t = 1,2, \ldots \end{cases} \qquad (11)$$

Total Surplus

- The insurer's total surplus at time t:

$$S(t)=\begin{cases} C_0+N_0(x_0)\cdot E\cdot a_{x_0}\cdot(1+l_p)-N_0(x_0)\cdot E\cdot a_{x_0} & t=0\\ A\,(t^-)-E\cdot N_0(x_0+t)-N_0(x_0+t)\cdot E\cdot a_{x_0+t} & t=1,2,\ldots \end{cases} \quad (12)$$

Basic Optimization Problem

$$\underset{\omega_1,\omega_2,\, N_0(x_0)}{Maximize} \quad S(t^*) \qquad (13)$$

Subject to

$$VaR_\alpha[S(t)] \geq \text{R, t=1,2,}\ldots, t^*$$
$$\omega_1 + \omega_2 = 1$$
$$0 \leq \omega_i \leq 1, i = 1{,}2$$
$$N_0(x_0) > 0$$
$$Duration\ Constraint$$

- ω_1: weight in 30-year Japanese government bond
- ω_2: weight in corporate bond index.

Duration Constraint

- Duration strategy 1

  - $D_0^S = 0$ (14)

  where

$$D_t^S = \begin{cases} \omega_1 D_t^{M_{30}} + \omega_2 D^{M_{BI}} - D_t^{M_L}, for\ Modified\ duration \\ \omega_1 D_t^{V_{30}} + \omega_2 D^{V_{BI}} - D_t^{V_L}, for\ Vasicek\ duration \end{cases} \quad (15)$$

- Duration strategy 2

  - $\gamma_1 \leq D_t^S \leq \gamma_2,\ t=0,1,\ldots,\quad t^*$ (16)

  where γ_1 and γ_2 are constants that control for the size of duration mismatching in subsequent periods.

Numerical Illustration

C_0	1000
l_p	0.18
E	1
$VaR_\alpha[S(t)] \geq \mathrm{R}$	$VaR_{0.01}[S(t)] \geq 0$
c	0.8%
r_0	-0.2%
t^*	10
Average maturity of corporate bond index	4
Average annual coupon rate of corporate bond index	1.36%

Parameter Estimation of Lee-Carter Model

- Data: Japanese male population mortality tables (1950 - 2017) from the Human Mortality Database.

- Estimates

g	-0.5375
σ_k^2	0.2949

Parameter Estimation of Vasicek Model

- Data: monthly yield data of Japanese government zero coupon bonds with 1-month,1-, 5-, 7-, 10-, 15-, 20-, and 30-year maturities from April 2010 to April 2020.

- Estimates

a	0.0970^{***}
b	0.0120^{***}
σ_r	0.0028^{***}

Parameter Estimation of Geometric Brownian Motion

- Data: monthly total return index of Bloomberg Barclays Asia-Pacific 3-5 year corporate bond index from April 2010 to April 2020.

- Estimates

μ	**0.0527**
σ_k	0.1119

- The correlation of the Brownian motions between the Japanese government zero coupon bond index and Bloomberg Barclays Asia-Pacific 3-5 year corporate bond index

ρ	0.1784

Difference in the Size of Duration Mismatching

Table 8: Vasicek Optimal Solution

Vasicek	$N_0(x_0)$	ω_1	ω_2
strategy 1	198.9856	0.4837	0.5163
strategy 2	181.4330	0.4201	0.5799

Duration Mismatching

[-0.375,0.375]

Table 9: Modified Optimal Solution

Modified	$N_0(x_0)$	ω_1	ω_2
strategy 1	159.6332	0.3209	0.6791
strategy 2	155.0015	0.2972	0.7028

Duration Mismatching

[-1.100,-0.260]

Difference in Portfolio Weights

Table 8: Vasicek Optimal Solution

Vasicek	$N_0(x_0)$	ω_1	ω_2
strategy 1	198.9856	0.4837	0.5163
strategy 2	181.4330	0.4201	0.5799

Table 9: Modified Optimal Solution

Modified	$N_0(x_0)$	ω_1	ω_2
strategy 1	159.6332	0.3209	0.6791
strategy 2	155.0015	0.2972	0.7028

Weight in corporate bond index

Weight in 30-year Japanese government bond

ω_1 and ω_2 denote the weights in 30-year Japanese government bond and corporate bond index, respectively.

Difference in Annuity Units

Table 8: Vasicek Optimal Solution

Vasicek	$N_0(x_0)$	ω_1	ω_2
strategy 1	198.9856	0.4837	0.5163
strategy 2	181.4330	0.4201	0.5799

Annuity units

Table 9: Modified Optimal Solution

Modified	$N_0(x_0)$	ω_1	ω_2
strategy 1	159.6332	0.3209	0.6791
strategy 2	155.0015	0.2972	0.7028

$N_0(x_0)$ denotes the annuity units.

Conclusions

- Our empirical analysis shows that a decrease in interest rates in the negative interest rate environment produces a much more serious consequence on life insurers than that in the positive interest rate environment.
- We propose an optimization framework to derive the optimal decision of a life insurer with a duration constraint based on the Vasicek interest rate model.
- The life insurer will assume more risk and will suffer a higher downside risk and greater duration mismatching if they use a modified duration constraint to implement their ALM.

Thank You !