Article from:

Risks and Rewards Newsletter

August 2005 – Issue No. 47
This article is based on research by the author for the purpose of completing his professional development project as a partial fulfillment of his FSA requirements. The views expressed in this article are those of the author alone and do not reflect the asset-liability management strategies of his employer.

**Introduction**

There are two main techniques for evaluating the financial impact of interest rate movements on insurance companies: duration measures and computer simulations. In many ways these approaches provide complementary viewpoints.

The duration approach to evaluating interest rate risk is based on a relatively simple theoretical model. The basic goal of the duration approach is to evaluate the impact of interest rate movements on the market values of assets and liabilities. Although the model has evolved since its description by Redington, its fundamental properties have remained unchanged—this model is essentially based on market value measures. On the other hand, earnings (GAAP, statutory and tax) are based on book value measures, such as net investment income and change in reserves. The duration approach, even at its most complicated, can therefore only approximate the impact of interest rate movements on earnings.

In contrast to the duration approach, computer simulation based asset-liability management (ALM) models typically take a large amount of asset and liability data and apply borrowing and reinvestment strategies to project financial experience. Although these simulation models will often produce market value outputs, their focus tends to be on a balance sheet and income statement presentation. In other words, instead of concentrating on a duration model, computer simulations can be said to have a book value focus.

Given the complementary nature of the existing models, what benefit is there in a new model for understanding the financial impact of interest rate movements? The answer to this question lies in the nature of theoretical models as compared to computer simulations.

Computer simulations tend to take all of the information available for inputs, use arbitrarily complex assumptions, and produce reams of data as output. These models attempt to approximate reality by using as much information as is available. On the other hand, these models tend to be extremely time-consuming.

---

1) See, for example, chapter 3 of Panjer (1998).
consuming to maintain, and produce no simple explanation of their results (if correct) or the source of errors in their results (if incorrect). Invariably, we turn to theory in order to convince ourselves of the sensibleness of simulations’ results.

In contrast, theoretical models attempt to approximate reality by using a few well-chosen, but dramatically simplifying, assumptions. These models are therefore able to organize the mass of inputs into just a few aggregate items. This creates a simpler understanding of the nature of the model results, and therefore, hopefully, of the real world. On the other hand, a theoretical model will generally have few outputs. For example, for all its strengths, the duration approach is limited to its market value focus.

This article discusses an attempt to build an earnings focused, theoretical model to complement the two approaches discussed above. A more mathematical development was presented recently at the 2004 Investment Actuary Symposium and AFIR Colloquium.

Why Should ALM Focus on Book Value Measures?

In comparing approaches there is a temptation to over-simplify and say that the market value focus captures the underlying “true” economics, while the book value focus is “only” based on accounting. In reality, of course, the situation is more complicated. On a practical level the book value focus is the preferred focus of regulatory agencies and equity analysts and, therefore, of senior management. Furthermore, balance sheets are often managed to book value specifications; for example, there may be a desire to hold the book value surplus at some multiple of risk-based capital.

Also, note that the fundamental goal of a market value based ALM model is to measure the potential volatility of the market value of surplus. The market value of surplus is typically calculated by subtracting the present value of liability cash flows from the market value of assets. An alternative calculation, however, is to define the market value of surplus to be the present value of projected portfolio earnings. An ALM model that produces earnings as output can therefore also be used to evaluate market value of surplus.

Finally, it is important to recognize that general account cash flows are often determined by book-value calculations. An obvious example of such a cash flow is federal income tax, which is based on tax reserves and asset book value calculations. Another example (illustration follows) is distributable earnings. Book value based cash flows can have a profound impact on traditional duration target calculations. Consider the following two cases:

**Case 1: Simple fixed liability cash flows**

Given a set of liability cash flows with no optionality (e.g., a portfolio of GICs or payout annuities), one simply calculates the liability duration and sets the asset duration target equal to the liability duration.

**Case 2: Include distributable earnings as a liability cash flow**

Any portfolio of assets and liabilities will throw off distributable earnings. Assume that these distributable earnings are in fact distributed. This represents a real cash flow out of the general account and should be included in the liability duration calculation. In order to estimate the size of these cash flows a simple ALM simulation model can be built, using the current assets and liabilities as inputs, and the distributable earnings under various interest rate scenarios can be calculated. These interest sensitive cash flows can then be added to the liability benefit cash flows and the liability duration can be computed. Unfortunately, this procedure leads to a liability duration that is exactly equal to the current asset duration, whatever the current asset duration is. In other words, a careful calculation, which includes book value based cash flows, can lead to a nonsensical tautology when attempting to find a target asset duration.


3) These two methods can be shown to yield equivalent results, provided one uses the appropriate interest rates in the present value calculation. See Girard (2000).

4) This can be seen in a simple spreadsheet by assuming a flat yield curve, and projecting distributable earnings for any simple portfolio. The market value (and hence duration) of liabilities plus earnings is calculated using an interest rate equal to the asset yield rate.
A Book-Value Measure of Asset-Liability Mismatch

In each of the two cases above, what is the risk minimizing asset portfolio? The risk-minimizing portfolio in Case 1 is clearly the classic cash-matched portfolio (dedication). To find the risk-minimizing portfolio in Case 2, consider the following points:

- If distributable earnings are in fact distributed, then the book value of the assets will be constrained to be equal to the book value of liabilities.

- If the asset portfolio is, therefore, designed so that the asset rollover is equal to the liability rollover, then the book value of assets will naturally equal the book value of liabilities and no reinvestment or borrowing will occur. Hence, this asset portfolio is the risk-minimizing portfolio.

Given the risk-minimizing portfolio, the asset-liability mismatch can be defined as the distance from the risk-minimizing portfolio. That is, define an asset-liability mismatch function to be the projected asset rollover minus the projected liability rollover. This asset-liability mismatch measure is called gap analysis and is commonly used in the banking industry.

A simple example can help elucidate the kind of information contained in the gap analysis measure. Consider a portfolio in which the only liability is a $100 coupon paying GIC with a 2-year term, and the only asset is a $100 corporate bond with a four-year term. The asset-liability mismatch function is shown in Figure 1.

There are three points that can be made about this asset-liability mismatch measure:

1. The asset-liability mismatch shown in Figure 1 is a snapshot. As time progresses, this graph will change. For example, if no other liabilities are added to the portfolio and no asset action is taken, then the mismatch function above will drift to the left.

![Figure 1: Gap analysis asset-liability mismatch function](image-url)

---

5) Here “risk minimizing” means the portfolio of assets that allows us to assume no future reinvestment or borrowing, hence no exposure to changes in future interest rates.

6) The book value of liabilities will depend on the context, but can be more complicated than the liability reserves. For example, in a GAAP context the book value of liabilities is the GAAP reserve minus any deferred tax asset and DAC asset, etc. Capital may or may not be included.

7) I would like to thank Jean-Francois Boulier for bringing to my attention the fact that the asset-liability mismatch function I discuss in Freedman (2004) is identical to gap analysis. More details on how gap analysis is used in banks can be found in the report of the Basel Committee on Banking Supervision (2004).
2. This asset-liability mismatch function emphasizes the timing of potential future earnings volatility. In the case of Figure 1, it is clear that interest rate movements will not impact earnings over the next two years. At the end of year two, however, the portfolio must either sell assets or write new liabilities, and the earnings beyond year two will be impacted by the interest rates at year two.

3. This asset-liability mismatch function does not provide any information about the overall level of earnings that can be expected. In the case of Figure 1, the mismatch function will not change regardless of whether the asset earns 50 or 100 basis points more than the liability credits.

Before proceeding, it is important to mention some of the limitations of this measure:

“[G]ap analysis ignores differences in spreads between interest rates that could arise as the level of market interest rates changes (basis risk). In addition, it does not take into account any changes in the timing of payments that might occur as a result of changes in the interest rate environment. Thus, it fails to account for differences in the sensitivity of income that may arise from option-related positions.”

These limitations, while important, are not overwhelming. This type of analysis is clearly not suitable for all asset portfolios and all types of liabilities, but for portfolios in which cash flow optionality is not significant, (e.g., payout annuities backed predominantly by non-callable bonds) gap analysis is very appropriate and also much less work than a full-fledged simulation.

An Earnings Focused ALM Model

From the perspective of asset-liability management, not all earnings are created equal. In particular, it is appropriate to separate earnings that will vary with future interest rate movements from those that will not. Already discussed is the fact that if the gap analysis measure is zero along the curve, then future earnings will be insensitive to interest rate movements. This section will focus on the level and volatility of earnings that arise due to a nonzero gap analysis measure.

Continuing the example above, recall that initially a two-year GIC was backed with a four-year bond. Assume now that at the end of year two, when the GIC matures, a decision is made to raise the required cash by issuing a floating rate liability. Future earnings can then be projected. There are three main sources of these earnings:

1. **Credit risk.** The liability will credit a floating rate (for example, 90-day LIBOR + 10 bp) based on the credit quality of the issuer. The bond backing the liability will earn a spread above LIBOR (for example, LIBOR + 45 bp) based on the credit risk of the asset.

2. **ALM risk.** The floating rate liability is backed with a two-year bond. The current earnings impact of this mismatch is equal to the current difference between the two-year LIBOR swap rate and the current 90-day LIBOR rate (for example, 100 bp).

3. **Results of previous ALM decisions.** In this example, the two-year bond backing the floating rate liability was actually purchased two years previously as a four-year bond. The coupon rate on this bond is therefore not likely to be the same as the current two-year rate. The earnings pickup in this case will be based on a combination of historical interest rate movements and historical yield curve shapes. (For example, the bond might have a coupon rate that is 60 bp above the currently available two-year coupon rate for an asset of similar credit quality.)

In this example the current earnings from the portfolio are:

\[35 \text{ bp (credit risk)} + 100 \text{ bp (ALM risk)} + 60 \text{ bp (Prior ALM)} = 195 \text{ bp.}\]

---

From an ALM perspective, the most interesting of these earning sources is clearly the 100 bp from ALM risk since the other sources of earnings will not be affected by interest rate movements. Note also that while the 95 bp from credit risk and prior ALM decisions are dependent on the details of the portfolio, the 100 bp from ALM risk does not depend on the details of the portfolio. It therefore seems that for the purpose of evaluating the risk-reward tradeoffs of ALM risk one does not need to carefully model the details of a specific portfolio; instead the problem can be treated generally and one can develop simplified earnings based risk and return measures.

In fact, the previous example can be generalized and evaluated mathematically. Defining the earnings that emerge from ALM risk as “mismatch-earnings,” it is possible to show that if one knows 1) the current and future gap analysis measure and 2) the current and future yield curve, then the future mismatch-earnings can be projected (given a series of simplifying assumptions). Below is a closed form formula showing projected mismatch-earnings:

\[ e_{\text{mismatch}}(t) = \int_0^t dt' \delta_r(t') C_r(t') + \int_0^t dt' \int_{\tau=0}^t dt'' \delta_r(t'') \left( \frac{\partial}{\partial t'} - \frac{\partial}{\partial \tau} \right) C_r(t') \]

where,

- \( e_{\text{mismatch}}(t) \) is the level of earnings at future time \( t \) due to current and future asset-liability mismatches.
- \( \delta_r(t) \) is the gap-analysis mismatch function at future time \( t \).
- \( C_r(t) \) is the yield curve (coupon rates) at future time \( t \).

A brief explanation of the mismatch-earnings projection formula is as follows. The first term in the calculation of \( e_{\text{mismatch}}(t) \) represents the earnings at time \( t \) from the asset-liability mismatch chosen at time \( t \). The second term represents the earnings at time \( t \) from the asset-liability mismatch chosen from times 0 to \( t \). In particular, the term \( \frac{\partial}{\partial t'} \) represents the earnings arising from the shape of the yield curve from time 0 to \( t \), while the term \( \frac{\partial}{\partial \tau} \) represents earnings arising from yield curve shifts from time 0 to \( t \).

Further explanations and analysis are given in Freedman (2004).

Using this formula it is clearly possible to calculate the expected value and standard deviation of future mismatch-earnings due to a series of stochastic interest rate scenarios.

Conclusions

Admittedly the mismatch-earnings formula previously shown does not have the clean and obvious form that one would like in an analytical model. However, there are two key results that can be derived from the existence of this formula:

1. Gap analysis is a useful framework for understanding a portfolio’s exposure to earnings volatility.
2. One does not need a full-blown simulation model to understand the impact of interest rate movements on a portfolio’s earnings. Instead the focus should be on deriving simple models (either analytical or spreadsheet) to project the expected earnings and volatility of earnings due to ALM risk (ignoring the other components of earnings).

Clearly there is much work to be done, but I am hopeful that earnings focused asset-liability management is both possible and useful.

References


