AN APPLICATION OF AN INTEREST RATE DIFFUSION MODEL TO THE VALUATION OF MORTGAGE RATE INSURANCE

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ABSTRACT

The federal government of Canada has introduced a program whereby a mortgage borrower can purchase insurance which gives protection against interest rate rises at mortgage renewal. A diffusion model of interest rates is applied to the valuation of mortgages, and the resulting partial differential equation is solved numerically. The boundary conditions of this problem necessitate a novel solution method which is likely to have applications in other areas. Estimates of the appropriate net premium are given.

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As a result of pressure from homeowners renewing mortgages at the high interest rates which prevailed in the early 1980s, the federal government of Canada introduced on March 1, 1984 the Mortgage Rate Protection Program. The Program is administered through Canada Mortgage and Housing Corporation, a federal agency. In Canada, residential mortgages are usually amortized over a 25 year period, but the mortgage is periodically renewed at prevailing interest rates. The inter-renewal period is usually 1, 2, 3 or 5 years.

Under the Mortgage Rate Protection Program, up to $70,000 of the principal at inception can be insured. For the rate rise a deductible of 2% per annum applies, and the portion of renewal mortgage payment resulting from an increase in rates of over 12% is not insured. The principal at the end of the insured period is calculated using the market rate rather than the rate reduced by the effect of the insurance. The insurance only pays 75% of the increase in monthly payments resulting from a rate rise.

As an example[1] of the operation of the insurance, consider the case of an insured mortgage with an amortization period of 25 years, when mortgage rates have risen 5% in the 5 year period from the date of mortgage inception and insurance purchase:

Original terms of mortgage:

Fixed interest term	5 years
Interest rate convertible semi-annually	12.5% p.a.
Principal	$50,000
Insurance single premium (1.5%)	750
Monthly principal and interest payments	534
Outstanding principal at time 5 years	47,887

5% market rate increase at renewal after 5 years:

New interest rate	17.5%
New market monthly payment	$ 699
New monthly payment if increase by 2% deductible	598
Monthly insurance payout (.75×(699-598))	76
Net monthly payment by borrower	623
Principal outstanding at time 10 years	45,612

The single premium paid for the insurance is 1.5% of insured principal, regardless of the inter-renewal term. Thus the premium structure takes no account of either the longer period of potential payout under the longer inter-renewal periods, or the higher probability that substantial rate increases will have occurred over a longer inter-renewal period [9]. At mortgage renewal at time 5 years in the example it would be possible to again purchase the insurance to protect against higher monthly payments between times 10 years and 15 years. However, the insurance would then be based on the 17.5% per annum market rate which prevails at time 5 years.

In this paper a method is presented by which the value of the insurance can be calculated. Work on the valuation by a different method of other types of mortgages without insurance has been reported by Dunn and McConnell [4], while the valuation of bond options has been studied by Dietrich-Campbell and Schwartz [3]. Sharp [10] examines the $1\frac{1}{2}$% premium charged under the Mortgage Rate Protection Program in the light of the hypothetical historical payouts under the insurance had the Program been in effect since 1938, and notes that for the first 30 years of the period considered no payouts would have occurred because of the relative stability of interest rates in the period 1938-1967.

Section I of this paper considers the partial differential equation which results from the application of an interest rate diffusion model. Section II deals with the transformation of the equation to a form suitable for numerical analysis and presents estimates of the volatility of rates in various historical periods. Section III presents the method of numerical solution of the partial differential equation, while section IV discusses the treatment of the boundary conditions. The numerical results are presented in Section V. Section VI contains some additional comments on the results.

I. Diffusion Interest Rate Model Applied to Mortgages

$r_i(t)$, the instantaneous expected rate of return at time t on an investment in a mortgage is assumed by analogy with the model of Brennan and Schwartz [2] to satisfy

$$dr_i = \beta(r_i)dt + \eta(r_i)dz \tag{1}$$

where dz is a Wiener process with $E(dz) = 0$ and $E(dz^2) = dt$. The rate $r_i(t)$ is measured as a force of interest. The functions $\beta(r_i)$ and $\eta(r_i)$ correspond with the drift and the volatility of r_i. Then by Ito's Lemma ([7], p. 81) the market value $V(r_i,t)$ of a mortgage with principal at inception \$1 satisfies

$$\frac{dV}{V} = \frac{1}{V}\left(\beta(r_i)\frac{\partial V}{\partial r_i} + \frac{1}{2}\eta^2(r_i)\frac{\partial^2 V}{\partial r_i^2} + \frac{\partial V}{\partial t}\right)dt + \frac{1}{V}\eta(r_i)\frac{\partial V}{\partial r_i}dz \tag{2}$$

It should be noted that for the purposes of this model it is assumed that the mortgage value V is a function only of the two variables r_i and t. Thus other variables such as the long rate of interest and the rate of inflation are assumed to affect $V(r_i,t)$ only to the extent that they are reflected in $r_i(t)$.

The monthly payments by the mortgage borrower are approximated as being paid continuously at a rate c per annum per dollar of initial principal. Measuring time t from mortgage inception and denoting by $r_c(t)$ the market rate at inception of a mortgage of inter-renewal period m years and amortization period n years, that is $r_c(t)$ is the rate used in calculation of the amount of periodic payments, we have

$$c = \frac{1}{\bar{a}_{\overline{n}|}(r_c(t))} \tag{3}$$

Then since $E(dz) = 0$ the coefficient of dt in equation (2) can be equated to the expected instantaneous rate r_i after adjustment for the continuous payments:

$$\frac{1}{2}\eta^2(r_i)\frac{\partial^2 V}{\partial r_i^2} + \beta(r_i)\frac{\partial V}{\partial r_i} + \frac{\partial V}{\partial t} - Vr_i + c = 0. \tag{4}$$

The boundary conditions on (4) are important. As r_i approaches infinity the discounted value of the payment streams must approach 0, hence

$$\lim_{r_i \to \infty} V(r_i,t) = 0 \quad r_i \geq 0, \quad 0 \neq t \neq m. \tag{5}$$

In order that the process (1) be constrained to produce only non-negative rates $r_i(t)$, it is necessary that $\eta(0) = 0$. Thus by substituting $r_i = 0$ in equation (4) a natural boundary condition is obtained:

$$\beta(0)\frac{\partial V}{\partial r_i}(0,t) + \frac{\partial V}{\partial t}(0,t) + c = 0 \quad t \geq 0 \tag{6}$$

The value of the mortgage rate insurance will be obtained as the difference of the time 0 values of two mortgages. Mortgage A is not insured and is renewed at time m, when its market value must equal the outstanding principal:

$$V_A(r_i(m),m) = \frac{\bar{a}_{\overline{n-m}|}(r_c(0))}{\bar{a}_{\overline{n}|}(r_c(0))} \tag{7}$$

Mortgage B is insured, and for the purpose of calculation it is appropriate to assume that the mortgage lender is also the insurer. Thus the market value at inception of mortgage B will be less than that of mortgage A because if rates rise sufficiently between times 0 and m, the periodic mortgage payments between times m and $2m$ will be lower under mortgage B. Denoting by $\hat{r}_c$ the mortgage rate which under the insurance plan is the maximum which can be charged to the borrower for the period m to $2m$, we have at time m

$$V_B(r_i(m),m) = \frac{\bar{a}_{\overline{n-m}|}(r_c(0))}{\bar{a}_{\overline{n}|}(r_c(0))}\min(1,\frac{I(r_i(m))}{\bar{a}_{\overline{n-m}|}(\hat{r}_c)} + P(r_i(m))\frac{\bar{a}_{\overline{n-2m}|}(r_c(m))}{\bar{a}_{\overline{n-m}|}(r_c(m))}) \tag{8}$$

where $I(r_i(m))$ denotes the value at time m of continuous payments of $1 per annum between times m and $2m$ and $P(r_i(m))$ denotes the value at time m of $1 principal payable at time $2m$. It will be observed that this formula takes account of the fact that the insurance limits only the periodic payments and not the outstanding principal at time $2m$.

It is of interest to note that from the mortgage borrower's point of view the purchase of the insurance is somewhat analogous to the purchase of a put option. If under the insurance the principal outstanding at time $2m$ were limited to that which would be calculated using the inception rate plus the deductible then the analogy with a put option is straightforward. The security underlying the put option would be a contract to make a series of payments from time m to $2m$ and to pay the outstanding principal at time $2m$, all calculated using the inception rate of mortgage interest plus the deductible. The exercise price would equal the principal outstanding at time m. However, the principal outstanding at time $2m$ actually depends under the insurance on the market mortgage rate at time m, so no such simple analogy with a well-defined put option can be made.

II Preliminary Transformation and Data Analysis

In view of the difficulty in handling numerically the semi-infinite range of r_i, it is useful to use the transformation

$$u = \frac{1}{1+r_i} \tag{9}$$

and to define $v(u,t) = V(r_i,t)$, $\beta_*(u) = \beta(r_i)$ and $\eta_*(u) = \eta(r_i)$. Then the transformed version of the partial differential equation (4) for the mortgage value v is

$$\frac{1}{2}u^4\eta_*^2(u)\frac{\partial^2 v}{\partial u^2} + (-u^2\beta_*(u) + u^3\eta_*^2(u))\frac{\partial v}{\partial u} - \frac{(1-u)}{u}v + c + \frac{\partial v}{\partial t} = 0 \tag{10}$$

The boundary conditions (5) and (6) apply to both mortgage A and mortgage B and under the transformation they become respectively

$$v(0,t) = 0 \qquad 0 \neq t \neq m \tag{11}$$

$$\beta_*(1)\frac{\partial v}{\partial u}(1,t) + \frac{\partial v}{\partial t}(1,t) + c = 0 \qquad t \geq 0 \tag{12}$$

The boundary conditions (7) and (8) remain effectively unchanged if one identifies $V_A(r_i(m),m)$ with $v_A(u(m),m)$ and $V_B(r_i(m),m)$ with $v_B(u(m),m)$.

Following Brennan and Schwartz [2] the process (1) is simplified by making the specifications $\beta_*(u) = 0 = \beta(r)$ and $\eta(r_i) = \sigma r_i$, hence

$$\eta_*(u) = \sigma \frac{1-u}{u} \tag{13}$$

In practical situations η tends to dominate the drift β, and the chosen form for η reflects the intuition which is supported by the data that changes in rates will be on a proportional basis (see e.g. [3]).

The values found for the insurance depend initially on the rate volatility parameter σ, which has to be estimated from the data. The process (1) is continuous, but Ananthanarayanan [1] has shown that parameter estimates found by linearization correspond very closely to those found by analytic solution of the stochastic process. Therefore monthly data was used for mortgage rates[2], and denoting by $\Delta r_c(k) = r_c(k+1) - r_c(k)$ the month to month change in mortgage rates, σ was estimated over an n month period as

$$\sigma = \left[\frac{12}{n-1} \sum_{j=1}^{n} \left(\frac{\Delta r_c(k)}{r_c(k)} - \left(\overline{\frac{\Delta r_c}{r_c}} \right) \right)^2 \right]^{\frac{1}{2}} \tag{14}$$

It should be noted that the available data is for mortgage rates r_c rather than the expected instantaneous rate of return r_i, but as will be shown the difference between the two is so slight that there is no practical difference between the resulting values of σ.

Table 1 displays the calculated values of σ. In view of the monthly differencing, the value for 1956-1960 for instance is calculated using data from December 1955 to December 1960 inclusive. As would be expected, the values for the volatility σ are particularly high if periods which include the early 1980's are considered.

III Method of Numerical Solution

With the specializations of the stochastic processes (1) (equation (13)) the partial differential equation (10) becomes

$$\frac{1}{2}\sigma^2(1-u)^2u^2\frac{\partial^2 v}{\partial u^2} + \sigma^2(1-u)^2u\frac{\partial v}{\partial u} - \frac{(1-u)}{u}v + c + \frac{\partial v}{\partial t} = 0 \tag{15}$$

where c is given by equation (3) and the boundary conditions (7), (8), (11) and (12) apply. Boundary condition (12) simplifies to

$$\frac{\partial v}{\partial t}(1,t) + c = 0 \qquad t \geq 0 \tag{16}$$

Thus the values $v(1,t) = 1-ct$ immediately result for the mortgage value at a zero interest rate r_i, and over the range of t and c considered no negative values of v will be encountered. Use is made of an interval h for the u co-ordinate and g for the t co-ordinate and of the notation $v(u,t) = v(ih,jg) = v_{ij}$. The finite difference approximations

$$\frac{\partial v_{ij}}{\partial u} = \frac{v_{i+1,j}-v_{i-1,j}}{2h} + O(h^2) \tag{17}$$

$$\frac{\partial^2 v_{ij}}{\partial u^2} = \frac{v_{i+1,j}-2v_{i,j}+v_{i-1,j}}{h^2} + O(h^2) \tag{18}$$

$$\frac{\partial v_{ij}}{\partial t} = \frac{v_{i,j+1}-v_{i,j}}{g} + O(g) \tag{19}$$

are used. It will be noticed that since the calculation proceeds from the known boundary conditions at, for instance, $t = m$ towards the desired value of v at $t = 0$, the expression (19) leads to an implicit form for the difference equation. The implicit form is preferable to the explicit form in view of its better stability ([6], p. 176).

The finite difference approximation to equation (15) can then be expressed as

$$\begin{pmatrix} e_1 & f_1 & 0 & \cdots & 0 & 0 & 0 \\ d_2 & e_2 & f_2 & \cdots & 0 & 0 & 0 \\ 0 & d_3 & e_3 & \cdots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & & d_{p-1} & e_{p-1} & f_{p-1} \\ 0 & 0 & 0 & \cdots & 0 & d_p & e_p \end{pmatrix} \begin{pmatrix} v_{1,j} \\ v_{2,j} \\ v_{3,j} \\ \cdot \\ \cdot \\ \cdot \\ v_{p-1,j} \\ v_{p,j} \end{pmatrix} = \begin{pmatrix} v_{1,j+1} \\ v_{2,j+1} \\ v_{3,j+1} \\ \cdot \\ \cdot \\ \cdot \\ v_{p-1,j+1} \\ v_{p,j+1} \end{pmatrix} + \begin{pmatrix} cg \\ cg \\ cg \\ \cdot \\ \cdot \\ \cdot \\ cg \\ cg \end{pmatrix} \tag{20}$$

where $ph = 1$ and

$$d_i = \frac{g}{2} i(1-ih)^2\sigma^2(1-i) \qquad i = 2,3,4,\ldots,p \tag{21}$$

$$e_i = 1 + g\frac{(1-ih)}{ih} + gi^2(1-ih)^2\sigma^2 \qquad i = 1,2,3,\ldots,p \tag{22}$$

$$f_i = -\frac{g}{2} i(1-ih)^2\sigma^2(1+i) \qquad i = 1,2,3,\ldots,p. \tag{23}$$

At zero interest rates, $i = p$, one has $d_p = f_p = 0$ and $e_p = 1$, so the pth row of equation (20) corresponds to the equation (16). At infinite interest rates one has by equation (11) a zero value for v so there is no necessity to include a row in equation (20) corresponding to $i = 0$.

The tridiagonal system (20) can be solved by standard techniques ([8], p. 32) to give the value at time 0 of the uninsured mortgage A. The starting point of the iteration is at time m, when for a given mortgage rate $r_c(0)$ the boundary condition is given by equation (7). For the insured mortgage B the boundary condition (8) requires special treatment, and this is discussed below.

IV Treatment of Boundary Condition

Boundary condition (8) for the insured mortgage A presents novel problems since for a given mortgage rate $r_c(0)$ the mortgage rate $r_c(m)$ at renewal at time m is not known. Thus the attempt to use the tridiagonal system (20) to iterate back in time from $t = m$ to $t = 0$ is hindered by lack of a set of values of $v(u,m)$. This problem is circumvented through consideration of equation (8). For the period $m \leq t \leq 2m$ the tridiagonal system (20) is solved to give values at $t = m$ of $I(r_i(m))$, (unit income) and $P(r_i(m))$ (unit principal at $t = 2m$). This is accomplished by suitable modification of the value of c in equations (16) and (20) and by requiring at $t = 2m$ that $v_I(u,2m) = 0$ for the values of the unit income and $v_p(u,2m) = 1$ for the value of the unit principal. Then the equation

$$\frac{1}{a_{\overline{n-m}|}(r_c(m))}(I(r_i(m)) + P(r_i(m))a_{\overline{n-2m}|}(r_c(m))) = 1 \tag{24}$$

is for every $r_i(m)$ under the finite difference scheme for u solved through the method of successive bisection ([5], p. 258) to give a mortgage rate value $r_c(m)$. In other words, for every possible expected instantaneous rate of return at time m, $r_i(m)$, there is found the corresponding mortgage rate $r_c(m)$. The two values differ only slightly. For example, in the case of the volatility $\sigma = 0.099085$ found for the period 1961-1985 an uninsured mortgage with an inter-renewal period $m = 5$ years and amortization period $n = 25$ years being renewed 5 years after inception when $r_i(5) = 0.10092$ (i.e. 10.092% per annum continuous) would, in order for the market to value the mortgage at the amount of the outstanding principal, be required to yield an income calculated using a mortgage rate $r_c(5) = 0.10055$ (i.e. 10.055% per annum continuous).

Having thus calculated the values $r_e(m)$ corresponding to every $r_i(m)$ considered under the differencing scheme, it is possible to use equation (8) to find the value at time m of the insured mortgage B. The value is the minimum of the outstanding principal and the market value at each $r_i(m)$ of the future principal at time $2m$ and the income payments from time m to $2m$. The income payments are "capped" by the application of the insurance so that the maximum mortgage rate $r_e(m)$ which is used to calculate the income payable under the mortgage is

$$\hat{r}_e = 2\ln(\exp(\frac{r_e(0)}{2} + .01)) \tag{25}$$

In equation (25) account has been taken of the fact that under the Mortgage Rate Protection Program the 2% per annum deductible is calculated on a semi-annually compounded basis, while $r_e(t)$ is measured as a force of interest. The maximum of 12% in the covered rise in rates is not taken into account in view of the very low probability for appropriate values of σ that the process (1) will over the relevant period give such an increase.

Thus boundary condition (8) for the insured mortgage B can be put into effect, and for a given mortgage rate at inception $r_e(0)$ the market value at inception $v_B(u,0)$ can be found through solution of the tridiagonal system (20).

It is suggested that this method of treatment of the boundary conditions of this problem may have applications in the valuation of other types of bond and mortgage.

V Results

Runs of the program were made for an amortization period of 25 years and inter-renewal periods m of 1, 2, 3 and 5 years. The interest rate scale $0 \leq u \leq 1$ was discretized into 120 intervals i.e. $h = 1/120$. After some testing, a time interval was chosen of $k = m/1280$. Results were produced for mortgage rates at inception $r_c(0)$ of 0.05 (i.e. 5% per annum continuous), 0.10 and 0.15 and for a range of volatility values σ (on an annual basis) corresponding to those found for various historical periods.

Table 2 gives the values of the initial instantaneous expected rate $r_i(0)$ at which for the given mortgage rate $r_c(0)$ the initial market value of the uninsured mortgage A is 1. The values of $r_i(0)$ were found from the 120 values of $V_A(r_i(0),0)$ produced by the program. The solution $r_i(0)$ of $V_A(r_i(0),0) = 1$ was found by the method of inverse interpolation with unequal intervals ([5], p. 116). It can be seen that the correspondence between $r_i(0)$ and $r_c(0)$ is, as might be expected, fairly close. The correspondence is less close for large values of m and the volatility σ.

Table 3 gives the main results of the analysis, i.e. the net single premium value of the insurance at inception. The value of the insurance is calculated for each $r_i(0)$ as the difference between the values of the uninsured mortgage A and insured mortgage B. Then using interpolation of the logarithm the insurance value corresponding to the $r_i(0)$ of Table 2 was derived. The Mortgage Rate Protection Program single premium of 1.5% of principal to cover 75% of the mortgage corresponds to a premium per principal incurred of 0.015/0.75 = 0.02. This value can be compared with those presented in Table 3.

VI Conclusions

The values of volatility σ used in producing Table 3 are intended to represent the range likely to be chosen as representing the most likely future volatility of interest rates. The value $\sigma = 0.187164$ found for 1979-1983 was the highest found for any five year period, though certainly higher values were observed for some shorter periods. It will be seen that none of the insurance values of Table 3 is as high as the gross premium of 0.02 charged under the Mortgage Rate Protection Program.

Taking into account the probable risk aversion of the mortgage borrower, the purchase of the insurance could be appropriate in the case of an inter-renewal period of 3 or 5 years if a period of high interest rate volatility is anticipated. In general, however, the Mortgage Rate Protection Program appears to be over-priced.

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FOOTNOTES

1. The example is adapted from the booklet "Mortgage Rate Protection Program", document number NHA 5813 84/08, Canada Mortgage and Housing Corporation, Ottawa.

2. The source of the mortgage rate data was the Statistics Canada CANSIM B14024 series - "Conventional Mortgage Lending Rate".

TABLE 1

Values of mortgage rate volatility σ found for selected historical periods

Period	σ
1956-1960	0.032287
1961-1965	0.026201
1966-1970	0.041347
1971-1975	0.072745
1976-1980	0.149146
1981-1985	0.138046
1979-1983	0.187164
1976-1985	0.144068
1961-1985	0.099085
1952-1985	0.086539

TABLE 2

EXPECTED INSTANTANEOUS RETURN AT INCEPTION, $r_i(0)$

MORTGAGE RATE ($r_c(0)$)	INTER-RENEWAL PERIOD (m)	$r_i(0)$ for given σ				
		$\sigma = 0.026201$ (1961-1965)	$\sigma = 0.072745$ (1971-1975)	$\sigma = 0.099085$ (1961-1985)	$\sigma = 0.144068$ (1976-1985)	$\sigma = 0.187164$ (1979-1983)
0.05	1	0.05000	0.05000	0.05000	0.05001	0.05001
	2	0.05000	0.05001	0.05002	0.05003	0.05006
	3	0.05000	0.05002	0.05003	0.05007	0.05013
	5	0.05000	0.05005	0.05009	0.05020	0.05034
0.10	1	0.10000	0.10001	0.10002	0.10003	0.10006
	2	0.10000	0.10003	0.10006	0.10013	0.10022
	3	0.10001	0.10007	0.10014	0.10029	0.10049
	5	0.10002	0.10019	0.10035	0.10076	0.10130
0.15	1	0.15000	0.15002	0.15004	0.15008	0.15013
	2	0.15000	0.15007	0.15014	0.15029	0.15050
	3	0.15002	0.15016	0.15030	0.15063	0.15107
	5	0.15005	0.15040	0.15075	0.15161	0.15275

TABLE 3

INSURANCE VALUE AT INCEPTION

MORTGAGE RATE ($r_c(0)$)	INTER-RENEWAL PERIOD (m)	INSURANCE VALUE FOR GIVEN σ				
		σ = 0.026201	σ = 0.072745	σ = 0.099085	σ = 0.144068	σ = 0.187164
		(1961-1965)	(1971-1975)	(1961-1985)	(1976-1985)	(1979-1983)
0.05	1	0.00000	0.00000	0.00001	0.00005	0.00015
	2	0.00000	0.00002	0.00008	0.00037	0.00096
	3	0.00000	0.00007	0.00025	0.00101	0.00229
	5	0.00000	0.00025	0.00082	0.00273	0.00518
0.10	1	0.00000	0.00006	0.00018	0.00067	0.00142
	2	0.00001	0.00035	0.00100	0.00285	0.00510
	3	0.00002	0.00086	0.00221	0.00542	0.00881
	5	0.00005	0.00209	0.00451	0.00908	0.01300
0.15	1	0.00000	0.00025	0.00071	0.00197	0.00348
	2	0.00003	0.00122	0.00280	0.00613	0.00950
	3	0.00007	0.00246	0.00498	0.00964	0.01385
	5	0.00019	0.00428	0.00748	0.01233	0.01576

CANADIAN MORTGAGE RATES 1951-1986

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1951	5.00	5.00	5.00	5.25	5.50	5.50	5.62	5.62	5.75	5.75	5.75	5.75
1952	5.70	5.70	5.70	5.70	5.80	5.80	5.85	5.85	5.75	5.80	5.80	5.80
1953	5.90	5.90	5.90	5.90	5.90	5.95	5.95	5.95	6.05	6.05	6.10	6.10
1954	6.05	6.05	6.05	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
1955	6.00	6.00	6.00	6.00	5.75	5.75	5.75	5.70	5.80	5.90	5.95	5.95
1956	5.95	5.95	6.00	6.00	6.00	6.05	6.15	6.35	6.40	6.55	6.65	6.65
1957	6.70	6.75	6.75	6.75	6.75	6.85	6.85	6.90	7.00	7.00	7.00	6.95
1958	6.95	6.90	6.80	6.75	6.75	6.75	6.75	6.75	6.75	6.80	6.80	6.80
1959	6.85	6.85	6.85	6.80	6.80	6.85	6.85	6.95	7.20	7.20	7.25	7.25
1960	7.30	7.30	7.30	7.30	7.25	7.25	7.15	7.15	7.10	7.00	7.00	7.00
1961	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
1962	7.00	7.00	7.00	6.90	6.80	6.95	7.00	7.00	7.00	7.00	7.00	7.00
1963	7.00	7.00	7.00	6.94	6.91	6.91	6.91	7.00	7.00	7.00	7.00	7.00
1964	7.00	7.00	7.00	6.95	6.88	6.88	6.88	7.00	7.00	7.00	7.00	7.00
1965	6.90	6.85	6.82	6.82	6.83	6.83	7.02	7.13	7.15	7.25	7.29	7.40
1966	7.38	7.45	7.46	7.48	7.51	7.57	7.68	7.80	7.84	7.87	7.91	7.95
1967	7.93	7.89	7.83	7.80	7.77	7.88	8.02	8.05	8.10	8.49	8.52	8.52
1968	8.83	8.84	8.96	9.20	9.23	9.18	9.14	9.12	9.03	9.01	9.09	9.10
1969	9.45	9.45	9.48	9.52	9.46	9.69	9.90	9.99	10.11	10.21	10.30	10.50
1970	10.58	10.54	10.58	10.60	10.58	10.53	10.38	10.40	10.36	10.35	10.28	10.16
1971	9.94	9.72	9.28	9.20	9.25	9.34	9.46	9.53	9.55	9.55	9.26	9.10
1972	9.04	8.93	8.97	9.03	9.16	9.37	9.41	9.41	9.38	9.35	9.30	9.22
1973	9.09	9.02	9.07	9.15	9.30	9.52	9.71	9.91	10.13	10.13	10.08	10.02
1974	10.02	10.01	10.04	10.70	11.26	11.37	11.60	11.85	12.05	12.05	12.00	11.88
1975	11.81	10.95	10.65	10.67	10.99	11.23	11.35	11.52	11.94	12.15	11.97	11.89
1976	11.84	11.80	11.90	12.03	11.99	11.93	11.86	11.83	11.76	11.60	11.56	11.27
1977	10.75	10.25	10.25	10.25	10.38	10.35	10.40	10.33	10.32	10.34	10.34	10.33
1978	10.32	10.31	10.33	10.42	10.43	10.32	10.31	10.31	10.67	10.93	11.25	11.53
1979	11.28	11.25	11.11	11.05	11.06	11.16	11.20	11.80	12.25	13.50	14.46	13.58
1980	13.26	13.50	14.69	16.94	13.99	12.92	13.09	13.44	14.50	14.87	15.00	15.60
1981	15.17	15.27	15.75	16.45	17.82	18.55	18.90	21.30	21.46	20.54	18.80	17.79
1982	18.21	18.97	19.41	19.28	19.11	19.10	19.22	18.72	17.49	16.02	14.79	14.34
1983	14.05	13.60	13.45	13.26	13.16	12.98	13.08	13.57	13.88	13.10	12.84	12.55
1984	12.55	12.52	12.82	13.51	14.26	14.53	14.96	14.45	13.99	13.72	13.25	12.74
1985	12.44	12.57	13.43	12.77	12.38	11.89	11.75	11.77	11.85	11.96	11.75	11.61
1986	11.67	11.94	11.66	11.12	10.60	10.87	11.06	11.00	11.10	11.25	11.25	11.17