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Chairperson's Corner

by Josephine E. Marks

The Investment Section continues to be a favorite with actuaries. Our membership stands at 4,126 (the largest of all SOA Sections), and our financial position is strong. We do, however, wish to solicit your input and involvement to ensure that the Section meets your needs. Please contact any member of the Investment Section Council with your ideas and suggestions.

Elections for Section Council for the year 2000-2001 will take place in July and we welcome your suggestions for possible candidates. Please submit nominations prior to April 15 to any member of the Investment Section Council.

Activities for 2000 include sessions at the Las Vegas and San Diego spring meetings in May and June and at the Chicago annual meeting in October. Topics include risk management, asset

(continued on page 11, column 3)

Dynamically Hedging Insurance Product Risk

by Marshall C. Greenbaum

Insurers face new and unfamiliar risks today as they race to design and distribute innovative insurance and annuity products with strong customer appeal. For instance, with the increasing popularity of equity-indexed and variable annuities (VA) products, insurers now confront substantial exposure to equity market risk. Given the typically thinner margins in these products, balancing profitability with prudent risk management is a particularly challenging task. This article focuses on demonstrating the effective management of equity market risk inherent in VA product using a dynamic hedging program. A case study is presented in which the costs (reduced expected cash flow) and the benefits (reduced cash flow variability) of a dynamic hedging program are compared to both a reinsurance alternative and to a no-risk management alternative. The relative effectiveness of each strategy is graphically illustrated. The conclusions reached in this article are equally valid for other insurance or annuity products including other capital market features.

Equity Market Risk Exposure

Equity market risk in a VA arises from two main sources. First, the bulk of the revenue of the product is achieved by charging the policyholder a mortality and expense (M&E) fee, assessed as a basis-point charge against account value. Therefore, if the equity markets move down, the insurer collects a basis-point charge applied against a commensurately reduced account value. The total dollar amount collected, therefore, fluctuates in relation to equity market levels. The second main source of VA equity market risk originates from policy guaranteed minimum death benefits (GMDB). At

(continued on page 4, column 1)

In This Issue

	page		page		page
Chairperson's Corner		The Funds Rate Target and Interest Rates		Bond Prices, Yields, and Convexity	
by Josephine E. Marks.....	1	by Daniel L. Thornton	12	by Macro Advisors	16
Dynamically Hedging Insurance Product Risk		Extracting Inflation Expectations from Bond Yields		Investment Section Council Meeting in San Francisco	17
by Marshall C. Greenbaum ...	1	by Frank A. Schmid	13	A Message from the President-Elect...Think NAAJ	
Editor's Column		Fear and Loathing in Swaps		by Rob Brown.....	19
by Nino Boezio.....	2	by Jim Sweeney.....	14	Own the Past.....	20
Projecting Budget Surpluses		1996-97 Redington Prize Awarded	15		
by Carl E. Walsh.....	9				

Dynamically Hedging Insurance Product Risk
continued from page 1

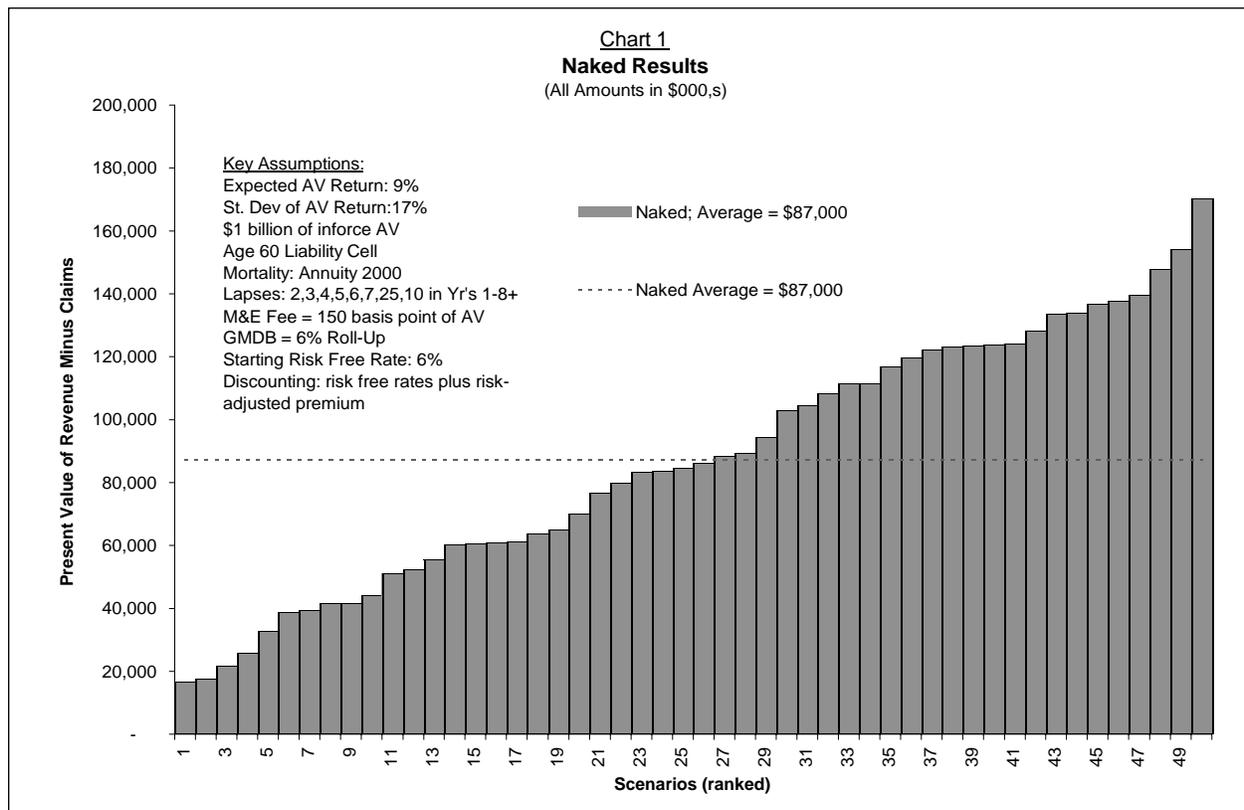
policyholder death, if the account value is less than this guaranteed value, the insurer commits to making up the difference. These GMDBs exist today in many different forms. The design illustrated in this article is commonly referred to as a “6% roll-up.” The policyholder is guaranteed to receive all deposits increased with 6% interest per annum as a minimum death benefit. While this is a relatively “rich” design from a policyholder’s standpoint, other more attractive designs are appearing. Together, as the market declines an insurer has exposure to increasing GMDB claims in addition to reduced M&E revenue. The case study performed focuses on these two items: M&E revenue and GMDB claims.

Going ‘Naked’

Today, many insurers have either not assessed their equity market exposure or

have decided that it is within an acceptable tolerance range. Failure to hedge the embedded cashflows of a product/financial instrument is referred to as going “naked” in the financial community. Using a Monte-Carlo simulation process, Chart 1 illustrates the equity risk variability of a \$1 billion block of VA account values. The stochastic scenarios employed include stochastic equity market movements as well as stochastic interest rates and stochastic market volatility (i.e., the volatility that drives equity market movements is itself stochastic). The present value (PV) of 15 years of net cashflows, M&E revenue minus GMDB claims, for 50 random scenarios are ranked from the worst outcome to the best outcome. The key assumptions noted on the chart are intended to be reasonable, although an alternative assumption set could be viewed as equally valid. The

expected account value return of 9% reflects the expected return for a policyholder’s account that contains a mixture of equity and fixed income investments. The discount rate is risk-adjusted, i.e. a spread is added to risk-free rates, to appropriately account for the variability of cashflows. (A discussion of this risk-adjusted rate is beyond the scope of this paper.) Effectively, the average present value calculated over the scenario set is identical to a market value of the product’s net cashflows that would be calculated using option-pricing techniques. The average present value or market value is \$87 million. However, tremendous variability in the cashflows exists. The present value ranges from \$17 million to \$170 million over all 50 scenarios.



Reinsuring the GMDB

While the marketplace for reinsuring the GMDB claims has become limited today, reinsuring the GMDB is a feasible strategy for eliminating some portion of the VA's equity market risk. The next alternative reinsures the 6% roll-up GMDB claims for a 60 basis-point charge per annum on AV under a 100% quota-share arrangement. The price is illustrative but intended to be indicative of the price for this benefit where no annual, per life or treaty caps are enforced. All GMDB claims are to be covered by the reinsurer. Chart 2 displays the results. One can see that the reinsurance cost has reduced the average PV of net cashflows from \$87 million to \$63 million. The difference can be viewed as net revenue (net of GMDB claims) to the reinsurer. The variability of results has been reduced, as demonstrated by the shape of the profile,

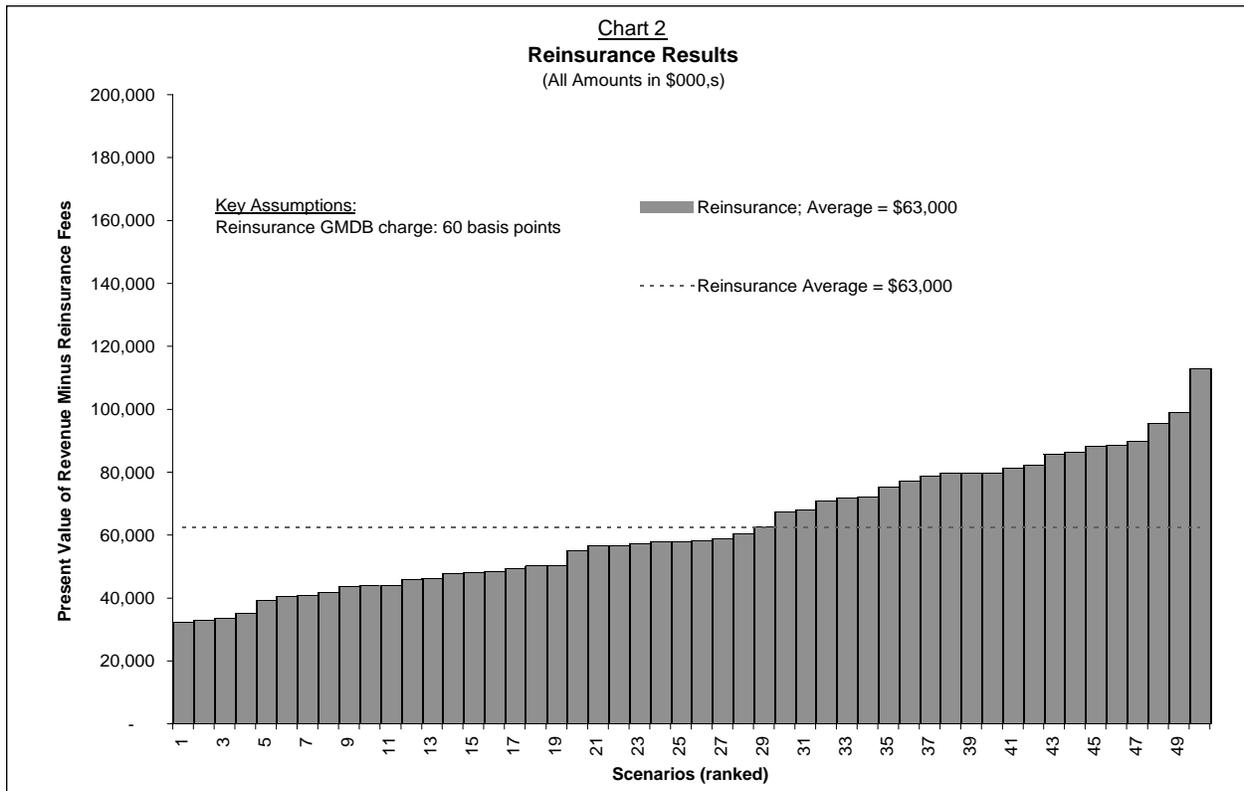
i.e. it's "flatter" and thus can be viewed as being less "risky" to the insurer. The re-insurance strategy results range from \$32 million to \$113 million. Of course, the height of the profile is dependent on the reinsurance cost. A lower charge would result in a higher average PV, while a higher charge would result in a lower average PV for the direct writer. It should be noted that the same 50 scenarios were used for all alternatives, and the results illustrated are ranked ordered. Therefore, the worst scenario for each alternative does not necessarily represent the same scenario.

Dynamically Hedging the VA Cashflows

Dynamic hedging is an effective risk management alternative to the reinsurance approach for direct writers interested in hedging the dynamics of the

VA product. Reinsurers can also use this approach where the quantity to be hedged would be the PV of the GMDB reinsurance premiums minus the reinsurer's GMDB claims. Direct writers can also use dynamic hedging in conjunction with reinsurance where reinsurance is deployed to cover a certain aspect or "layer" of the VA equity market risk with dynamic hedging covering the residual piece. Dynamic hedging's goal is to utilize liquid financial instruments to provide the necessary offsetting impact on net PV. A program that attempts to eliminate all variability in the PVs resulting from equity market risk would expect to realize the expected PV of net cashflows for all scenarios. Using our example, the expected PV of net cashflow of \$87 million would be achieved on all 50 stochastic scenarios.

The strategy is "dynamic" because it



(continued on page 6, column 1)

Dynamically Hedging Insurance Product Risk

continued from page 5

entails adjusting the initial portfolio of financial instruments as current economic conditions warrant at each point in a scenario projection. This is in contrast to a “static” hedge program, which purchases instruments under a “set and forget” approach. In general, the dynamic hedging strategy employed uses index futures contracts to hedge changes in the PV with respect to changes in the account value, known as *delta* hedging. In other words, the quantity being hedged is the PV of net cashflows and this is accomplished by offsetting the delta of the PV of expected cashflows with a portfolio of index futures contracts. Because the delta of the futures portfolio will move out of alignment with the PV of net cashflows as capital markets change, rebalancing will be required. The following is the general procedure for the

and recalculate the expected PV, labeled PV₋. Then, calculate an effective delta in an analogous manner to an effective duration calculation as follows:

$$\text{Effective Delta} = (PV_+ + PV_-) / (2 * \text{shock percentage} * AV_t)$$

- Step 2) Go long/short the appropriate number of futures contracts so that the delta of the hedge plus any existing futures contract from a prior period plus the delta of the PV quantity equals zero. We would then have a delta-neutral portfolio and theoretically be indifferent to any changes in AV or the equity markets. Any increase/decrease in our net PV should be exactly offset by changes in our

- Step 3) Move forward to the next rebalancing period and repeat.

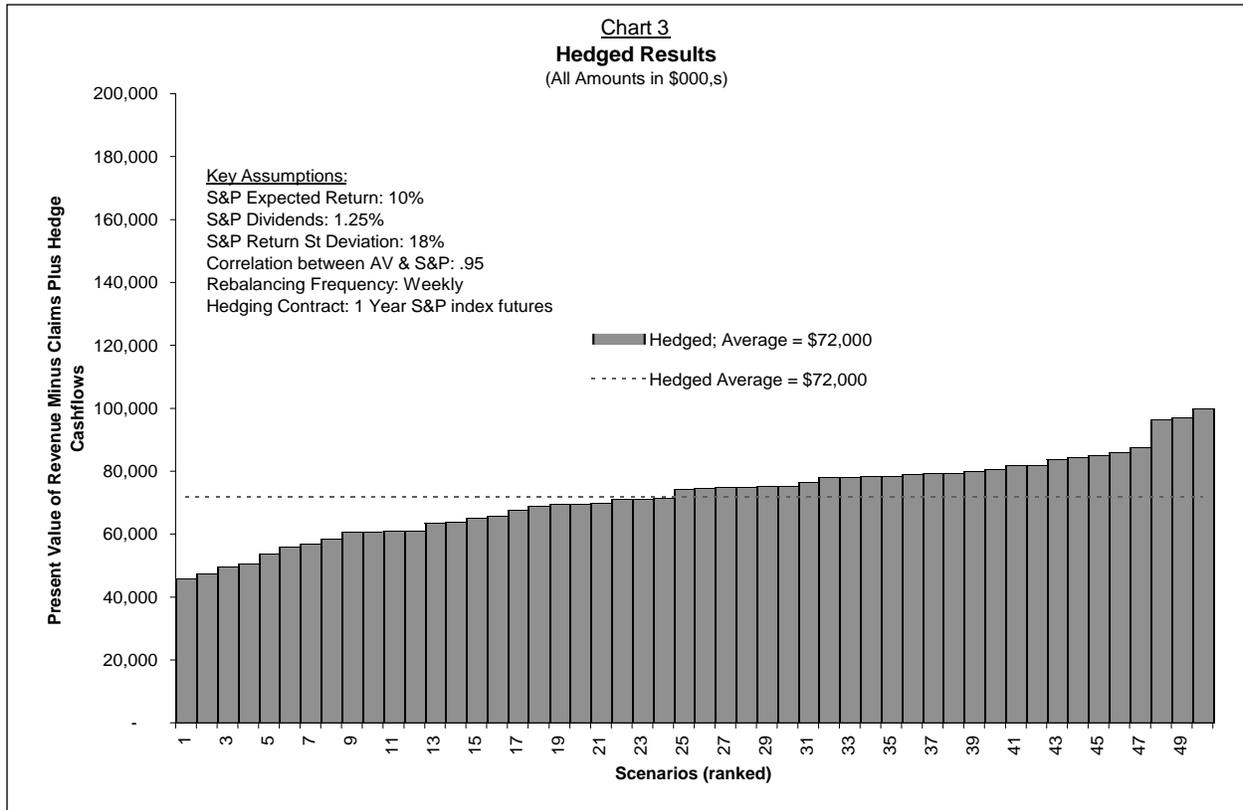
Chart 3 represents the results of the dynamic hedging alternative. The rebalancing period used was weekly (15 years of weekly rebalancing or 780 times per scenario). The results now include the present value of any cashflows resulting from the futures contracts including settlement and interest costs associated with a futures margin account. The key assumptions are included on the chart. Again, the goal of the dynamic hedging program again is to produce a risk profile so that all scenarios return the expected \$87 million amount. One can see that this does not quite occur. The average PV is now \$72 million, higher than the reinsurance strategy but lower than the naked results. Also, while we have eliminated much risk, residual risk or “slippage” has occurred. This is due to a number of factors. First, there are transaction costs on the futures contract. Futures contract prices are quoted via bid-ask spreads that need to be reflected in addition to any flat-dollar costs. Second, the rebalancing period was limited to weekly. A more frequent rebalancing period such as daily would improve the results. Third, “basis” risk exists because the S&P 500 index, which determines the hedge payoffs, is not 100% correlated with the AV. The adjustment discussed above minimizes this risk but does not completely eliminate it. The PV results range from \$46 million to \$100 million, a significant improvement over going “Naked.”

“The strategy is ‘dynamic’ because it entails adjusting the initial portfolio of financial instruments as current economic conditions warrant at each point in a scenario projection.”

delta hedging program along any scenario at a point in time:

- Step 1) Calculate the delta of the PV of expected cashflows at time *t*. This is done by calculating a market value of net cashflows or the expected PV of cashflows over all scenarios based on the current AV at time *t*. Shock the current AV up by (1 + shock percentage) and recalculate the expected PV, which we will label PV₊. Shock the AV down by (1 – shock percentage)

futures account. In this case study, one final adjustment was made. Because the example uses a S&P 500 index futures contract, which had an assumed correlation with the AV of 0.95, an adjustment was made by multiplying the delta of the PV by the beta of the AV with the S&P 500. For a discussion of this adjustment see Hull, *Options, Futures, & Other Derivatives*, 4th Edition, Prentice Hall, pages 65-67.



Conclusions

Chart 4 contrasts the results of the three risk management alternatives. It is important to note that the relative shape of the reinsurance strategy and the dynamic

hedging program are similar only by chance. The reinsurance strategy is a hedge against the GMDb claims only. The dynamic hedging strategy attempts to eliminate the variability of both the

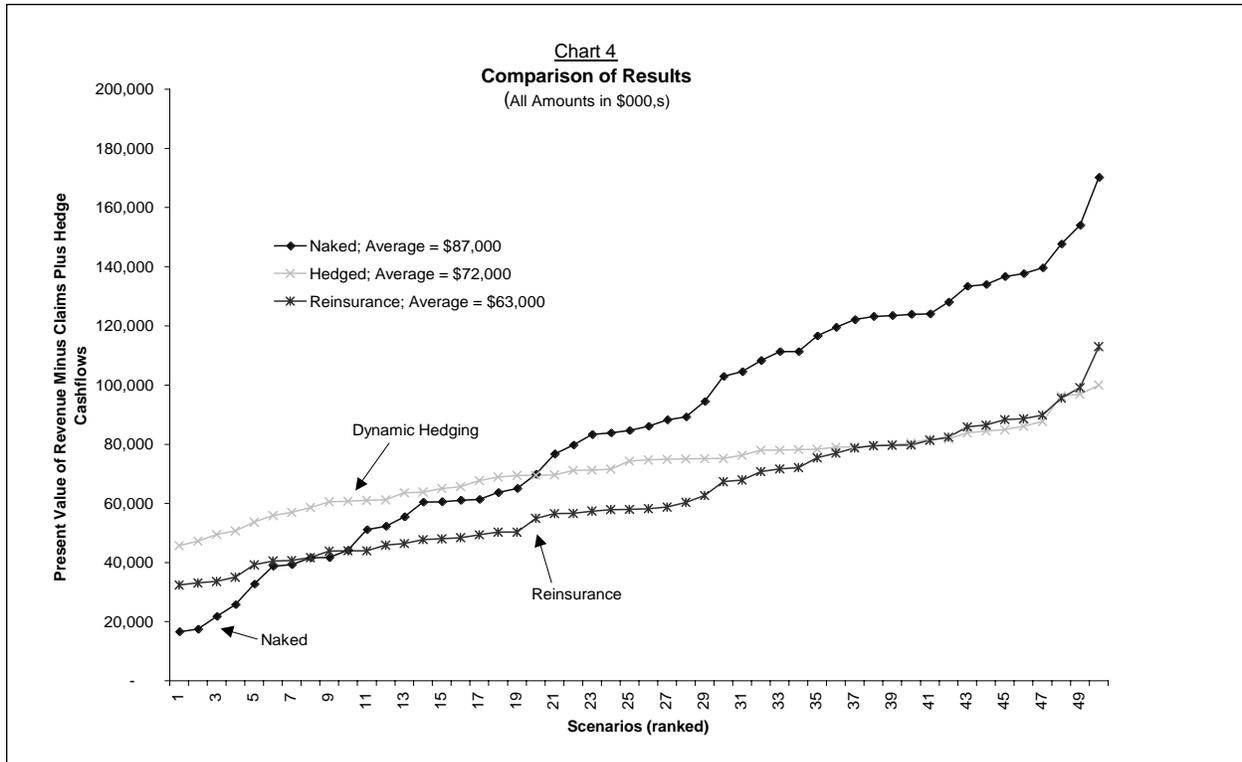
GMDb claims and the M&E revenue. Also, if the reinsurance agreement were to include annual/lifetime caps, the risk profile curve would be steeper or more “risky.” In addition, the hedging program relied upon weekly rebalancing. A rebalancing program with more rebalancing periods would improve results in terms of the shape of the risk profile (i.e., flatter profile would be expected) as well as improve the expected PV (i.e., the level of the profile would be higher).

“A dynamic hedging program can be an effective solution to insurance company equity market risk management problems.”

(continued on page 8, column 1)

Dynamically Hedging Insurance Product Risk

continued from page 7



The dynamic hedging strategy used in this article is conceptually simple. Alternative strategies might include hedging other so-called “Greek” parameters that measure sensitivities to changes in other risk elements besides equity market-induced changes in AV such as implied volatilities (vega), interest rates (rho) and changes in delta (gamma). They would potentially include the simultaneous sale/purchase of multiple index options, interest rate futures contracts, and index futures contracts to match a combination of Greek parameters. These strategies would expect to flatten the risk profile shown above. They would also prove to be particularly beneficial during stress test

scenarios such as an October 1987 market drop scenario.

The results illustrate that a dynamic hedging program can be an effective solution to insurance company equity market risk management problems. The strategy can offer potential cost savings over reinsurance approaches. Also, the strategy is flexible in that it can be employed on a stand-alone basis or in conjunction with reinsurance where dynamic hedging might cover any “tail” risk not covered by a reinsurer. Finally, the strategy can offer attractive synergies with FAS 133 development efforts, since both critically rely on the ability to mark-to-market derivative positions (in both

assets and liabilities). As demonstrated in this article, the long-run risk management implications of dynamic hedging are extremely positive.

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