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The Lowly Loss Ratio

by Paul Margus

"There are more things in heaven and earth, Loss Ratio, than are dreamt of in your philosophy."

The loss ratio has been around for a long time. A properly formulated loss ratio tells us what portion of premium income is set aside for claims, over a fairly short interval, over many years, or over the lifetime of an entire block. Yet despite its many modifications to accommodate diverse lines of business, it's still a very blunt instrument.

Loss ratios began as a casualty insurance concept. In auto and homeowners' insurance, renewal periods are short. The loss ratio is just the aggregate claims paid, divided by the aggregate premiums collected. This works because the premiums and claims are confined to a short period, usually one year or less. The timing of the premiums collected roughly matches the timing of the cash claim payouts.

Subsequently, the loss ratio was embraced by the group and individual health insurance business. In those lines, claims can extend much longer, necessitating long-tail claim reserves and the "incurred claims" concept.

With Long Term Care and Individual Disability Income things get more complicated. These lines use issue age and level premiums, an idea borrowed from life insurance. Active life reserves and investment income complicate the picture, requiring further refinements, and recognition of the time value of money. Here, our definition is more properly the *present value* of (expected) future claims, divided by present value of future premiums. Or equivalently, it's the *accumulated value* of (actual) past claims, divided by the accumulated value of past premiums. Despite all those elaborations, loss ratio calculations are based solely on aggregate data and are easy to calculate.

As we will see, these last enhancements have, by necessity, made the loss ratio sensitive to policy persistency. Thus, it's no longer a pure measurement of claims.

The loss ratio is often used for regulatory purposes. For new rate filings in some lines, companies must demonstrate that the loss ratio, calculated under reasonable assumptions, is expected to meet a legal minimum. In addition, they must monitor emerging experience on their existing business in force. If claims are lower than expected, they may have to decrease premiums or increase policyholder dividends. But this article will concentrate on using the loss ratio as an internal management tool. The methods analyzed here may not precisely match the legal definitions.

What makes a good tool for financial analysis?

- Ease of use—The traditional loss ratio is easy to calculate, because it's based solely on aggregate data, namely premiums, claims and reserves. If possible, any refinement should preserve this advantage. But, as we'll see below, we must sacrifice some simplicity to understand loss ratio dynamics and tie it into other financial measures.
- Drill-down capability—If we subdivide our data, it should be possible to get loss ratios for various underwriting and occupation classes, geographical regions and markets. This can help us monitor the experience of important subgroups, and estimate our pricing adequacy.
- Consistency—To be useful, a measurement must be consistent. Having adopted a benchmark, we should be able to judge how we're doing in relation to it. In other words, if our experience is exactly as originally assumed, the loss ratio should remain constant throughout the life of the business.

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… loss ratio calculations are based solely on aggregate data and are easy to calculate.

The remainder of this article will explore alternative definitions and their mathematical underpinnings. We'll discuss the modifications for level premium lines of business, chiefly Long Term Care and Individual Disability Income, harping again and again on the importance of interest adjustments. The loss ratio concept will be extended to expenses and profit. Limited pay plans and Life Insurance will be briefly explored. Finally, we'll examine the strengths and limitations of the loss ratio, and how they can be remedied. This will entail linking the loss ratio to gain and loss analysis.

In the next two sections, we deal separately with numerator and denominator.

The Numerator: Claims

For very short-duration claims, it may be sufficient simply to use cash payments. But if claim payouts extend beyond the expected period of the loss ratio, we will have to include the claim reserve. We have two methods of addressing this.

1. The simpler method is to include the *initial claim reserve at the moment of inception,* and ignore all subsequent activity on that claim. For our loss ratio, the entire claim obligation is discharged in one lump sum. Thus, the loss ratio responds to actual claim incidence, but not to deviations from our assumed claim termination.

2. Another method is to count *cash payouts, plus the increase in claim reserve.*

Initially, Method 2 is equivalent to the Method 1. At claim inception, the claim reserve *instantaneously* jumps from zero to its initial value. And in this infinitesimal span, we haven't had enough time to make any payment. But subsequently, Method 2 makes mid-course corrections as the actual claim terminations deviate from expected. To see this, consider the familiar recursive formula for the annuity function.

Equation 1

$$\left(1-h_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t} = \left(1+i_t\right) \times \left(\ddot{a}^{(i)}_{[x]+t-1}-1\right)$$

Here, $\ddot{a}^{(i)}_{[x]+t}$ is the claim reserve, $h_{[x]+t-1}$ is the claim termination rate expected in the claim reserve calculation, i_t is the reserve interest rate, and the periodic claim payout is $1.

Actual claim termination always differs from expected. Let the actual termination rate during claim year t be $\hat{h}_{[x]+t-1}$. Then, subtracting $\left(\hat{h}_{[x]+t-1}-h_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t}$ from both sides of Equation 1, we get

Equation 2

$$\left(1-\hat{h}_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t} =$$

$$\left(1+i_t\right) \times \left(\ddot{a}^{(i)}_{[x]+t-1}-1\right) - \left(\hat{h}_{[x]+t-1}-h_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t}$$

At the beginning of the period, the aggregate inforce is $1 and the aggregate claim reserve is $(1) \times \left(\ddot{a}^{(i)}_{[x]+t-1}\right)$. At the end of the period, we're left with $\left(1-\hat{h}_{[x]+t-1}\right)$ of aggregate claims in force, bearing an aggregate reserve of $\left(1-\hat{h}_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t}$.

$\left(\hat{h}_{[x]+t-1}-h_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t}$ is the amount of "actuarial gain from claim termination." If aggregate claims in the period are exactly as expected, then the actuarial gain is zero. If terminations are bigger than expected, the gain is positive. If they're less than expected, the gain is negative, and we have a loss from claim termination.

Thus, on any closed block of claims, we can subtract actual claim payouts from the last period's aggregate claim reserve. Then we adjust for interest, taking into account the actual timings. If this overstates the current aggregate claim reserve, then the amount of the overstatement represents the actuarial gains for the period.

So, in many practical applications, $\left(\hat{h}_{[x]+t-1}-h_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t}$ is just the balancing item. But as I will explain later, it may be useful to invest additional effort to calculate it explicitly.

For other than annual payouts with one-year loss ratios, the above math is more complex; but the result is the same, as long as we let our interest adjustments reflect the actual timing of the payments.

We now return to our definition of "incurred claims": cash payouts, plus the increase in claim reserve.

Equation 3

$$\begin{pmatrix}\textit{Incurred}\\ \textit{Claims}\end{pmatrix} = \begin{pmatrix}\textit{Cash}\\ \textit{Payouts}\end{pmatrix} + \begin{pmatrix}\textit{Increase in}\\ \textit{Claim Reserve}\end{pmatrix}$$

$$= \quad \{(1)\} \quad + \quad \left\{ \frac{\left(1 - \hat{h}_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t}}{1 + i_t} \quad - \quad \left(\ddot{a}^{(i)}_{[x]+t-1}\right) \right\}$$

To make things work out neatly in Equation 4, I have applied an interest adjustment to the reserve increase in Equation 3. In the real world, the claim payout isn't concentrated at the beginning, so it may need some sort of discounting, too. In practice, that's all there is to calculating the aggregate incurred claims. But let's see what we're actually calculating.

Equation 4

$$\begin{pmatrix} Incurred \\ Claims \end{pmatrix} = \{(1)\} + \left\{ \frac{\left(1 - \hat{h}_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t}}{1 + i_t} - \left(\ddot{a}^{(i)}_{[x]+t-1}\right) \right\}$$

$$= \frac{\left(1 - \hat{h}_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t}}{1 + i_t} - \left(\ddot{a}^{(i)}_{[x]+t-1} - 1\right)$$

$$= \frac{\left(1 - \hat{h}_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t} - (1 + i_t) \times \left(\ddot{a}^{(i)}_{[x]+t-1} - 1\right)}{1 + i_t}$$

$$= - \frac{\left(\hat{h}_{[x]+t-1} - h_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x]+t}}{1 + i_t}$$

$$= - \begin{pmatrix} Discounted \\ Actuarial \\ Gains \end{pmatrix}$$

As mentioned above, at the moment of claim inception, Method 2 is identical to Method 1. Thereafter, Method 2 records deviations from expected terminations as actuarial gains. These gains (and losses) serve as mid-course corrections to the initial claim reserve, which occur only as the experience unfolds.

Method 1 is simpler, and it confines the claim experience to the period of the loss ratio, while neglecting the mid-course corrections. Method 2 scrupulously adjusts for under- or over-reserving over time. But it blends prior claims into the calculation of the current loss ratio. Thus, each method has its advantages and disadvantages.

In the above derivations, we have assumed that our claim reserve is a quasi life annuity calculation. But these principles are equally valid for claim triangles. In any event, if claim durations are potentially long, we need an interest element in the reserve and incurred claim calculations. (Loss reserves should be discounted.)

As a practical matter, under Method 2, the incurred claims are calculated using the fundamental definition: cash payouts, plus the (interest-adjusted) increase in claim reserve. The sole purpose of our derivations was to show that incurred claims are exactly:

- the claim reserve at the moment of claim inception, and
- the negative of actuarial gains for any subsequent period.

The Denominator: Premiums

Premiums should be recognized only when due. "Incurred Premiums" represent what we'll collect over the period, if everyone pays exactly on time. This is just the Cash Collections over the period, plus the increase in "Premiums Due and Unpaid," minus the increase in "Premiums Paid in Advance."

Equation 5

$$\begin{Bmatrix} Incurred \\ Premiums \end{Bmatrix} = \begin{Bmatrix} Cash \\ Premiums \end{Bmatrix} + \left(\begin{Bmatrix} Due\ \& \\ Unpaid \end{Bmatrix}_1 - \begin{Bmatrix} Due\ \& \\ Unpaid \end{Bmatrix}_0 \right) - \left(\begin{Bmatrix} Ad- \\ vance \end{Bmatrix}_1 - \begin{Bmatrix} Ad- \\ vance \end{Bmatrix}_0 \right)$$

A further refinement is to use the "Earned Premium," which represents what we would collect if premiums were paid continuously, and always exactly on time. Thus, over a four-month period, we show $\frac{4}{12}$ of an annual premium, regardless of when the policy anniversary occurs. (Otherwise, for a block of policies paying annually in February, we could be dividing by zero if we tried to do a loss ratio for just the summer months. And a first quarter loss ratio would be understated because it would reflect a whole year's premium.) The "earned premium" is the "incurred premium," minus the increase in unearned premiums.

Equation 6

$$\begin{Bmatrix} Earned \\ Premiums \end{Bmatrix} = \begin{Bmatrix} Incurred \\ Premiums \end{Bmatrix} - \left(\begin{Bmatrix} Un- \\ earned \end{Bmatrix}_1 - \begin{Bmatrix} Un- \\ earned \end{Bmatrix}_0 \right)$$

For loss ratios taken over an extended period, we must adjust for interest. This means taking the present or accumulated value of premiums. In addition, it seems appropriate to interest-adjust all of the "Due and Unpaid," "Advance" and "Unearned" accruals. (See the Active Life Reserves section)

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Active Life Reserves

The above formulation is consistent with a mid-terminal Active Life Reserve. If t represents the time (in years) since the last anniversary (between 0 and 1), then it's good enough to interpolate the Active Life Reserve linearly between anniversaries:

Equation 7

$$ALR_t = (1-t) \times ALR_0 + (t) \times ALR_1$$

This differs somewhat from custom as follows:

- The unearned premium is *omitted* from the reserve;
- In Equation 6 above, we *subtracted* it from the premiums.

If our reported premiums and reserves follow a different convention, we should adjust them for loss ratio calculations. (In the case of Life Insurance, premiums should exclude any increase in deferred premium. For the casualty and group lines, we end up with no active life reserve at all.)

To get meaningful loss ratios, we'll want our reserves to be as realistic as possible. Usually, GAAP benefit reserves are the best candidate. To the extent possible, the margins for adverse deviation should be removed, perhaps using a simple multiple.

Long Term Care and Individual Disability Insurance specify level premiums, payable for the term of the coverage or for a limited period. Because the premium is level and claim costs are increasing, the premiums and claim costs are mismatched. Without some adjustment for active life reserves, the loss ratios will be meaningless. They will start out unrealistically low, but would ultimately attain astronomical levels. To remedy this, we recognize the increase in active life reserves as part of the claim cost for the current period. Two definitions are popular.

Equation 8

$$\begin{pmatrix} Loss \\ Ratio \end{pmatrix} = \frac{Claims + [ALR_1 - ALR_0]}{Earned \quad Premiums}$$

Equation 9

$$\begin{pmatrix} Loss \\ Ratio \end{pmatrix} = \frac{Claims + \left[\frac{1}{1+i} \times ALR_1 - ALR_0\right]}{Earned \quad Premiums}$$

As always, the 0 subscript refers to the beginning of the period, while 1 means the end.

Equation 9 is the better choice. As we will show in section entitled Doing the Math, the loss ratio works out to be the valuation net premium for the benefit reserve, divided by the gross premium, minus actuarial gains (as a percent of premium). Thus, it meets the "consistency" criterion discussed in the introduction of this article.

If you accept the previous assertion for now, then Equation 8 fails the "consistency" criterion. Even in the absence of actuarial gains, the loss ratio won't be level. In the early policy years, when the active life reserve is small, it will be almost right. It will increase artificially as the missing interest adjustment becomes significant. It will peak at some point, and then decrease back to normal at the end of time. Back in the real world, when the loss ratio increases, we won't know whether to blame bad claims or chalk it up to the natural behavior of an aging block.

Over an extended period, Equation 9 is better written as:

Equation 10

$$\begin{pmatrix} Loss \\ Ratio \end{pmatrix} = \frac{PV\begin{pmatrix} Incurred \\ Claims \end{pmatrix} + PV\begin{pmatrix} Int - Adj \\ Incr \quad in \\ ALR \end{pmatrix}}{PV\begin{pmatrix} Earned \\ Premiums \end{pmatrix}}$$

where *PV(whatever)* is the n-year present value at the mth policy year, or

$$\sum_{t=0}^{n-1} v^t \, {}_t\hat{p}_{[x]+m} \times whatever_{[x]+m+t}$$

Now, we define a few symbols.

x	=	Issue Age.
t	=	Policy Year.
${}_tV_x$	=	Active life reserve per unit in force at the end of policy year *t*.
${}_0V_x$	=	Active life reserve on the policy issue date, which is zero.
${}_nV_x$	=	Active life reserve at the end of coverage. P_x is chosen so that this comes out to zero.
i_t	=	Valuation interest rate for policy year t.

$\hat{w}_{[x]+t-1}$	=	Actual Lapse rate for policy year t.
$\hat{q}_{[x]+t-1}$	=	Actual Mortality rate for policy year t.
$1	=	Actual Amount in force at *beginning* of policy year t.
$1-\hat{w}_{[x]+t-1}-\hat{q}_{[x]+t-1}$	=	Actual Amount in force at the *end* of policy year t.
$1.0000 \times {}_{t-1}V_x$	=	Aggregate Reserve at *beginning* of policy year t.
$\left(1-\hat{w}_{[x]+t-1}-\hat{q}_{[x]+t-1}\right) \times {}_tV_x$	=	Actual Aggregate Reserve at *end* of policy year t.

Therefore,

$$PV\begin{pmatrix} Int-Adj \\ Incr \quad in \\ ALR \end{pmatrix} = \sum_{t=0}^{n-1} v^t \, {}_t\hat{p}_{[x]+m} \times \left\{ \frac{1}{1+i} \times ALR_1 - ALR_0 \right\}$$

$$= \sum_{t=0}^{n-1} v^t \, {}_t\hat{p}_{[x]+m} \times \left\{ \frac{\left(1-\hat{w}_{[x]+m+t}-\hat{q}_{[x]+m+t}\right) \times {}_{m+t+1}V_x}{1+i} - (1) \times {}_{m+t}V_x \right\}$$

$$= \sum_{t=0}^{n-1} v^{t+1} \, {}_{t+1}\hat{p}_{[x]+m} \times {}_{m+t+1}V_x - \sum_{t=0}^{n-1} v^t \, {}_t\hat{p}_{[x]+m} \times {}_{m+t}V_x$$

$$= \sum_{t=1}^{n} v^t \, {}_t\hat{p}_{[x]+m} \times {}_{m+t}V_x - \sum_{t=0}^{n-1} v^t \, {}_t\hat{p}_{[x]+m} \times {}_{m+t}V_x$$

$$= v^n \, {}_n\hat{p}_{[x]+m} \times {}_{m+n}V_x - v^0 \, {}_0\hat{p}_{[x]+m} \times {}_{m+0}V_x$$

$$= \frac{{}_n\hat{p}_{[x]+m} \times {}_{m+n}V_x}{(1+i)^n} - {}_mV_x$$

At policy year m, we have $1 in force with an aggregate reserve of ${}_{m+n}V_x$. Then n years later, ${}_n\hat{p}_{[x]+m}$ remains in force, and the aggregate reserve is ${}_n\hat{p}_{[x]+m} \times {}_{m+n}V_x$.

Substituting into Equation 10, we get

Equation 11

$$\begin{pmatrix} Loss \\ Ratio \end{pmatrix} = \frac{PV\begin{pmatrix} Incurred \\ Claims \end{pmatrix} + \left(\frac{{}_n\hat{p}_{[x]+m} \times {}_{m+n}V_x}{(1+i)^n} - {}_mV_x \right)}{PV\begin{pmatrix} Earned \\ Premiums \end{pmatrix}},$$

which looks a lot like Equation 9. From this, we draw some conclusions.

- Equation 9 applies over any period of time that we choose, as long as we properly adjust for interest.
- The foregoing derivation does not in any way use the reserve valuation assumptions. But the section Doing the Math does.
- Reserves matter only at the endpoints. Intermediate reserves have no effect on the loss ratio.
  - o Within the $PV\begin{pmatrix} Incurred \\ Claims \end{pmatrix}$ term, the claim reserve increases (Method 2) telescope in the same way.
  - o For m = 0 (new business) and $n=\infty$ (the end of time), the reserve increase becomes Zero minus Zero. Similarly, the accruals in Equation 5 and Equation 6 go to zero, so the lifetime loss ratio is

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Equation 12

$$\begin{pmatrix} Loss \\ Ratio \end{pmatrix} = \frac{PV\begin{pmatrix} Cash \\ Claims \end{pmatrix}}{PV\begin{pmatrix} Cash \\ Premiums \end{pmatrix}},$$

which is similar to the loss ratio that we file for a new policy form.

- o As mentioned previously, the premium accruals need interest adjustments of the form

$$\frac{\{Due \,\&\, Unpaid\}_{m+n}}{(1+i)^n} - \{Due \,\&\, Unpaid\}_m$$

$$\frac{\{Advance\}_{m+n}}{(1+i)^n} - \{Advance\}_m$$

$$\frac{\{Unearned\}_{m+n}}{(1+i)^n} - \{Unearned\}_m$$

- For analyzing past results, we calculate retrospective accumulated values rather than present values. Instead of inserting persistency and mortality assumptions into the Equation 11 summations, we simply accumulate aggregate the historical premiums and claims with interest. At the endpoints, we use actual reserves with the interest adjustment.

Doing the Math

The active life reserve funds benefits over the term of the coverage. During policy year t, it changes as follows:

Equation 13

$$\left(1 - w_{[x]+t-1} - q_{[x]+t-1}\right) \times {}_tV_x$$

$$= \left(1+i_t\right) \times \left({}_{t-1}V_x + P_x\right) - r_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]}$$

where

P_x = Net level premium.

$r_{[x]+t-1}$ = Valuation Claim incidence rate for policy year t.

$\ddot{a}^{(i)}_{[x+t]}$ = The present value of benefits at inception, under a claim starting in policy year t. For individual disability income, it's the familiar claim annuity.

$r_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]}$ = The net annual claim cost for claims starting in policy year t.

$w_{[x]+t-1}$ = Valuation Lapse rate for policy year t.

$q_{[x]+t-1}$ = Valuation Mortality rate for policy year t.

Here, the net level premium is set at a level that funds the benefits over the term of the policy, assuming that claim costs, interest, lapse and mortality occur exactly as assumed. Thus, the Active life reserve starts and ends at zero. At intermediate times, if $r_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]}$ generally increases with t, the reserve is greater than zero.

Actual Experience never follows our script.

$\hat{r}_{[x]+t-1}$ = Actual Claim incidence rate for policy year t.

$\hat{r}_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]}$ = The actual net annual claim cost for claims starting in policy year t. To keep it simple, I'm using the Method 1 definition of "incurred claims."

If 1.0000 = Amount in force at the beginning of policy year t, then;

$\left(1 - w_{[x]+t-1} - q_{[x]+t-1}\right) \times {}_tV_x$ = Expected Aggregate Reserve at end of policy year t.

$1 - w_{[x]+t-1} - q_{[x]+t-1}$ = Expected Amount in force at the end of policy year t.

Then, we can transform Equation 13 as follows:

$$\left(1 - w_{[x]+t-1} - q_{[x]+t-1}\right) \times {}_tV_x =$$
$$\left(1+i_t\right) \times \left({}_{t-1}V_x + P_x\right) - r_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]}$$

Add $\left(w_{[x]+t-1} + q_{[x]+t-1}\right) \times {}_tV_x$ (expected decrements) to both sides …

$${}_tV = \left(1+i_t\right) \times \left({}_{t-1}V_x + P_x\right) - r_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]}$$
$$+ w_{[x]+t-1} \times {}_tV_x + q_{[x]+t-1} \times {}_tV_x$$

Subtract $\left(\hat{w}_{[x]+t-1} + \hat{q}_{[x]+t-1}\right) \times {}_tV_x$ (actual decrements) from both sides …

$$\left(1 - \hat{w}_{[x]+t-1} - \hat{q}_{[x]+t-1}\right) \times {}_tV_x = \left(1+i_t\right) \times \left({}_{t-1}V_x + P_x\right) - r_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]}$$
$$+ w_{[x]+t-1} \times {}_tV_x + q_{[x]+t-1} \times {}_tV_x$$
$$- \hat{w}_{[x]+t-1} \times {}_tV_x - \hat{q}_{[x]+t-1} \times {}_tV_x$$

Collect terms …

$$= (1+i_t) \times ({}_{t-1}V_x + P_x) - r_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]} - (\hat{w}_{[x]+t-1} - w_{[x]+t-1}) \times {}_tV_x - (\hat{q}_{[x]+t-1} - q_{[x]+t-1}) \times {}_tV_x$$

On the right hand side, add and subtract $\hat{r}_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]}$ (actual claims) …

$$= (1+i_t) \times ({}_{t-1}V_x + P_x) - \hat{r}_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]} - \left\{ \begin{array}{l} (r_{[x]+t-1} - \hat{r}_{[x]+t-1}) \times \ddot{a}^{(i)}_{[x+t]} \\ + (\hat{w}_{[x]+t-1} - w_{[x]+t-1}) \times {}_tV_x \\ + (\hat{q}_{[x]+t-1} - q_{[x]+t-1}) \times {}_tV_x \end{array} \right\}$$

Finally, we can write

Equation 14

$$(1 - \hat{w}_{[x]+t-1} - \hat{q}_{[x]+t-1}) \times {}_tV_x - (1+i_t) \times {}_{t-1}V_x = (1+i_t) \times P_x - \hat{r}_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]} - \left\{ \begin{array}{l} (r_{[x]+t-1} - \hat{r}_{[x]+t-1}) \times \ddot{a}^{(i)}_{[x+t]} \\ + (\hat{w}_{[x]+t-1} - w_{[x]+t-1}) \times {}_tV_x \\ + (\hat{q}_{[x]+t-1} - q_{[x]+t-1}) \times {}_tV_x \end{array} \right\}$$

Equation 14 says

$$\left\{ \begin{array}{c} Interest\text{–}Adjusted \\ Reserve \\ Increase \end{array} \right\} = \left\{ \begin{array}{c} Interest\text{–}Adjusted \\ Earned \\ Net\ \ Premiums \end{array} \right\} - \left\{ \begin{array}{c} Actual \\ Claims \end{array} \right\} - \left\{ \begin{array}{c} Actuarial \\ Gains\ \ during \\ the\ \ period \end{array} \right\}$$

The actuarial gains represent deviations from expected claims, lapses and mortality. They can be either positive (favorable) or negative (unfavorable).

This motivates a definition for Interest-Adjusted Loss Ratio:

Equation 15

$$LR = \frac{\left\{ \begin{array}{c} Actual \\ Claims \end{array} \right\} + \left\{ \begin{array}{c} Interest\text{–}Adjusted \\ Reserve\ \ Increase \end{array} \right\}}{\left\{ \begin{array}{c} Interest\text{–}Adjusted \\ Earned\ \ Gross\ \ Premiums \end{array} \right\}}$$

$$= \frac{\left\{ \hat{r}_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]} \right\} + \left\{ (1 - \hat{w}_{[x]+t-1} - \hat{q}_{[x]+t-1}) \times {}_tV_x - (1+i_t) \times {}_{t-1}V_x \right\}}{(1+i_t) \times G_x} \quad \textbf{Equation 15}$$

$$= \frac{\left\{ \hat{r}_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]} \hat{r}_{[x]+t-1} \right\} + \left\{ (1+i_t) \times P_x - \hat{r}_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]} - \left(\begin{array}{l} (r_{[x]+t-1} - \hat{r}_{[x]+t-1}) \times \ddot{a}^{(i)}_{[x+t]} \\ + (\hat{w}_{[x]+t-1} - w_{[x]+t-1}) \times {}_tV_x \\ + (\hat{q}_{[x]+t-1} - q_{[x]+t-1}) \times {}_tV_x \end{array} \right) \right\}}{(1+i_t) \times G_x}$$

Equation 16

$$= \frac{P_x}{G_x} - \frac{(r_{[x]+t-1} - \hat{r}_{[x]+t-1}) \times \ddot{a}^{(i)}_{[x+t]} + (\hat{w}_{[x]+t-1} - w_{[x]+t-1}) \times {}_tV_x + (\hat{q}_{[x]+t-1} - q_{[x]+t-1}) \times {}_tV_x}{(1+i_t) \times G_x}$$

$$= \left\{ \begin{array}{c} Net-to-Gross \\ Ratio \end{array} \right\} - \frac{\left\{ \begin{array}{c} Actuarial\ \ Gains \\ during\ \ the\ \ period \end{array} \right\}}{\left\{ \begin{array}{c} Interest\text{–}Adjusted \\ Earned\ \ Gross\ \ Premiums \end{array} \right\}}$$

In practice, the loss ratio is calculated from aggregated data. Therefore, actual calculations use the Equation 15 definition. Equation 16 shows that the loss ratio is our established Net-to-Gross ratio, adjusted for experience over the period.

Expense Ratios, Combined Ratios and Profit Margin

In GAAP accounting, we establish a deferred acquisition cost asset. The DAC asset is simply the negative of an "expense reserve." From year to year, the expense reserve progresses in a manner similar to Equation 13:

Equation 17

$$(1 - w_{[x]+t-1} - q_{[x]+t-1}) \times {}_tV^{(e)}_x = (1+i_t) \times ({}_{t-1}V^{(e)}_x + P^{(e)}_x - e_t),$$

where

e_t = Expected incurred expense at the beginning of policy year *t*.

$\hat{e}_t$ = Actual incurred expense at the beginning of policy year *t*.

$P^{(e)}_x$ = Expense Net level premium.

${}_tV^{(e)}_x$ = Expense Reserve per unit in force at the end of policy year *t*. It's generally negative. Negating it gives us the positive DAC asset.

Then we define an expense ratio as follows:

Equation 18

$$LR^{(e)} = \frac{\left\{ \begin{array}{c} Actual \\ Expenses \end{array} \right\} + \left\{ \begin{array}{c} Interest\text{–}Adjusted \\ Expense\ \ Reserve\ \ Increase \end{array} \right\}}{\left\{ \begin{array}{c} Interest\text{–}Adjusted \\ Incurred\ \ Gross\ \ Premiums \end{array} \right\}}$$

Skipping a lot of math that's very similar to our transformation of Equation 13, we get:

Equation 19

$$= \frac{P^{(e)}_x}{G_x} - \frac{(1+i_t) \times (e_t - \hat{e}_t) + (\hat{w}_{[x]+t-1} - w_{[x]+t-1}) \times {}_tV^{(e)}_x + (\hat{q}_{[x]+t-1} - q_{[x]+t-1}) \times {}_tV^{(e)}_x}{(1+i_t) \times G_x}$$

$$= \left\{ \begin{array}{c} GAAP \\ Amortization \\ \% \end{array} \right\} - \frac{\left\{ \begin{array}{c} Actuarial\ \ Expense\ \ Gains \\ during\ \ the\ \ period \end{array} \right\}}{\left\{ \begin{array}{c} Interest\text{–}Adjusted \\ Incurred\ \ Gross\ \ Premiums \end{array} \right\}}$$

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Actual calculations use the Equation 18 definition. Equation 19 shows that it works out to our GAAP amortization percentage, adjusted for experience over the period.

Equation 19 is the deferrable expense analogue of Equation 16. Adding:

- the net-to gross ratio from Equation 16 (covering benefits),
- the GAAP Amortization percentage from Equation 19, and
- some provision for nondeferrable expenses results in the "combined ratio," another concept borrowed from casualty insurance. And 100 percent minus the combined ratio is the profit margin.

Lines of Business

The Equation 9 definition of the loss ratio modifies the basic concept to fit issue age level premium plans, chiefly Long Term Care and Individual Disability Income.

In the early years, LTC claims are very small. Even significant percentage deviations will not register significantly in the loss ratio calculation. The actuarial gains from claim experience are small compared to the other components (Net-to-gross ratio, and mortality and lapse gains). High early lapses could make a new block of LTC look very profitable. But the remaining insureds may be less healthy, and subsequent claim experience may be unfavorable. You should always examine your loss ratio results critically, and understand what's driving them.

Limited-pay plans (e.g., 10-pay) present special problems. Applying our usual formulas, we're dividing by a very large premium in the early years. Later on, we're faced with the prospect of dividing by zero. None of this matters if we don't have much limited pay in force, or if we have a mature distribution by policy year. But the scheme is fairly popular; and most policies are probably still in their premium-paying period. One solution may be to restate the active-life reserve as lifetime pay, and treat the excess as unearned premium. Thus:

- the interest-adjusted increase in the lifetime pay portion would be added to claims in the numerator, and
- the increase in the interest-adjusted excess would be subtracted from the premiums in the denominator.

This doesn't sound very practical. If the limited-pay block is small, you can spare yourself the effort.

Equation 20

$$LR = \frac{P_x}{G_x} - \frac{\left(q_{[x]+t-1} - \hat{q}_{[x]+t-1}\right) \times \left({}_tF - {}_tV_x\right) + \left(w_{[x]+t-1} - \hat{w}_{[x]+t-1}\right) \times \left({}_tCV - {}_tV_x\right)}{\left(1+i_t\right) \times G_x},$$

We can apply these concepts to life insurance. The life insurance analog of Equation 16 is
where

$_tF$ = Face Amount for policy year *t*

$_tCV$ = Cash Value for policy year *t* (usually zero for term insurance)

In life insurance the law doesn't require loss ratio calculations. There is no consensus on acceptable values, especially for Cash Value Whole Life, although they may be helpful for term insurance. They may also be useful if your parent company is a casualty insurer.

Limitations and Food for Thought

The loss ratio is easy to apply, based solely on aggregate premiums, claims and reserves. Using modern data warehouse technology, we can examine separate loss ratios for various underwriting and occupation classes, geographical regions and markets.

Of course, we must confine our examination to subsets that produce statistically significant results. That entails some combination of choosing sufficiently large subsets or sufficiently long study periods.

But Equation 16 indicates one obvious area where the information is incomplete. We see that the major component of the loss ratio is the ratio of valuation net premium to gross premium (and this ratio may be similar to what we originally filed with the states). In the loss ratio calculation, actuarial gains, expressed as a percentage of premium, are implicitly subtracted.

So, if our loss ratio is higher than expected, is it because of excess claim incidence or insufficient lapses? (And if we're using method 2 to calculate our incurred claims, are low claim terminations to blame?) Thus, the loss ratio alone gives us an incomplete picture.

The solution is to calculate the individual actuarial gains explicitly. For example, Equation 16 contains $\left(r_{[x]+t-1} - \hat{r}_{[x]+t-1}\right) \times \ddot{a}^{(i)}_{[x+t]}$, the gain from incidence. The

$\hat{r}_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]}$ component is the sum of all new claim reserves established during the period. And $r_{[x]+t-1} \times \ddot{a}^{(i)}_{[x+t]}$ requires doing a summation over all active lives in force. Once we get those figures, we can divide $(r_{[x]+t-1} - \hat{r}_{[x]+t-1}) \times \ddot{a}^{(i)}_{[x+t]}$ by the interest-adjusted earned premiums, giving us the gain from incidence as a percentage of premium.

Similar analysis gives us the remaining actuarial gains. Then, we can calculate the aggregate net-to-gross ratio, subtract the actuarial gains percentages, and hope that they add up to the aggregate loss ratio.

This is a bit tedious, because we are required to go beyond merely taking ratios of aggregate quantities. But our reward is that we can split our loss ratio into the expected net-to-gross value and all deviations from expected. For example, assume that we priced for a 55 percent loss ratio, which is the net-to-gross premium ratio. During the period, if incidence gains are +2 percent, lapse gains are -7 percent, and termination gains are +1 percent, then our total loss ratio is

$$55\% - (2\% - 7\% + 1\%) = 59\%$$

The loss ratio is higher than we priced for, and yet claim incidence and termination are fine. The problem is "insufficient" lapses.

We can perform similar analysis of expenses (see Section 6). Then the "deferrable" expense ratio will be the GAAP amortization ratio, minus the actuarial gains. Here, the "insufficient" lapses may translate into an actuarial gain, somewhat offsetting the disappointing loss ratio.

All of the foregoing, plus the nondeferrable expenses and profit margin, add up to 100 percent. The moral of the story is that not all deviations are created equal. A higher than expected loss ratio does indicate an unanticipated level of claim payout. But if it's solely because of low lapses, then our expense amortization offers some mitigation.

GAAP reserves generally include some margin for adverse deviation. As mentioned earlier, we can have a more meaningful exhibit of actuarial gains if we devise an adjustment that removes them.

If the net-to-gross ratio varies by issue age, sex, underwriting class, etc., then the aggregate net-to-gross will gradually shift with variations in lapses and mortality. This is another pitfall to examining loss ratios in isolation. We can overcome this by splitting the loss ratio into its basic net-to-gross ratio, minus actuarial gains. §