Life Contingencies Algorithms for the Programmable Calculator

by

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Introduction

The specification for the new life contingencies text include (Adams, 1978) "a commitment to the following features:

1. stochastic treatment of risk processes,
2. computer algorithms for developing actuarial values,
3. full treatment of life, health, and group applications,
4. at least elementary casualty mathematics and coverages."

This paper explores the possibility of using the programmable calculator as the vehicle for presenting algorithms for developing actuarial values. The adaptability of the programmable calculator is tested by implementing algorithms from a hopefully representative set of subject areas scheduled to be included in the new text.

Algorithmic Approach to Calculating Actuarial Values

The algorithmic approach to calculating actuarial values has been a feature of actuarial texts for some time. In older texts, the algorithmic approach takes the form of principles of worksheet design. Gershenson's Measurement of Mortality (1961) gives advice on the construction of worksheets:

1) complete the worksheet column by column to simplify the work (p. 23),
2) minimize the number of columns of the worksheet (p. 23),
3) nonrecursive approaches are preferrable to recursive ap proaches so as not to propagate arithmetic errors (p. 61),
4) each line of the worksheet should be self checking (p. 61), and,
5) all of input should come from a single line of a source document (pp. 67-68).

A recent actuarial text, Kellison's Numerical Methods (1975) has a much stronger commitment to the algorithmic approach; it discusses algorithm explicitly in Chapter 1, Chapter 6, and in Appendices A, E, G, and H. It emphasizes that speed and accuracy are the primary criteria for evaluating algorithms. Early in the text raises issues related to accuracy, including:

1) relative error and absolute error propagation in arithmetic,
2) truncation errors, and
3) roundoff error.

The issue of algorithm speed is treated primarily through an emphasis on recursive formulas in the problems at the end of each chapter.

The emphasis of the althorithmic approach has clearly shifted in a number of respects: 1) accuracy is discussed more quantitatively, 2) the replacement of the actuarial clerk by the computer means that the need for accuracy does not constrain the choice of algorithm very much, today the actuary can consider a wide range of algorithms, well beyond the capacity of a clerk using one of the old Friden or Monroe "coffee grinders," and 3) the importance of speed, as indicated by the emphasis on recursive formulas, is much greater. The current actuarial student faces a much broader and more complex range of choices in algorithms, than did his predecessors of just a few years ago.

Use of Programmable Calculators by Actuarial Students

In this context, the programmable calculator represents a useful supplement to the use of computer implemented, procedure oriented languages like BASIC or FORTRAN in the study of algorithms. The virtues of the programmable calculator are 1) mathematical capabilities almost identical to those of BASIC and FORTRAN, 2) unrivaled accessibility (there would be no physical problem in allowing every student to bring a programmable calculator into the examining room), and 3) unrivaled affordability --the very best can be purchased for $375 including the cost of a printer. Programmable calculators also have some shortcomings which are not as easy to evaluate: 1) limited storage, 2) limited input/output capabilities, 3) lack of subscripted variables, 4) lack of matrix operations, and 4) slow speed of operation.

Some Test Cases

To evaluate the usefulness of the programmable calculator in presenting algorithms for calculating actuarial values and guage its advantages and shortcomings, a hopefully representative benchmark set of applications has been programmed for the TI59 and SR-56 calculators. The algorithms selected for presentation include:

1. force of mortality approximations,
2. tables of commutation functions,
3. binomial distribution for stochastic life table model,
4. variance of the expectation of life,
5. Newton's method for Esscher approximation in risk theory,
6. the Capital Asset Pricing Model algorithm for aggregate pricing of casualty coverages.

While these algorithms show a broad range of applications, they certainly are not a complete inventory of algorithms for the proposed two volume life contingencies text.

Presentation of Algorithms

The algorithms in this report are given in the following format:

1. a short narrative with a reference,
2. a flowchart,
3. a set of operating instructions for the programmable calculator algorithm,
4. a programmable calculator listing, and
5. a numerical example.

The flowcharts are drawn at a high enough level that the user could easily code the algorithm in a high level, procedure oriented computer language, e.g., BASIC, as well as in a programmable calculator language.

Concluding Observations

The case studies examined here suggest that the advantages of programmable calculator were, if anything, understated. The operating instructions for each of the case studies show that the programmable calculator allows greater flexibility in the control of the algorithm than is usually possible with an algorithm written in a conventional procedure oriented language.

The anticipated disadvantages (limited storage, limited i/o capability, lack of subscripted variables, lack of matrix operations, and slowness) were not at all constraining in the applications tested. Indeed it seemed that the rollover logic using the EXC key gives the programmable calculator capabilities equivalent to subscripting. The other limitations, however, are real, and they would show up in applications like linear programming or Whittaker-Henderson graduation.

For the most part, the capabilities of the programmable calculator seem adequate for most actuarial algorithms. Its unique accessibility and affordability give the programmable calculator some significant advantages.

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7. Texas Instruments, Texas Instruments Programmable Slide-Rule Calculator SR-56 Applications Library, Dallas, Texas: Texas Instruments, 1976.

Force of Mortality Approximations: μ_x

Approximation 1:

$$\mu_x = 1/2\ (\text{colog}\ p_{x-1} + \text{colog}\ p_x)$$
$$= 1/2\ \log \left(\frac{l_{x-1}}{l_{x+1}}\right)$$

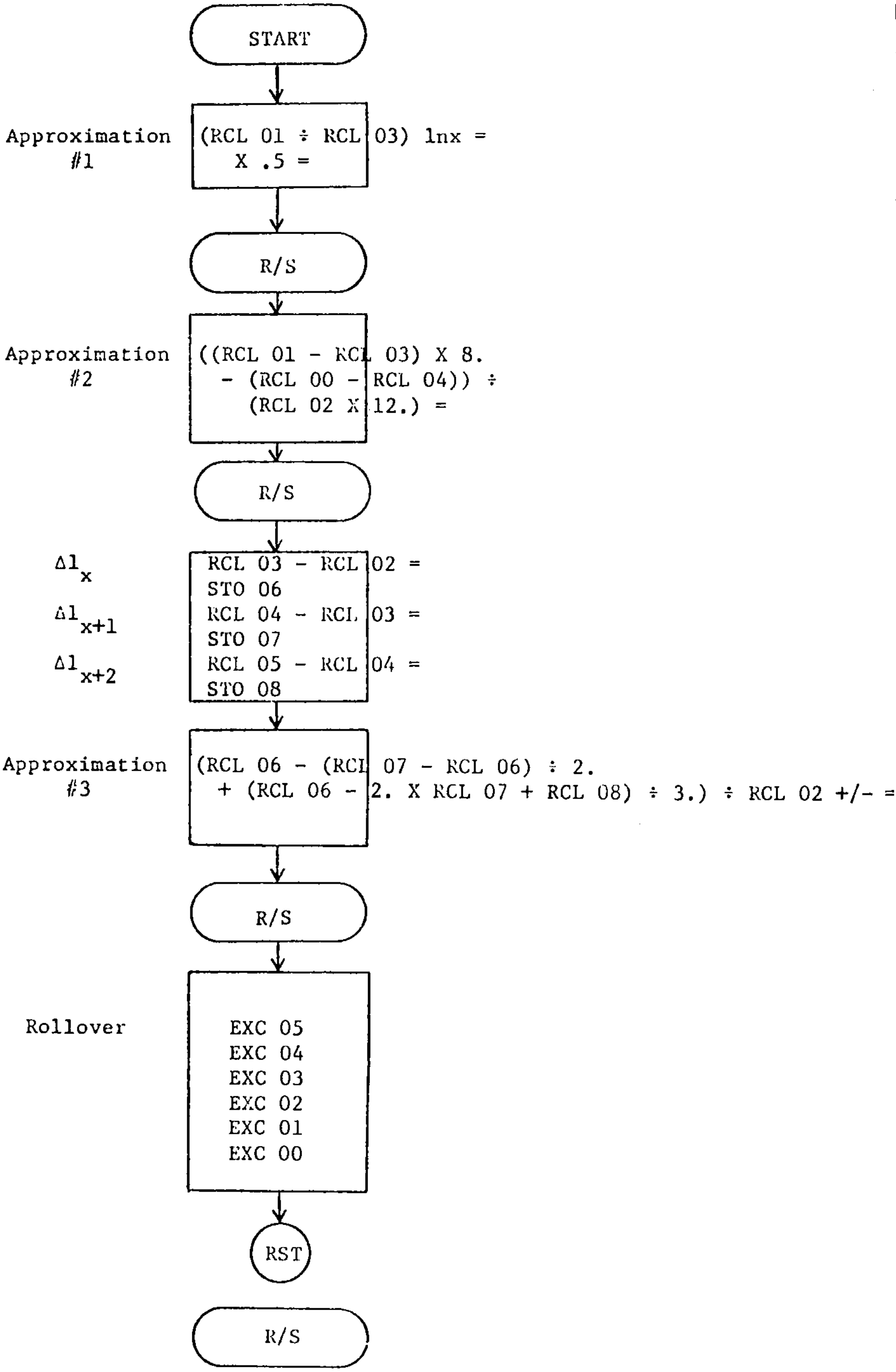
Approximation 2:

$$\mu_x = \frac{8(l_{x-1} - l_{x+1}) - (l_{x-2} - l_{x+2})}{12 \cdot l_x}$$

Approximation 3:

$$\mu_x = \frac{-1}{l_x}\left(\Delta l_x - \frac{1}{2}\Delta^2 l_x + \frac{1}{3}\Delta^3 l_x - \ldots\right)$$

Register Usage		Operating Instructions			
nn	R(nn)	Push	Enter	Push	Display
00	l_{x-2}	RST CMS	l_{x-2}	STO 00	l_{x-2}
01	l_{x-1}		l_{x-1}	STO 01	l_{x-1}
02	l_x		l_x	STO 02	l_x
03	l_{x+1}		l_{x+1}	STO 03	l_{x+1}
04	l_{x+2}		l_{x+2}	STO 04	l_{x+2}
05	l_{x+3}		l_{x+3}	STO 05	l_{x+3}
06	Δl_x			R/S	μ_x #1
07	Δl_{x+1}			R/S	μ_x #2
08	Δl_{x+2}			R/S	μ_x #3
			new l_{x+3}	R/S	μ_x #1
				etc.	


START
Approximation #1
(RCL 01 ÷ RCL 03) lnx =
X .5 =
R/S
Approximation #2
((RCL 01 - RCL 03) X 8.
- (RCL 00 - RCL 04)) ÷
(RCL 02 X 12.) =
R/S
Δl x
Δl x+1
Δl x+2
RCL 03 - RCL 02 =
STO 06
RCL 04 - RCL 03 =
STO 07
RCL 05 - RCL 04 =
STO 08
Approximation #3
(RCL 06 - (RCL 07 - RCL 06) ÷ 2.
+ (RCL 06 - 2. X RCL 07 + RCL 08) ÷ 3.) ÷ RCL 02 +/- =
R/S
Rollover
EXC 05
EXC 04
EXC 03
EXC 02
EXC 01
EXC 00
RST
R/S

Force of Mortality Approximations

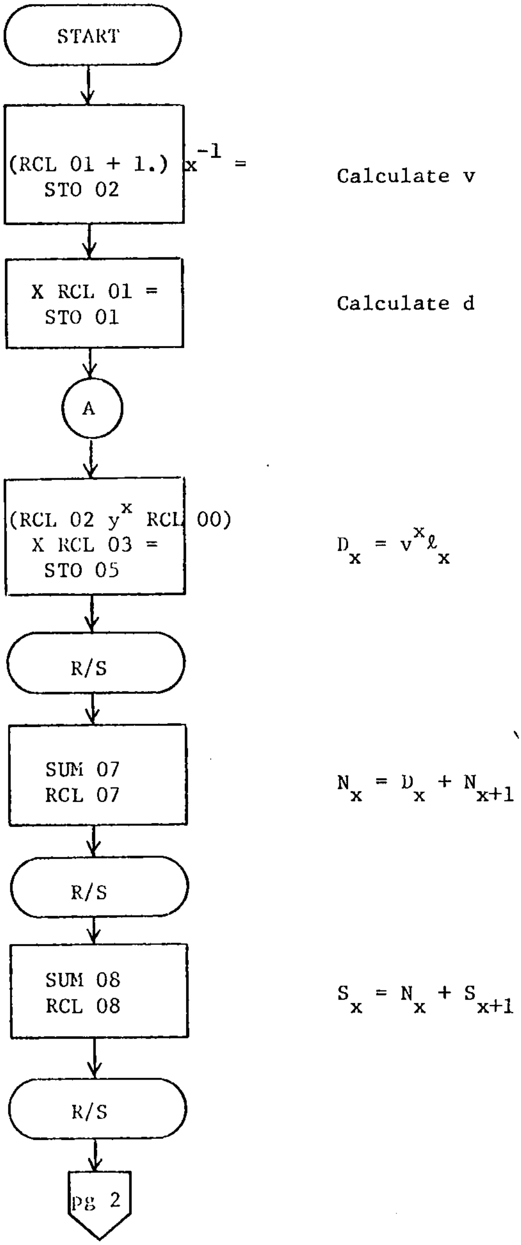
μ_x (TI 59 Listing)

LOC	CODE	KEY	LOC	CODE	KEY	LOC	CODE	KEY
000	53	(	040	01	1	080	54	)
001	43	RCL	041	02	2	081	55	÷
002	01	01	042	93	:	082	02	2
003	55	÷	043	54	)	083	93	:
004	43	RCL	044	95	=	084	85	+
005	03	03	045	91	R/S	085	53	(
006	54	)	046	43	RCL	086	43	RCL
007	95	=	047	03	03	087	06	06
008	23	LNX	048	75	-	088	75	-
009	95	=	049	43	RCL	089	02	2
010	65	x	050	02	02	090	93	:
011	93	:	051	95	=	091	65	x
012	05	5	052	42	STO	092	43	RCL
013	95	=	053	06	06	093	07	07
014	91	R/S	054	43	RCL	094	85	+
015	53	(	055	04	04	095	43	RCL
016	53	(	056	75	-	096	08	08
017	43	RCL	057	43	RCL	097	54	)
018	01	01	058	03	03	098	55	÷
019	75	-	059	95	=	099	03	3
020	43	RCL	060	42	STO	100	93	:
021	03	03	061	07	07	101	54	)
022	54	)	062	43	RCL	102	55	÷
023	65	x	063	05	05	103	43	RCL
024	08	8	064	75	-	104	02	02
025	93	:	065	43	RCL	105	94	+/-
026	75	-	066	04	04	106	95	=
027	53	(	067	95	=	107	91	R/S
028	43	RCL	068	42	STO	108	48	EXC
029	00	00	069	08	08	109	05	05
030	75	-	070	53	(	110	48	EXC
031	43	RCL	071	43	RCL	111	04	04
032	04	04	072	06	06	112	48	EXC
033	54	(	073	75	-	113	03	03
034	54	)	074	53	(	114	48	EXC
035	55	÷	075	43	RCL	115	02	02
036	53	(	076	07	07	116	48	EXC
037	43	RCL	077	75	-	117	01	01
038	02	02	078	43	RCL	118	48	EXC
039	65	x	079	06	06	119	00	00
						120	81	RST
						121	91	R/S

Commutation Functions
D_x, N_x, S_x, C_x, M_x, R_x

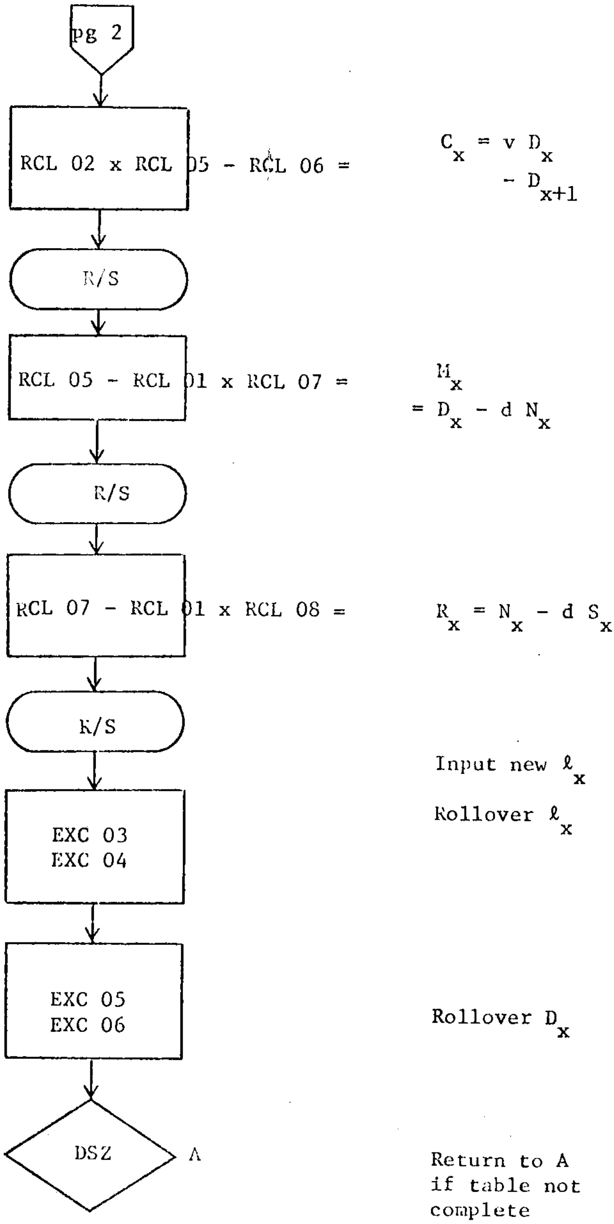
Register Allocation

nn	R(nn)
00	w - 1
01	i, d
02	v
03	ℓ_x
04	ℓ_{x+1}
05	D_x
06	D_{x+1}
07	N_x
08	S_x



Commutation Functions (Page 2)

D_x, N_x, S_x, C_x, M_x, R_x



Commutation Functions
Operating Instructions

Push	Enter	Push	Display
CMS RST	w - 1	STO 00	w - 1
	i	STO 01	i
	ℓ_{w-1}	STO 03	ℓ_{w-1}
		R/S	D_{w-1}
		R/S	N_{w-1}
		R/S	S_{w-1}
		R/S	C_{w-1}
		R/S	M_{w-1}
		R/S	R_{w-1}
	CLR ℓ_{w-2}	R/S	D_{w-2}
		etc.	

Commutation Functions
Listing (TI 59)

Loc	Code	Key
000	53	(
001	43	RCL
002	01	01
003	85	+
004	01	1
005	93	:
006	54	)
007	35	1/X
008	95	=
009	42	STO
010	02	02
011	65	X
012	43	RCL
013	01	01
014	95	=
015	42	STO
016	01	01
017	53	(
018	43	RCL
019	02	02
020	45	Y^X
021	43	RCL
022	00	00
023	54	)
024	65	x
025	43	RCL
026	03	03
027	95	=
028	42	STO
029	05	05
030	91	R/S
031	44	SUM
032	07	07
033	43	RCL
034	07	07
035	91	R/S
036	44	SUM
037	08	08
038	43	RCL
039	08	08
040	91	R/S
041	43	RCL
042	02	02
043	65	x
044	43	RCL
045	05	05
046	75	-
047	43	RCL
048	06	06
049	95	=
050	91	R/S
051	43	RCL
052	05	05
053	75	-
054	43	RCL
055	01	01
056	65	x
057	43	RCL
058	07	07
059	95	=
060	91	R/S
061	43	RCL
062	07	07
063	75	-
064	43	RCL
065	01	01
066	65	x
067	43	RCL
068	08	08
069	95	=
070	91	R/S
071	48	EXC
072	03	03
073	48	EXC
074	04	04
075	48	EXC
076	05	05
077	48	EXC
078	06	06
079	97	DSZ
080	00	00
081	00	00
082	17	17
083	91	R/S

Commutation Function Table Example

1958 CSO 3% Male Lives

x	l_x	D_x	N_x	S_x	C_x	M_x	R_x
95	97,165	5,861.0	13,112.1	25,677.3	1,998.652	5,479.119	12,364.196
96	63,037	3,691.7	7,251.1	12,565.2	1,435.657	3,480.467	6,885.077
97	37,787	2,148.5	3,559.4	5,314.1	1,018.801	2,044.810	3,404.610
98	19,331	1,067.1	1,410.9	1,754.7	692.218	1,026.009	1,359.800
99	6,415	343.8	343.8	343.8	333.791	333.791	333.791

Stochastic Life Table Model*

The probability distributions of the main biometric functions are useful in calculating the expected value, variance, and covariance of derived biometric functions, e.g., life expectancy or commutations functions. The distributions of some of the basic functions are:

$\ell_i | \ell_o \sim$ binomial

$$f(\ell_i = k | \ell_o) = \binom{\ell_o}{k} P_{oi}^{K} (1-P_{oc})^{\ell_o - k}$$

for $k = 0, 1, \ldots, \ell_o$
$i = 0, 1, \ldots, W$

$I_i | \ell_o \sim$ binomial

$$f(d_i = k | \ell_o) = \binom{\ell_o}{k} (P_{oi} q_i)^k (1 - P_{oi} q_i)^{\ell_o - k}$$

for $k = 0, 1, \ldots, \ell_o$
$i = 0, 1, \ldots, w-1$

*Chin Long Chiang, "A Stochastic Study of the Life Table and Its Applications: I. Probability Distributions of the Biometric Functions" Biometrics (Dec., 1960) pp. 618-635.

Binomial Distribution
(for Stochastic Life Table)*

$$f(k;N,p) = \binom{N}{k} p^{k} (1 - p)^{N-k} \qquad k = 0, \ldots, N$$

$$P(k) = \sum_{x=0}^{k} f(x;N,p)$$

Algorithm

$$f(0) = (1 - p)^{N}$$

$$f(i + 1) = f(i) \frac{p}{1 - p} \times \frac{N - i}{i + 1} \qquad i = 0, \ldots, N - 1$$

$$P(0) = f(0)$$

$$P(i + 1) = P(i) + f(i + 1)$$

Storage Allocation

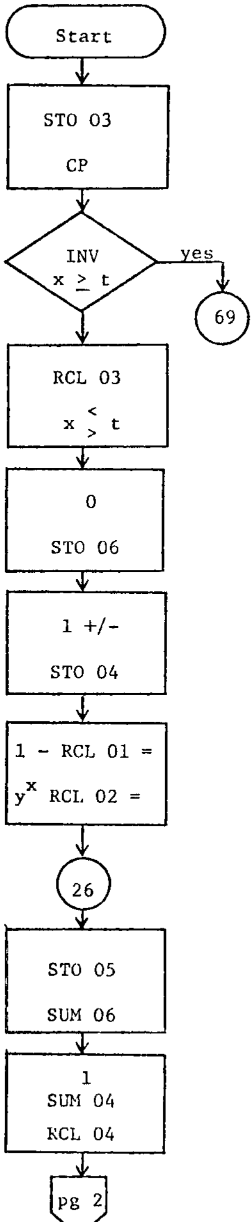
nn	R(nn)
1	P
2	N
3	k
4	i
5	f(k;N,p)
6	P(k;N,p)

Operating Instructions

Push	Enter	Push	Display
CMS RST	p	STO 01	p
	N	STO 02	N
	k	R/S	f(k;N,p)
		R/S	P(k;N,p)
	1	RST R/S	f(1)
		R/S	P(1)
	2	RST R/S	f(2)
		R/S	P(2)
		etc.	

*Algorithm adapted from TI Programmable Slide-Rule Calculator SR-56 Applications Library, Dallas TI, 1976, pp. 52-55.

Binomial Distribution
(for Stochastic Life Table)



Set R(3) = k

Zero out t register

If k < 0 go to 69
for output

Set x = k

exchange x and t register

Zero out P(k)

accumulator

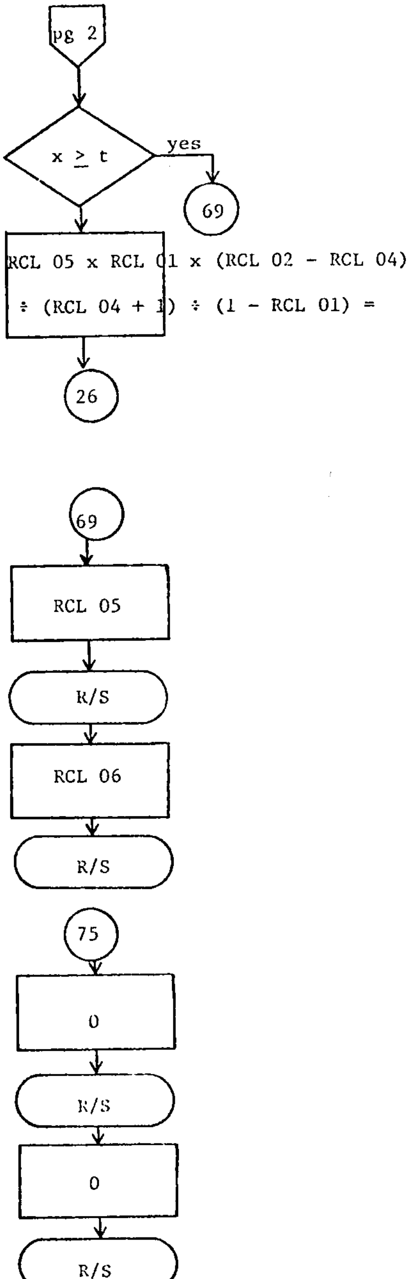
Set counter i to -1

$f(0) = (1 - P)^N$

Set R(5) = f

Add f to P

i = i + 1



Ready for output?

$$f(i = 1) = f(i) \frac{p}{1 - p} \frac{N - i}{i + 1}$$

Normal output segment

Display f(k)

Display P(k)

Output when $k < 0$

$f(k) = 0$

$P(k) = 0$

Variance of the Expectation of Life*

$$\sigma^2_{e_{x_i}} = p^2_{x_i} \cdot \sigma^2_{e_{x_{i+1}}} + \left[e_{x_{i+1}} + (1 - a_{x_i}) \cdot n_{x_i}\right]^2 \cdot \sigma^2_{p_{x_i}}$$

where

e_{x_i} = the expectation of life at age x_i

$\sigma^2_{e_{x_i}}$ = the variance of the expectation of life at age x_i

p_{x_i} = probability of surviving from age x_i to age x_{i+1}

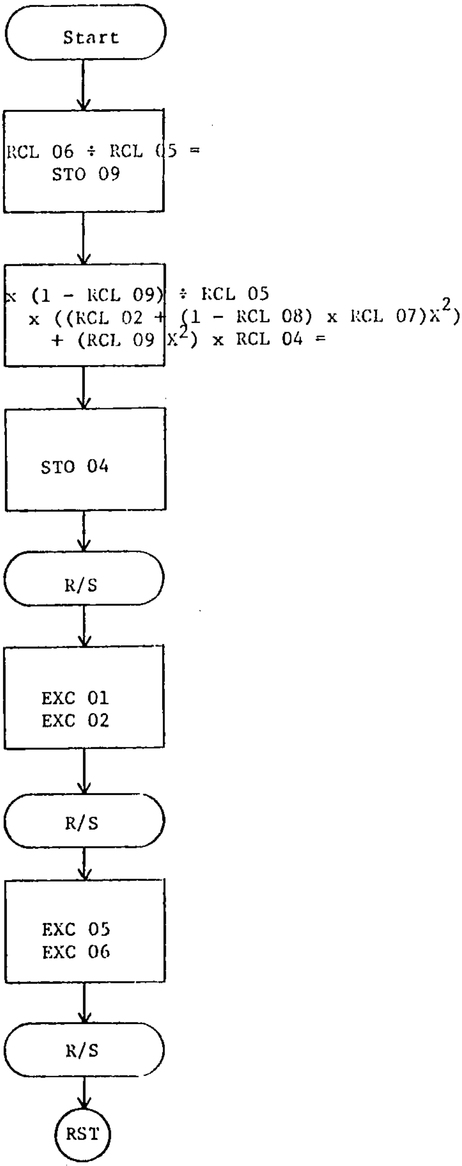
$X + a_{x_i} \cdot n_{x_i}$ = average age of those who die between age x_i and x_{i+1}.

n_{x_i}: Number of years in intervals between ages x_i and x_{i+1}

a_{x_i}: Fraction of years lived between x_i and x_{i+1}

*Adapted from Chin Long Chiang, "A Stochastic Study of the Life Table and Its Applications: I Probability Distributions of the Biometric Functions," Biometrics (Dec., 1960), pp. 618-635.

Variance of the Expectation of Life
Flowchart



$$p_{x_i} = \frac{\ell_{x_{i+1}}}{\ell_{x_i}}$$

$$\sigma^2_{e_{x_i}} = \left[e_{x_{i+1}} + (1 - a_{x_i}) \cdot n_{x_i}\right]^2 \cdot \sigma^2_{p_{x_i}} + p^2_{x_i} \cdot \sigma^2_{e_{x_{i+1}}}$$

Input new e_{x_i} and rollover

Input new ℓ_{x_i} and rollover

Variance of the Expectation of Life

Storage Allocation

nn	R(nn)
1	e_{x_i}
2	$e_{x_{i+1}}$
3	
4	$\sigma^2_{x_{i+1}}, \sigma^2_{x_i}$
5	l_{x_i}
6	$l_{x_{i+1}}$
7	n_{x_i}
8	a_{x_i}
9	p_{x_i}

Operating Instructions

Push	Enter	Push	Display
RST CMS	e_{w-1}	STO 01	e_{w-1}
	l_{w-1}	STO 05	l_{w-1}
	1	STO 07	1
	.5	STO 08	.5
		R/S	$\sigma^2_{e_{w-1}}$
	e_{w-2}	R/S	e_w
	ℓ_{w-2}	R/S	l_w
		R/S	$\sigma^2_{e_{w-2}}$
	etc.		

Example: Variance of the Expectation of Life

DAYS $x_i - x_{i+1}$	l_{x_i}	e_{x_i}	$\sigma_{e_{x_i}}$
60 and over	13	2.50	----
55-60	34	4.41	.417
50-55	76	5.59	.436
45-50	130	7.23	.433

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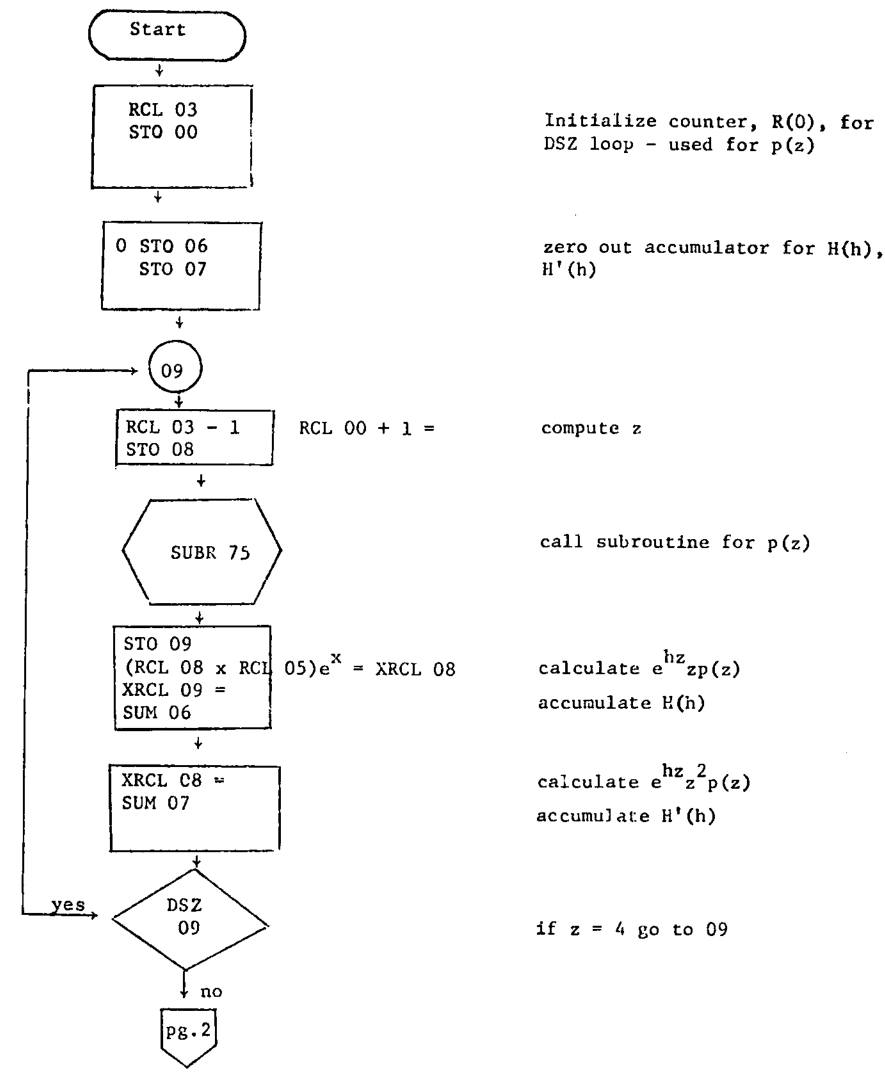
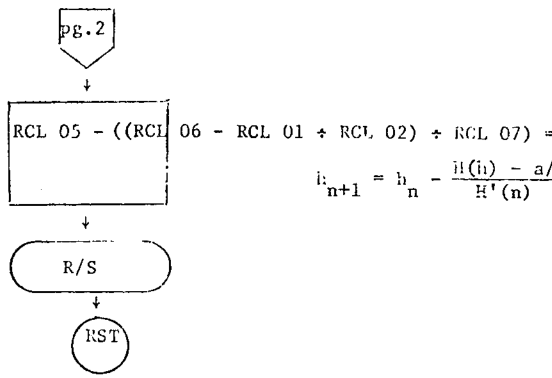
life table for adult male Dropsophila melangogasteo taken from Chin Long Chiang, "A Stochastic Study of the Life Table and its Applications: II. Sample Variance of the Observed Expectation of Life and Other Biometric Functions," Human Biology Vol. 32 No. 3 (September, 1960) p. 227.

* fruitfly

Newton's Method for Esscher's Approximation*



*Adapted from John C. Wooddy, Part 5 Study Notes on Risk Theory, 55.1.71 (1973 ed.), Chicago: Society of Actuaries, 1973.



$$h_{n+1} = h_n - \frac{H(h) - a/m}{H'(h)}$$

Newton's Method for Esscher Approximation

Storage	Allocation
nn	R(nn)
1	a
2	m
3	u
5	h_n
6	$H(h_n)$
7	$H'(h_n)$
8	z
9	p(z)

Newton's Method for Esscher Approximation

Operating Instructions

Push	Enter	Push	Display
RST CMS	a	STO 01	a
	m	STO 02	m
	u	STO 03	u
	h_o	STO 05	h_o
		R/S	h_1
		STO 05 RST R/S	h_2
		etc.	

Newton's Method for Esscher Approximation
(SR-56) Listing

LOC	CODE	KEY	LOC	CODE	KEY
00	34	RCL	25	34	RCL
01	03	03	26	08	08
02	33	STO	27	64	X
03	00	00	28	34	RCL
04	00	00	29	05	05
05	33	STO	30	53	)
06	06	06	31	14	e^x
07	33	STO	32	94	=
08	07	07	33	64	x
09	34	RCL	34	34	RCL
10	03	03	35	08	08
11	74	-	36	64	÷
12	34	RCL	37	34	RCL
13	00	00	38	09	09
14	84	+	39	94	=
15	01	01	40	35	SUM
16	94	=	41	06	06
17	33	STO	42	64	x
18	08	08	43	34	RCL
19	57	SUBR	44	08	08
20	07	07	45	94	=
21	05	05	46	35	SUM
22	33	STO	47	07	07
23	09	09	48	27	DSZ
24	52	(	49	00	00

LOC	CODE	KEY
50	09	09
51	34	RCL
52	05	05
53	74	–
54	52	(
55	52	(
56	34	RCL
57	06	06
58	74	–
59	34	RCL
60	01	01
61	54	+
62	34	RCL
63	02	02
64	53	)
65	54	+
66	34	RCL
67	07	07
68	53	)
69	94	=
70	41	R/S
71	42	RST
72	00	
73	00	
74	00	
75	52	(
76	52	(
77	34	RCL
78	08	08
79	64	x
80	02	2
81	53	)
82	54	÷
83	52	(
84	52	(
85	34	RCL
86	03	03
87	43	x^2
88	53	)
89	74	–
90	34	RCL
91	03	03
92	53	)
93	53	)
94	94	=
95	58	RTN
96	00	
97	00	
98	00	
99	00	

Sample Problem

z	p(z)
1	1/10
2	2/10
3	3/10
u = 4	4/10

n	h_n
0	0
1	.12
2	.1006
3	.09993
4	.09993

$a = 14$

$m = 2$

$h_o = 0$

$u = 4$

Capital Asset Pricing Model (CAPM)
Rating (Monoline)*

$K' = EP_x[1 - \bar{R}_p + t\,\hat{\sigma}_p]$

K' = level of risk capital required

EP = estimated premium

R_p = combined expense/loss ratio

t = safety factor (usually taken from t table)

σ_p = standard deviation of combined loss/expense ratio.

$I' = i_r' \times (1 - e_i) \times (1 - t_r) \times K'$

I' = net after tax earnings on risk capital invested in risk-free instrument

e_i = expense rate on investments

t_r = corporate tax rate.

$I = i_r' \times (R) \times (1 - e_i) \times (1 - t_r)$

I = net after tax earnings on premium-generated reserves (unearned premium reserve, loss reserves, etc.)

R = premium-generated reserves.

$E_I = p + b\sigma_i$

E_I = predicted % return of efficient portfolio with a given level of risk

$= .047 + .316\,\hat{\sigma}_i$ per Cooper (p 66).

Initially assume

$E_I = i_r' = .0475$

that is all funds are invested in risk-free securities.

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*Adapted from Robert W. Cooper, Investment Return and Property-Liability Insurance Ratemaking, Homewood, Ill.: Irwin, 1974.

$Y_p = E_I \times K'$

= predicted dollar return on K'.

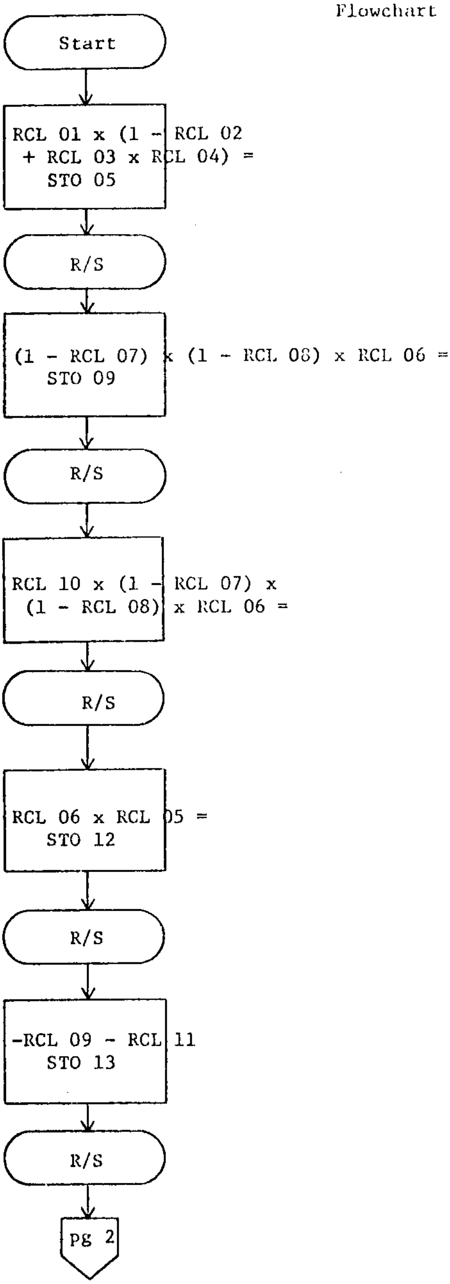
$U^* = Y_p - I - I'$

= underwriting profit

$u^* = U^*/EP$

u* underwriting profit as a % of EP.

CAPM Monoline Rating
Flowchart



$K' = EP \times [1 - \bar{R}_p + t \times \sigma_p]$
required capital

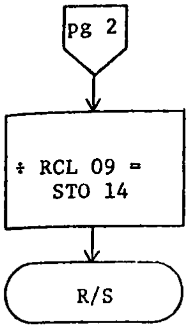
$I' = K' \times (1 - e_i) \times (1 - t_r) \times i'_r$
interest on required capital

$I = R \times (1 - e_i) \times 1 - t_r) \times i'_r$
interest on unearned premiums and other reserves

$Y_p = E_I \times K'$
CAPM required profit

$U^* = Y_p - I - I'$
CAPM required underwriting profit

CAPM Monoline Rating
Flowchart



u* = U*/EP
underwriting profit allowance

Storage Allocation

nn	R(nn)
1	EP
2	$\overline{R}_p$
3	t
4	σ_p
5	K^1
6	i'_r, E_I
7	e_i
8	t_r
9	I'
10	R
11	I
12	Y_p
13	U*
14	u*

Operating Instructions

Push	Enter	Push	Display
RST CMS	EP	STO 01	EP
	$\overline{R}_p$	STO 02	$\overline{R}_p$
	t	STO 03	t
	σe'	STO 04	σ_p
	i_r	STO 06	i'_r
	e_i	STO 07	e_i
	t_r	STO 08	t_r
	R	STO 10	R
		R/S	K^1
		R/S	I'
		R/S	I
		R/S	Y_p
		R/S	U*
		R/S	u*

Capital Asset Pricing Model (CAPM)
Monoline Rating

Listing (TI 59)

LOC	CODE	KEY	LOC	CODE	KEY
000	43	RCL	043	53	(
001	01	01	044	01	1
002	65	x	045	75	–
003	53	(	046	43	RCL
004	01	1	047	07	07
005	75	–	048	54	)
006	43	RCL	049	65	x
007	02	02	050	53	(
008	85	+	051	01	1
009	43	RCL	052	75	–
010	03	03	053	43	RCL
011	65	x	054	08	08
012	43	RCL	055	54	)
013	04	04	056	65	x
014	54	)	057	43	RCL
015	95	=	058	06	06
016	42	STO	059	95	=
017	05	05	060	42	STO
018	91	R/S	061	11	11
019	65	x	062	91	R/S
020	53	(	063	43	RCL
021	01	1	064	06	06
022	75	–	065	65	x
023	43	RCL	066	43	RCL
024	07	07	067	05	05
025	54	)	068	95	=
026	65	x	069	42	STO
027	53	(	070	12	12
028	01	1	071	91	R/S
029	75	–	072	75	–
030	43	RCL	073	43	RCL
031	08	08	074	09	09
032	54	)	075	75	–
033	65	x	076	43	RCL
034	43	RCL	077	11	11
035	06	06	078	95	=
036	95	=	079	42	STO
037	42	STO	080	13	13
038	09	09	081	91	R/S
039	91	R/S	082	55	÷
040	43	RCL	083	43	RCL
041	10	10	084	01	01
042	65	x	085	95	=
			086	42	STO
			087	14	14
			088	91	R/S

Assumptions for CAPM
Analysis

$EP = \$738.6M$

$\overline{R}_p = 1.009$

$t = ?$

$\sigma_p = .026$

$i_r = .0475$

$e_i = .044$

$t_r = .48$

$R = \$821.2M$

$E_I = i'_r = .0475$

Example for CAPM
Monoline Rating

$K' = 50.961 \times 10^6$

$I' = 1.203 \times 10^6$

$I = 19.391 \times 10^6$

$Y_p = 2.42 \times 10^6$

$U^* = -18.174 \times 10^6$

$u = -.0246$