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The Evolution of *Actuarial Mathematics*

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Summary: The second edition of the Actuarial Mathematics *textbook was published in the spring of 1997. The authors of the text discuss what was added to the new edition. They focus on what they did, why they did it, and why they stopped where they did.*

Mr. James C. Hickman: There were five authors on this book who were engaged in a project that took place largely in the late 1970s and early 1980s. A central theme was probability—to try to build a probability basis under what had been called life contingencies and to show that risk theory, that had often been taught as a separate subject, also had a fundamental probability basis, and that the distinctions made in more conventional expositions were no longer relevant. The idea of variance, of variability of results around expected values, was a key theme. The author team, of course, was Newt Bowers, Hans Gerber, Don Jones, Cecil Nesbitt, and I.

The first edition of this book was published in about 1986–87. Times have changed. Newt Bowers was involved in revising this book. Our friend, Hans Gerber, from the University of Lausanne in Switzerland, served as a consultant and contributed several magnificent problems. I was involved. My colleague Don Jones, from Oregon State, bore a heavy part of this responsibility. Our colleague Cecil Nesbitt from Michigan also contributed several good ideas and exercises.

We should also acknowledge the fact that a whole army of people, many of whom are in this room, supplied ideas, corrections, and comments. The fact that they weren't all recognized was because we weren't always smart enough to recognize the goal behind them. We should also recognize the contributions of the Society staff, who are basically responsible for the prize-winning design and for keeping us

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on track. This was such an interesting assignment that I'm afraid we got carried away at times with the stimulation rather than simply completing the work.

There are big factors that have influenced this exposition and influenced the new edition. One can trace actuarial science probably back almost to the beginning of human thought. We have chosen to start it in 1693 with Edmund Haley and the publication of the Bresaw Table. By the middle part of the last century, the ideas of variability and the idea of using the central limit theorem for probability statements was very common, particularly in central Europe. We've identified that with the name of Hausdorf and his famous theorem of 1869. He was not alone. In 1903, those remarkable Swedes, particularly Philip Lunberg, developed what we technically call "compound stochastic processes," but in those days it was called "collective risk theory," one of the most creative applications of stochastic processes.

Next comes the name of Harold Cramér, that remarkable Swede, who in 1930, published a book on the mathematical theory of risk, which in many ways captured the essence of *Actuarial Mathematics* up until that time. In 1955 Cramér published a small, marvelous monograph on the collective theory of risk, or what we mainly would call compound Poisson processes. Although it was not original, it explained the idea of probability and statistical approaches to life contingencies, which had begun to enter the mainstream. In 1964, I published a paper that applied these ideas in the multiple decrement theory. Little of it was original, but it was beginning to enter the mainstream.

In the 1980s, a variety of authors began to look not only at the time and the cause of decrement and the number of claims and the claim amounts as random variables, but also at interest rates as random variables. There are several routes to this stream of thought, and we will try to identify some of those routes in a few moments. And of course, still in the future is the idea of building stochastic expense rates. If you believe that expense rates are constant and invariant, I have a bridge to sell you.

This effort is also influenced by the history of calculating. *Actuarial Mathematics* is not an abstract subject. You have to produce numbers. In the history of calculating, one can go back to clay tables and scratchers and paper and pencils. By 1614, that amazing Scotsman along with his bones—referring not to his skeleton but his computing devices—developed logarithms. Then came slide rules. By the last part of the last century, we began to get four-function, automatic calculators. And of course, some of us, Rowland Cross and myself perhaps, entered the world in which this was our principle computing device. By the mid-1950s, high-speed electronic computers began to influence not only business practices, but actuarial calculations, enabling us to recognize more variables, and to at least approximate distributions of results. Some of us earned our living for some years using the IBM 650. In fact, in the early part of the 1950s, the Univac was installed in a couple of major life insurance companies.

By the early 1970s, we began to use hand-held calculators that had capabilities far in excess of the capabilities of the mechanical machines we were using just a few years before. Following that, the pace became very rapid with the variety of equipment and software and the ability to store tables. Subsequently, the use of books full of tables began to decrease.

Spreadsheets and portable PCs influenced the lives of us all. Not just in how we administered insurance and pension operations and how we communicated with each other, although those were important, but how we used our models. And of course, by the early 1990s, this had shrunk down to palm-size computers, that certainly had capabilities far in excess of the clay tablets. You could go about two-thirds of the way down that list of computing and hold in your palm equipment that could beat them.

Now these factors, both the march of probability and statistical thought, obviously have influenced *Actuarial Mathematics'* first edition as well as the second edition. It's also influenced by practical issues, such as changed products. Various flexible life insurance products, in which premiums or benefits hop around, at one time were not only rare, they didn't exist until the publication of these textbooks. Now they dominate much of actuarial practice. Fixed products are basically down. Likewise new products, like accelerated benefits and long-term care, require somewhat more sophisticated models than simply the model of time of decrement and cause of decrement. These new products have become important.

Pensions have changed. Defined-contribution (DC) plans have basically been up, and defined-benefit (DB) plans are down. If, in our educational process, we are to prepare our students for this world, this needs to be recognized. Those three big factors, the growth of the probability and statistical models, the change in computing, and the change in the world that we serve have influenced the history of actuarial textbooks. One can't underestimate the importance of George King. In many ways, King defined the subject. What he put in his first and second edition almost defined the chapter headings of Spurgeon, Manor, et al., and one of the books by C.W. Jordan, who died this summer. When you simply take a look at the number of new members of the Society since 1952 up until the mid-1980s and see that fantastic growth, if you remember any name in North American actuarial science, it should be that of C.W. Jordan.

In 1987 in the *Actuarial Mathematics* book, you'll notice that the life expectancies of actuarial textbooks are 15 years or so. However, the life expectancy of any educational material is now shortened because of the three factors we mentioned before—new ideas, new computing, and new needs. Don Jones will tell you a bit about the major categories of change.

Mr. Donald A. Jones: Jim gave us a history of why we were in the position we were in within the profession. Now we'll start with what was the thought at the time of the transition from edition 1 to edition 2. In the beginning of the book, we put an introduction to the second edition, in which we listed the major changes. There were 11 of them. We made two computing changes, six changes in the models we were using, and three applications changes.

There were two computing changes. First, commutation functions are gone. I'll mention their role in history, but there's not a single symbol in the textbook. In place of that, we put in recursive relations. We think they have replaced the commutation function for our students. Second, more general recursive calculation for compound distributions was added to Chapter 12. It covers compound Poisson distribution, compound negative binomial, and compound binomial distributions.

When we wrote the first edition in the early 1980s, PCs and spreadsheets were just coming in. In fact, there was a time when I used VisiCalc, which wouldn't let you do spreadsheets from the bottom up. It would only do the formulas top down, which was a real pain to an actuary because you had to either turn your ages upside down or you had to find out the value of the annuity at age zero or 20 and then go down the column. You couldn't just give it a value of omega and then go the other way. Lotus changed all that, but that's where we were in the early 1980s.

Storing calculators were just coming in during that time. We had some interesting author discussions about whether all the readers of the book would have an HP41 calculator with a little magnetic card that they could use to load their mortality tables, but that was considered too new at that time. So in the first edition, we stayed with the commutation functions and the four-function calculators as tools for the exercises. We just didn't think the PCs and the storing calculators were far enough along at that time.

Now if you look around the campuses, I would guess 50% of the students have a computer. And if they don't, they can go over to a lab where there are just tons of computers where they can use either a Macintosh or a PC, and I don't think they ever form a line to use one. The teachers are now expecting students to do homework with spreadsheets and do their papers with word processors nowadays, and that's just common on campus. When I went to work at Bankers Life, now The

Principal, for a summer job, I had a four-function Marchant on my desk that cost two-and-a-half times my monthly salary. Now a student can go to work and have a PC on his or her desk, and if it's worth 2.5 times his or her monthly salary, he or she has a very nice PC. So, we assume the students using our book have a desktop computer with a spreadsheet, word processor, and so on.

Graphing, storing, and programmable calculators are now even required in calculus courses, yet storing calculators were kind of rare 15 years ago, so we expect the students to do all their numerical exercises in a spreadsheet. That means that the recursive relations, which in theory are usually called difference equations, are what they're using on the spreadsheet. What goes around, comes around. I had a math professor at the University of Iowa for my actuarial teacher who was enamored with difference equations. He would spend a lot of time teaching compound interest and life contingencies with difference equations. Of course, the only thing we solved then were the compound interest problems when we summed a geometric series. Nowadays, getting numerical solutions is very possible.

To deal with this we instituted a chain of computing exercises from Chapters 3 to 8. The students build a mortality table and then apply it in calculations. There are a set of exercises where the student is told, "Do this on your spreadsheet," and you assume that they have one available to do that. This replaces the old idea of the commutation function in the back of the book and the four-function calculator for doing a problem.

What are recursive relations? Here's an example from Chapter 3, "What's the life expectancy?" The recursive relation is a first-order linear, one with nonconstant coefficients. Just as you do when you solve the differential equation, you have to have a starting value or initial condition of some kind, and with actuaries, we like to go down to the end of the table where we know it's easy to postulate a starting value. So you have e_x, for x = omega equal to zero, and you program the formula into your spreadsheet column and plug the zero into whatever you determine to be omega, and, bingo, have all the values and so on. Newt Bowers will follow up on these computing ideas.

Mr. Newton L. Bowers, Jr.: The first change in the model is the utility period change. We did it in the first chapter, essentially, in the first edition. We kept it in Chapter 1 and ended up with Arrow's Theorem, which kind of justified why insurance makes sense. We were a little more ambitious in the second edition. And we used this in Chapters 6 and 7 when we began to get monetary values to get the premium. The premium is a value such that the insurance company is indifferent to offering the policy at an annual premium or to offering the contract or not offering the contract, indifferent to using as the measure of indifference a particular utility of wealth function.

Now, the linear utility of wealth function is equivalent to what we've always done in actuarial science, so there's nothing new in that. But we also worked out the exponential utility function, where you have to have a positive first derivative and negative second derivative to determine what signs you put into the utility function. And you can calculate the same premium where you're indifferent, using the exponential utility function.

I'm doubtful as to how much use it has. It does reflect the risks of the policy. We have a demonstration where, for a particularly easy policy, the premium for a policy of 10 times the face amount turns out to be 11.5 times as large. This is nonlinear. The extra risk with the extra amount of risk there actually increases this indifference premium.

We did the same thing with reserves in Chapter 7. We cover the amount of money that the insurance company would have to transfer to the reinsurer if the reinsurer were to assume the risk. This is the amount of money that they would transfer if they were to be indifferent about continuing the risk or transferring it to a reinsurer.

Regarding random future lifetime, Jim kind of touched on this. When I'm using the normal distribution central limit here and making approximate probability statements, as long as I assume independence between the various lives and the time of death of various lives, then all I need is the expected value and the variance. Simply add those up and you have one method of doing probability statements.

We did touch on some of the other things in the chapter on development in the first edition. What we added in the second edition were illustrations of the probability density functions. At this point we reached somewhat of a limitation of the text. As in the first edition, we did not try to teach computers. I don't even know if this would be the current best technique to do convolutions by the fast Fourier transform, or whatever the current best method is. We make no pretense of telling you how to do that. But it is possible and with the probability distribution functions here, for these various individual loss variables, it is perhaps possible to perform convolutions and get the distribution of the losses for our portfolio of independent lives. It's the same kind of problem that we were facing in the first edition.

I always used to get to Chapter 7 at the end of the semester. Our particular schedule is that we teach it in the spring when everyone is getting kind of restless. It's a long chapter, there's no question about that, so we decided to break it up into two chapters.

The new Chapter 7 is really almost what the insurance department wants you to do to come up with reserves for standard old, level premium, level benefit policies. It has all the things such as apportionable premiums and all that semi-interesting stuff.

We moved the more interesting stuff, like nonlevel benefits that involve interim reserves to Chapter 8.

I never seem to quite get to the Hattendorf Theorem in Chapter 7. Our schedule at Drake was particularly handy because if I got that far, then for the fall semester Stuart Klugman or someone else would take over. Just don't teach the two courses in a row.

We did some work on joint dependent lives, which shows that independence is not a good assumption to use. Back in the first edition, and this probably looks almost random to the students, sometimes we'd write ${}_tp_{xy}$ equals ${}_tp_x$ times ${}_tp_y$. Now those two were almost interchangeable statements back in the Jordan book. In the first edition everything that we could do without assuming independence we did first. We then put in, "and now assuming independence of the times of death of the two lives" and we did whatever else had to be done there. So that part of this dependence has been covered.

For examples of distributions with dependence, we didn't do anything about that. These apparently were supposed to be God-given numbers that you would use if you wanted to look at a dependent situation. We looked at two different models. There's a common stock model. I don't know if my coauthors would agree, but it strikes me that this is perhaps a generalization of Makeham's law. You have the μ force of mortality equal to $A + Bc^x$. The A term consists of first accidental risks perhaps applicable to you alone, and there's also some small piece in there that is applicable to you and your significant other. For example, you're driving the Kansas Turnpike and plow into a bridge abutment; in other words, simultaneous disasters to two people who would have a reason for having a joint life policy of some form. We worked some of that in section 9.6.

In the mid-1960s, the issue of having two marginal distributions first came up. Statisticians would examine situations where there was a joint distribution where independence didn't hold. That's easy enough to do if independence holds. You just multiply them together. But if independence doesn't hold, you have to do some magic. The kind of magic that we looked at was a method by Frank Copula. It involves a constant alpha, which gives the measure of dependence. Now Jed Frees, Jacque Carriere, and Emil Valdez had some Canadian data and estimated this alpha value based on some joint and second-to-die experience. Using this Frank

Copula method, they showed that the annuity values for dependent lives (this is for a second to die, last-survivor annuity) were less than they would be if they assumed independent. The annuity values were reduced. This is a *real* problem.

Now for an unreal solution. By sheer accident we came up with two distributions. There's one with independence, where you simply have to multiply two marginals together, and one with dependence, for which you produce the same distribution for the joint life status. We used that to simply plow through that chapter to show the kind of calculations there are with a joint distribution, what you would need to do to get last-survivor probability and so on.
This really makes more sense, I think, going in the other direction. It indicates the problem better going the other direction. You observe a distribution of, let's say, the time of the first death, the time of decrement from a multiple decrement situation. Now it's impossible to follow the people after the first decrement has occurred. It's hard to ask what is the probability that somebody would die from cancer if they've already died from heart disease. You can't do that. You can't handle that. In this example in Chapter 9 we developed a situation where two different distributions were identical for the time of the first death. The probability of a decrement in each case was a half at any given point in time, and the marginals are essentially useless in that situation.

This probably should give pause to people who make decisions based on mortality data from a multiple decrement world. Insurance companies don't have death-only contracts. They can walk if they stop paying premiums, get disabled, and so on. If you can only follow them to the time of decrement, it can cause some problems. This is also true for patching two distributions together. You can assume that the two are independent, but that would seem false on the face of it.

Anyway, these are problems and we don't solve them. We don't solve nonidentifiability problems at all. We did a fair amount of work on this, but probably the most creative single change is the one that Jim here will describe. There's a good reason why he's describing it—he essentially wrote it for us.

Mr. Jones: Jim, with all that talk about the dependent lives and so forth, we have the author of a first-life contingency question here. Our book stated that the lives could be dependent, but we never gave them any examples. Rich came up with one and put it on one of his exams.

Mr. Hickman: Although the references that we give for the mathematics of this are from the last 10–15 years, if you read some of the pioneers, Makeham in particular, they knew that was out there. They didn't quite use the language that we use today, and the word "independence" was not very well defined, but no one can claim that we concocted the idea in the last 10 or 15 years. We simply made it a little bit more precise and put it in probablistic terms.

There's no question that it was the most difficult, philosophic problem, although we were also faced with what to do about stochastic models for interest. It is probably the least important practical problem because the area is moving so rapidly that no one will probably read what we did anyway. But philosophically this is extraordinarily important. We recognized from the first edition that to build a stochastic model for time until decrement, by cause of decrement, and number of claims and claim amounts, was only going part way. As long as investments and funds were concerned, another important variable had to do with interest, and we assumed that to be deterministic.

What I will do here is basically outline the path that we have in mind. It traces the intellectual path, I think, that actuaries have taken. In the definition of the problem, we take a vector of observed interest rates simply to show you that they hop around a lot, which is no big deal. We make a distinction between those models for interest that are statistical in nature, primarily based on an analysis of past data, and those models that are based on characteristics, either developed by theory or by observation of the capital markets. One might well take the second class as constraining the first.

In the next section we engaged in some notation and preliminaries. One of the fundamental preliminaries was the expected value of a discount factor greater than or equal to the discount factor, based on the expected interest rate. This gives us a chance to review Jensen's inequality, which plays an important role in Chapter 1.

Some of these basic ideas and the whole idea of the notation are in that section. We should have used capital I's to represent random interest rates. In the next section we deal with scenarios. A scenario for us is simply a sequence of interest rates. The first of these sequences that we study are deterministic sequences, or deterministic scenarios. This was quite common, even in the days that I was in actuarial practice. We would assume that interest rates might be high for a few years and then go down. Deterministic scenarios were used in regulation, particularly by the New York Department, for a kind of sensitivity test of financial reporting for certain contracts.

One could also attach probabilities to each of these scenarios. Once one does that, several consequences are immediate. First, a major consequence is that it illustrates that the risk management problem for random interest is much different from the risk management problem for individual risks. The risk management problem there can be solved by getting bigger or by reinsurance. Because the random interest scenario affects all of these policies, the risk management problem is quite different. That's illustrated in this random scenario section.

Then comes a couple of sections that are very much statistically based, starting, as you usually do in statistics, with the independent and identically distributed random variables. We've illustrated those with the log-normal model, which is carried forward into simple actuarial functions such as annuities and benefit premiums for single premium insurances, including means and variances. We then take a step upward in what I hope is a sensible progression going from a deterministic scenario, random scenarios, independent interest rates, and now dependent interest rates.

We chose to illustrate this with a simple moving average model for interest rates. Do we claim that this is a universal model? No. But it brings out the main issues in using a statistical-based model for interest scenarios. This implementation section makes the point that there are many other models, databased clearly within the autoregressive, integrated, moving average class—the ARIMA models. There are a great many different models that one might use depending on the data. The implementation section brings up the possibility of using simulation to approximate the distributions, whereas in an earlier world, the world that Newt was describing, you could actually display the distributions of loss variables.

The next section has to do with financial economic models. It introduces spot rates, forward rates, and some of the capital market concepts, the most important of which are the no-arbitrage requirement or assumption and the idea that economic information may be captured in current prices and maturities. This is the yield rate idea, that within that observable phenomena, there may be a consensus forecast of the future state of the economy, and that this information may be useful to the actuary in both designing products and managing risks.

That goes together, and we chose an autoregressive stochastic model in which the parameters of the model are constrained by assumptions about the capital markets, primarily the no-arbitrage assumption. At best, those can only illustrate some of these ideas and the models are restricted to quite simple ones, auto-regressivefor example. The last section tries to create a problem, namely, the management of interest-rate risk, and give some hints as to what some of the solutions might be. Section 21.6.1 is a classical section. In fact, it's a retreat to a more deterministic world, in which immunization rules are developed and commented on.

There is a short narrative section, largely building on work by Jed Frees, on the management of both mortality as a random variable and interest as a random variable. These are to give the student an idea of the risk management tools that might be available in managing this interest rate risk. There's an extensive problem section. Our decision on whether this should go into the problems or the narrative was a close one. I'm sure that many of you who take the time to go through it wish we had gone the other way, being that some of the important ideas are represented in the problems rather than in the narrative. One could have done equally well presenting the exposition in the narrative form.

Mr. Jones: We had some cooperation from the stock markets around the world to justify the addition of that chapter to the book to prove that investment rates are an important part of this world. On the other hand, Chapter 10 was probably the biggest victim of change because of its application of multiple decrement theory. It basically pinched an application. It was quickly out-of-date because a lot of the material in there had to do with U.S. integrated pension plans, and since then the integration rules have changed to permit disparity.

Also during this period, the DB plans that are described in there are losing favor and being replaced by DC plans. But now that the markets are going down, maybe DC plans will fall out of favor, and we'll be returning to employer-sponsored DB plans again.

Chapter 10 is no longer a pension plan chapter; its applications of multiple decrement theory were consolidated. We brought some topics on disability insurance, such as disability riders, from Chapter 16 of the first edition, back into this chapter, which is now Chapter 11. Newt said that 7 was split, so this is Chapter 11. There are more examples of multiple decrement ideas in this chapter now than before, and the special design plans that are really motivated by regulation have been deleted.

In the applications, there are some new insurance products in Chapter 17. We dropped out the retirement income policies. I remember one of the big three auto companies had a pension plan based on individual retirement endowment policies. We've taken those out. Instead, we have an accelerated benefits policy section and another section on long-term care policies. in Chapter 17.

Another thing about the applications for pricing and accounting Chapter 6 comprised single-decrement, level-benefit premiums. Then you moved to Chapter 7, where you still work with single-decrement and still-level premium. We studied reserves. We then went to Chapter 14 and added expenses to this single-decrement world and modified the reserves to accommodate the expenses. Then we went to Chapter 15 and added in another decrement and used asset shares for pricing. That was the way we presented this in the second edition; the first two ideas are there, but the last two have been modified.

Mr. Jones: We broke that up. Now we have multiple decrement and accounting in one chapter and more pricing models in the next chapter. There's the Anderson model for insurance pricing and the Hoskins model, so there is more on pricing but it's after they've had all of the other material. What used to be Chapters 14 and 15 have been scrambled all together and parceled out differently into two chapters. The regulation stuff has been deleted. The specific regulation stuff, the idea of standards that you have to meet, is in there, but you don't have to know the U.S. or the Canadian standards.

What are some of the notations and nomenclature changes that will annoy the teachers because you'll still be saying net single premium and so on? One thing we did change was the symbol for the force of mortality, which is very difficult. It's something that we haven't treated very well in North American actuarial textbooks over the years. We made the force of mortality symbol essentially select, which would imply then that everything else is select. Things got so out of hand that we decided let's let it kind of go by context. But we kept this select notation for the force of mortality. Instead of having μ_{x+t} we have $\mu_x(t)$ to separate the duration from an underwriting characteristic at the beginning, what we did was a compromise. It'll bug you at times, and maybe we should have done something else, but we got rid of net single premium in Chapter 4 on the insurances and called that actuarial present value and made it somewhat symmetric with the annuities. The actuarial present value there and those two things are the same idea. That's not really a net single premium because they don't talk about pricing. The net single premium idea, the idea of the actuarial present value as the net single premium, is in Chapter 6. That wording is actuarial present value, which is another nomenclature change.

We got rid of net level premium. We thought that was a benefit premium. This premium was only designed to cover the benefits. Net level was not quite descriptive. The implication is that if the reserve then is almost double the premium reserve, it's a benefit reserve, i.e., gross premium. Since gross is not a politically correct word these days, we have contract premiums instead of gross premiums.

This is sort of a summary of the big changes. What chapter had the least change? That would be old demography Chapter 18, which is now Chapter 19. All we did was change the numbers from 18 to19. The chapter with the most change was the one Jim just talked about, the stochastic interest, because it didn't exist in the first edition. It rose up out of nothing.

The biggest complaint or the most frequent complaint we had about the first edition was the size of the book. "I can't carry it," the students would complain. The new book has the same outside dimensions, but it has more material.

Mr. Jones: It has slightly smaller margins and slightly smaller print. And, of course, the price went up, but the cost per page went down. It was $65 for 624 pages; it's now $75 for 753 pages. The good news is that the 753 pages are squeezed into the same outside dimensions of the first one, so you can't say that it's heavier than the first edition.

Just last week, Barbara Simmons said that *Actuarial Mathematics* won an award, the best in its category, scholarly book publishing, at the 47th annual media show of the Chicago Book Clinic. Congratulations to Barb and her staff.

The first exam that it's going to be used for is November 13, 1997. That's the 151 exam. If you go back through the history of textbooks, you will see that each book was used for 30 or 35 years, and editions would be used for 10–15 years. I think that's now an upper bound. The world is changing much faster. The syllabus is getting rebuilt. I estimate the lifetime to be 3–15 years.

Mr. Jones: The first edition had the advantage of being used in study note form for three or four years. With students and professors finding mistakes, misprints, and typos, you'd think the second edition would be almost pure. But there was too much rewriting and it was never used as a study note, so the errata are starting to come in. The Society will maintain the current errata sheet on their Web site and instructions. You can see the yellow italics of the instructions, and I have copies of the current proof.

From the Floor: If we find more errata, where should we send them?

Mr. Jones: I'll give you my e-mail address.

Mr. Rowland Cross: An awful lot of things were slipping by that I couldn't really grasp, but one I noticed was that you dropped the Lidstone Theorem. Is there anything comparable to that? I want to know what the effect on reserves would be of a change in interest rate or mortality. Is there something similar to that or do you have another useful tool?

Mr. Jones: That's a good question. Our feeling was that the idea of the impact on reserves is so easily done now on the spreadsheet, that—

Mr. Cross: But you have to do it. You must have the machine to do it. If you're consulting a client and they ask you, "What's going to happen if you change the mortality table?" you can't say, "Just a minute, I'll get back to my computer."

Mr. Jones: Yes, you're right Rowland. Perhaps those corollaries that we had to Lidstone's Theorem should still be in there. I doubt if you'd use that critical function as you're sitting and talking to a client, but you probably would for the consequences of the solo effect of an interest change or of a mortality change. Four corollaries were in the first edition. They did not survive the second edition, but maybe they should have.

Mr. Cross: Is there thought that you might restore them?

Mr. Hickman: I think the main point, Rowland, was that the amount of time and space that you had to get Lidstone's Theorem, which involves a difference equation, subtracting, getting a function that kind of goes from zero to zero and a lot of assumptions you put on it. By the time you taught all that, what do you get out of it? Well, you could get a couple of corollaries that are directly interesting for the issue that he just brought up. But there are other ways to provide that insight, and this just may well be a mistake. But the idea was that it just took too much time and space to do all Lidstone's Theorem for the rather meager result. Not that it's an unimportant result, but there are other ways to provide that insight.

The Lidstone argument is obviously an interesting argument. It pops up in differential equations and several places and problems. But there are no longer two or three pages devoted to Lidstone's Theorem.

From the Floor: How do you treat state regulations on these subjects?

Mr. Hickman: There's a little of that multiple-state stuff back in the material on accelerated benefits and long-term care. It's far from a complete exposition because we do not have returns. It is basically taught as if it was multiple decrements theory, although there's a section that recognizes that with multiple-state models you can move back and forth between several models, which is perhaps a better way to do it. However, we did not take the time to do it. With respect to the nonidentifiability problem, clearly, you can specify the parametric distribution and perhaps get away from it. But as long as you think in terms of time until decrement and "cause" of decrement, this is a pretty neat problem that has not been recognized this is essentially an estimation problem.

It's not an applications problem. But it's very serious and when it really comes up is in the elderly because the natural models are to think of causes of death as independent. That's the way we all learned. But, obviously, that's only partially true with the young, and is surely not true with the elderly. Therefore, the value of cause of death information, as you project in the higher ages, is very questionable. It's not that you don't get any information, but while your standard models are assuming independence, there is no independence between these causes. And it probably isn't so in many employee benefit plans. Death, disability, layoff in a pension plan, almost on their face, are not independent.

Mr. Cross: What about small schools that have just a very modest program in actuarial science in their math departments?

Mr. Hickman: Yes, they want to incorporate this new edition. The split between the second edition of Jordan and the first edition of *Actuarial Mathematics* was a bigger jump than this one. That was the big jump. But, for example, we knew about nonidentifiability ten years ago when we wrote the multiple decrement chapter. The words are there, but they're fuzzy. And that was by design. But the world has gone on. More people are facing up to their problems. The language is better. We can be a little bit more explicit about it this time. A few years from now, we may have a richer class of multi-variate models to slap in there. They're out there now, but they are, in our view, not quite ready for this level. But the answer is yes. That solo teacher out there at a liberal arts school who's keeping one step ahead of the student and had a lot harder time going from Jordan to the first edition of *Actuarial Mathematics* is going to have a slightly harder time in going to this new one. But in some sense, that's the world, Rowland. Not only is that true in *Actuarial Mathematics*, but in almost any subject that you can imagine the pace has stepped up.

From the Floor: I'm from China. If you do not have easy access to a spreadsheet with the book now, are you completely stuck?

Mr. Hickman: Yes, the problems that were stated in terms of commutation functions, or where we expected them to use commutation functions aren't there, so somebody teaching, in the nonspreadsheet environment might have to make up some exercises and provide commutation functions.

Mr. Hickman: Yes, we did reduce our market on that decision. You're absolutely right.

Mr. Cross: The Society publishes a study note, doesn't it, on commutation functions?

From the Floor: Not yet.

Mr. Jones: The Joint Board might, Rowland.

Ms. Esther Portnoy: This is a question of commutation function versus the spreadsheet, especially using the recursion relations. I'm not teaching that material this semester. Philosophically, I can appreciate that, but in terms of what my students are doing, I've tried for years. I've pushed the recursion relations for years and they've resisted. It's that leap to understanding. I keep telling them, "This is easy." You just think what happens the first year, but after that, you don't have to memorize all those formulas. Still, they resisted. I'm very skeptical of how it's going to go over pedagogically.

Mr. Jones: Look to the spreadsheet to help you, I think. The fact that they're doing things on that spreadsheet, I think, will help them understand and get the motivation to learn the recursion relations—that's my reaction to it. My students liked the spreadsheet exercises. I used them last year because I set them up.

Mr. Hickman: Yes, I think you're right. And in justification, I'll just add two things to what Don said. More and more actuarial functions do not depend only on attained age. The pension people almost enacted a standard and said everything shall be select. They didn't quite enact it, but that idea was there. The idea that functions depend only on attained age, where you get most of the economy from commutation functions, is somewhat declining in practice.

Further, I'd just like to quote to you from 1961 in *The Journal of the Institute of Actuaries Student Society* a man named William Phillips, one of the real pioneers of computing, who said "Now we know that commutation columns, the formula for approximate regression and the approximate evaluation methods, are as dead as a do-do." And 36 years later, we're kind of catching up with where Phillips was then. By the way, I recommend that you read about this remarkable guy. It's in Jack Moorhead's book, *Our Yesterdays*. He was a century ahead of the rest of us.

Mr. Cross: But realistically, people are going to have to know about commutation functions. They're going to have to see developments in offices by other experts that use commutation functions. You can't just eliminate them by an act of God and say they don't exist. They are going to have to get it somewhere, aren't they?

Mr. Jones: I think Max Planck said something about that. He said a new idea comes along and it takes a generation for it to be accepted. You have to just wait until the people who use the old idea leave and the new people come along. I have a question for you. In the equation down here that Jim went over, what do we call this thing right here? The IRS calls that "best estimate." Right? It's the actuaries' expected value of their distribution for the interest rate. What does this equation tell us? The equation says that the expected value of the random liability present values is bigger than the one you calculate when you use your best estimate.

Mr. Hickman: This is kind of lesson one in stochastic interest.

Mr. Jones: Yes, so does this say that the IRS is telling us to underfund pension plans by using our best estimate in the interest rates rather than a distribution for the interest rate? I think that this innocuous looking thing could be a political statement, probably the only one in the book. But it's kind of an interesting thing. Does that say that the IRS is underfunding our DB pension plan?

Mr. Cross: What's the order of the inequality? It is dramatically bigger or is it just a little bit bigger?

Mr. Hickman: Work problem 21.1 will ask you for various distributions in interest rates and how far they are apart. The answer is yes. They can be quite a bit.

Mr. Cross: Like what? Fifty percent more?

Mr. Hickman: Oh, I don't remember. We asked them to evaluate those differences.

Mr. Jones: It's about 5% of some common values. Because they went in the other direction, it's basically offset by that dependency of the joint life calculations. If you happen to have a plan with joint life benefits in it, it may turn out even.

As far as timing what you cover in a semester, depending on where you started, you have to do a little bit with Chapter 1 now because of the use of utility theory and the pricing. We're used to starting on Chapter 3 and getting through 7 in a semester. You do have to do a little bit with utility now.

I don't have perfect experience to relate to you because I'm on quarters. I'm clearly through Chapter 7 by the second quarter, but where I would have been if I had been on a semester year, I never gave that a thought.

Mr. Hickman: I don't think you'll have to do all of Chapter 1 to do Chapter 6 and 7. You can do it with a backward look. For example, the Arrow Theorem does not appear again in Chapters 6 and 7, but the idea that you could build a coherent system of both pricing and reserving for other than linear utilities does pop up. If you chose quadratic utility or exponential utility, you could get a whole structure of premiums and reserves. The whole thing would pull together and make sense. That idea is there. If you can help the student know that indifference equation of the reserve of an expected utility of the future cash flows at time zero, you get the premium. At other times, you get the reserves. That's all they need to know. But they will be led through problems that illustrate both exponential utility and quadratic utility. Just illustrate that idea.

Linear utility isn't the world. You can build a whole structure on a different value system. And you can illustrate that idea. That trade-off is all you need to do—only about the first four or five pages of Chapter 1.