A PROOF OF THE SCHUETTE - NESBITT FORMULA FOR DEPENDENT EVENTS

Hans U. Gerber

Let $A_1, \ldots, A_n$ be certain events of interest, and let $P_{[k]}$ denote the probability that exactly k of these n events take place. Also let B_k be the sum, for all choices of k events out of the n, of the probabilities that k specified events will happen, irrespective of whether the other $n-k$ events occur. In their discussion of White and Greville's paper (TSA 11, 88-99) Schuette and Nesbitt prove the formula

$$P_{[o]} c_o + P_{[1]} c_1 + \ldots + P_{[n]} c_n = c_o + B_1 \Delta c_o + B_2 \Delta^2 c_o + \ldots + B_n \Delta^n c_o , \tag{1}$$

which is valid for arbitrary numbers $c_o, c_1, \ldots, c_n$. Their proof is only for the special case of independent events. The purpose of this note is to give a proof in the general case, and at the same time, to give some more publicity to this formula, which is useful in connection with some of the material covered in chapter 10 of Jordan.

For the proof, let X_i denote the indicator function of the event A_i, and let Y_k denote the indicator function of the event that exactly k of the n events $A_1, \ldots, A_n$ take place. We define

$$(2) \qquad \phi(E) = (X_1 E + 1 - X_1) \ldots (X_n E + 1 - X_n)$$

Note that $\phi(E)$ is a random - operator. After multiplying, we see that

$$(3) \qquad \phi(E) = Y_o + Y_1 E + \ldots + Y_n E^n .$$

If, on the other hand, we substitute $E = 1 + \Delta$ in (2) we see that

$$(4) \qquad \phi(E) = (1 + X_1 \Delta) \ldots (1 + X_n \Delta)$$

After multiplying and collecting coefficients of powers of Δ, we see that the expected value of $\phi(E)\, C_o$ is the right side in (1). From (3) we see that it equals the left side.

Q.E.D.