APPLICATIONS OF OPERATIONS-RESEARCH TECHNIQUES IN INSURANCE

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A literature review was conducted as part of a study of applications of operations-research techniques in insurance. This paper provides a cursory overview of some of the literature reviewed.

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1. INTRODUCTION

This paper is based upon a project which arose because of a concern voiced by actuarial students and many practicing actuaries that they had a problem conceptualizing practical applications for Operations Research, OR, in their daily work, particularly applications that are unique to the insurance business. This obviously is a serious problem. While some aspects of OR may be aesthetic, the primary thrust of the subject is the development of tools and techniques to solve practical problems.[2] This focus can be lost if the area is viewed as abstract or primarily academic.

The purpose of the project was to help alleviate this problem by providing a review of applications of OR techniques in insurance. A major resource, in this regard, was journals in operations research, actuarial science, insurance, and related fields of business. This paper gives a cursory overview of some of the literature reviewed.

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[1]This research was funded in part by a grant from the Actuarial Education and Research Fund.

[2]Some would regard this methodology-oriented view of OR as too narrow. Jewell (1980, p. 113), for example, would prefer to stress the system building opportunities and areas for constructive interaction within insurance, rather than tools and techniques of OR.

M. Goovaerts et al. (eds.), Insurance and Risk Theory, 129–143.

2. PREVIOUS STUDIES

There have been a number of studies of the application of OR techniques. Examples include Turban (1972), Fabozzi and Valente (1976), and Ledbetter and Cox (1977). These studies reported on the results of surveys of large companies including, but not limited to, some insurers.

There also have been a number of studies which have dealt specifically with the interaction of OR techniques and the insurance industry. These generally have taken two distinct routes: studies of OR applications directed at an insurance audience and studies of OR applications in insurance directed at an OR audience. Commonly referred to examples of the former are the studies of Zubay (1965, 1969), Denenberg (1968), and Wade et. al. (1970). Examples of the later are found in Sutton (1965), Halmstad (1974), and Jewell (1974).

A number of these studies reported that there was not a great deal that specifically related OR to insurance situations, and many of the applications alluded to had not yet been implemented. Part of the focus of this study is to see how the literature has changed in these regards.

3. GAME THEORY

Game theory involves competition or conflict between two or more decision makers, and is concerned with prescribing best strategies. Specific insurance applications include topics such as insurance purchases, sales, underwriting, management, expense allocation, reinsurance, and premium determination.

3.1. Insurance Consumption

Williams (1960) discussed the use of pure strategies in game theory for the evaluation of insurance consumption alternatives. The analysis was based on loss in utility associated with the decision of whether or not to buy fire insurance. The notion of a worry factor was used to help explain insurance purchases.

Further insight into the development of this model was contained in Williams and Dickerson (1966) and an empirical investigation of the model was reported in Neter and Williams (1973).

3.2. Optimal Insurance Conflict

While two-person, zero-sum games have serious shortcomings in many practical situations, owing primarily to the lack of total knowledge and zero-sum conditions, they can provide useful models in the proper circumstances. They set the stage for Miller (1972), for example, who used expected utility maximization to show the conflict between the insurer and insured insofar as the optimal type of insurance contract is concerned.

3.3. Insurance Sales Game

Bragg (1966) envisioned an insurance sales game played by three players: the insurer, the agent, and the buyer. The first phase of the game involved a coalition between the insurer and the agent against the buyer, while the last involved a conflict between the insurer and the agent.

The essential feature of the first phase was an attempt to maximize the marginal expectation of a policy sale, where the margin was the premium less cost other than commission, and the marginal expectation was the product of the margin and the probability of the sale at a given price. The insurer-agent conflict revolved about the proportion of the margin which should be allocated to commissions, and was viewed as a two-person cooperative game.

3.4. Life Insurance Underwriting

Lemaire (1980) addressed the problem of insurance underwriting within the context of a two-person non-cooperative game between an insurer (player 1) and potential policyholders (player 2), who are in either a healthy or nonhealthy state. The minimax criterion, which presupposes malevolent applicants, and the Bayes criterion, which assumes the applicant has adopted a fixed priori strategy, were considered.

3.5. Executive Game

One of the earlier models of a game for a property and liability insurance company was the executive game for officers and middle management suggested by McGuiness (1960).

The inputs to the model included basic decisions, assumptions, and data while the outputs of the model were reports to the players and an analysis of the effect of their decisions. Assumptions included those that were known to the players and those which described the environment and were not known to the players. The data included such information as economic activity, underwriting experience fluctuations, and cost of training agents. Two limitations of the game were that no investments were involved and rating bureaus did not play a role.

3.6. Cost Allocation

Lemaire (1984) discussed the application of game theory to the problem of allocating expenses among the departments of an insurance company when cooperation leads to economies of scale.

He first showed that the cost allocation problem was identical to the value of a game with transferable utilities, and then discussed the attributes of four cost allocation methods based on game theory. The criteria advocated are collectively rational, in the sense that no departments subsidize another, monotonic costs, in that all departments contribute to an increase in global costs, and additivity, in the sense that a subdivision of a department does not affect the cost allocation.

3.7. Reciprocal Reinsurance Treaties

A simple model for analyzing a reciprocal reinsurance treaty was discussed by Borch (1960) within the context of a two-person cooperative game. After reviewing basic axioms relating risk situations and showing that the quota share treaty will best reduce the variance of the portfolio of each company when the utility index depends only on the free reserves and the variance of the risk distribution, he goes on to discuss the nature of the quota ceded and the free reserves transferred.

3.8. The Actuary Against Nature

Bühlmann (1976) investigated a game which pitted the actuary against nature in the context of estimating a pure premium from sample information. The situation involved the actuary who chose a pair of estimators, nature who chose a parametric family of distribution functions and structure function, and a quadratic loss function. Both the non-parametric case, where the parametric family was not known, and the parametric case, where the parametric family was known, were considered.

4. STOCHASTIC DOMINANCE

Stochastic dominance (SD) provides a means for preference ordering when uncertain alternatives are involved. The three orders of SD are based on increasingly restrictive sets of utility functions: increasing, risk adverse, and decreasing absolute risk adverse. Potential areas of application include optimal insurance coverages and reinsurance.

4.1. Insurance and Risk Sharing

Doherty (1977) used second order SD to develop a model for investigating the appropriateness of a deductible from the vantagepoint of an individual risk-adverse insured. He later extended this approach (Doherty, 1980) to model the preferences of both the insured and the insurer insofar as the use of deductibles, coinsurance, the franchise form, and first loss.

4.2. A Reinsurance Example

Gandhi et. al. (1981), discussed a simple reinsurance application where the manager of a stock company must choose between two portfolios, one containing reinsurance, and the other not. The paper described in detail the characteristics of first, second, and third-order stochastic dominance, and discussed the superiority of stochastic dominance over mean-variance, coefficient of variation, and expected utility models.

5. LINEAR PROGRAMMING

Linear programming has long been recognized as one of the most important techniques of OR because of its versatility and power in resolving problems involving the allocation of scarce resources. Specific applications in insurance include insurance purchase optimization, insurer profits maximization and the bounding of stop-loss premiums.

5.1. Life Insurance Purchases

A perennial problem in the sale of life insurance is the optimal combination of various types of life insurance policies and alternate investments. Schleef (1980) showed how a linear programming model could be used to help resolve this problem.

The objective of the model was to maximize the present value, adjusted for the marginal tax rate, of future cash flows due to cash value recovery, loans, and other investments. The constraints of the model included a budget constraint, which provided for the payment of premium and loan interest and recognized alternative investment cash flows, a death benefit constraint, which provided for the desired level of death benefits, and non-negative constraints on the alternative investment fund, the cash value, the loan balance, the face amount, and all decision variables.

5.2. Profits of Property and Liability Insurers

Hofflander and Drandell (1969) used a linear programming model to maximize the profits of a property and liability insurer subject to operational and regulatory constraints. The purpose of their model was to find the optimum allocation of assets so that profits could be maximized.

5.3. Stop-Loss Premiums

One way of reinsuring is through a stop-loss arrangement, under which the reinsurer pays that portion of the loss in excess of a specified amount. A concern, in this regard, is the maximum value for a stop-loss premium.

Taylor (1977) used a linear programming technique to help isolate the upper bounds on stop-loss premiums. His approach was to generalize from an illustration which validates that if there was a discrete claim distribution, the maximal stop-loss distribution has its mass concentrated at the two extreme points of the range.

6. GOAL PROGRAMMING

The essential feature of goal programming is that it provides an opportunity to assign priorities to conflicting objectives and then minimizes deviations from those objectives. The method provides an opportunity for three types of analysis: input requirements, given a goal; relative goal attainment, given resources; and a test for an optimum solution. An important benefit of the model is its simulation capabilities.

6.1. Insurance Agency Management

Gleason and Lilly (1977) examined goal programming as an agency decision-making tool in the context of property and casualty insurance agency decisions regarding number of insurers to represent, cost reduction efforts, and expanded commercial lines. Rather than deal with all possible goals that might confront such an agency, they limit themselves to common goals.

6.2. Profit Maximisation in Property and Liabilities Companies

Klock and Lee (1974) argued that a linear programming approach to profitability, even if it is extended to include other utility function characteristics, while theoretically correct, is impractical. They suggested goal programming as an alternative.

The authors applied a goal-programming model to the H-D model (Hofflander & Drandell, 1969), with some modifications, to transform it from a linear programming format. They identified six goal constraints which are, in decreasing priority: liquidity, financial strength, profit, distribution of asset portfolio, distribution of premium portfolio, and use of capacity. The system constraints involve policyholders' surplus, capital, premiums written as a multiple of policyholders' surplus, total assets as a multiple of premium volume, premium balances as a proportion of premium volume, distribution of assets, proportion of miscellaneous liabilities, proportion of loss reserves, proportion of unearned premium reserves, total assets, and equality of assets and liabilities plus net worth.

One of the obvious questions to be resolved insofar as goal programming is concerned, is the similarity between it and linear programming. In this regard, Drandell (1977) demonstrated that the goal programming model of Klock and Lee produced results which were equivalent to the H-D model. The basis of the comparison was a formulation of goals into liquidity, stability, and profit.

6.3. Capital Budgeting in Property Liability Companies

Lawrance and Reeves (1982) formulated the capital budgeting in a property liability insurance company as a 0-1 goal programming problem involving multiple goals and indivisible activities. They viewed the model as being more representative of reality since it was not constrained by a single objective or continuous activities, while being easy for decision makers to understand and use.

For the purpose of examples, seven projects were considered which cover three time periods and a number of regions of operations. The constraints of the model included seven sets of goal constraints corresponding to the objectives. Additionally, there were constraints to reflect indivisibility, single period nature, and independence and/or dependence of the projects.

7. QUADRATIC PROGRAMMING

Quadratic programming is concerned with optimizing a quadratic objective function subject to linear constraints. Areas of application include portfolio analysis, from both a financial and insurance perspective, and international reinsurance.

7.1. Portfolio Analysis

One of the most common uses of quadratic programming is in the resolution of questions related to portfolios. The origin of this approach was the work of Markowitz (1952), who suggested using probability estimates of future security performance to develop an efficient set of portfolios which could then be matched with an investor's preference.

Sharpe (1963) discussed an efficient resolution of the problem which greatly simplified the analysis.

7.2. Portfolio Theory and Non-Life Insurers

Markle and Hofflander (1976) applied the Markowitz model to non-life insurers under the assumption that the goal is to maximize returns for given levels of risk subject to institutional solvency constraints. The objective function is the underwriting and investment profits.

For this purpose, the overall return for a given line was assumed to be an average return for that line over the period of investigation, as were the expected returns from securities.[3]

7.3. International Reinsurance

Louberge (1983) used quadratic programming to model international reinsurance operations within a mean-variance context. In addition to addressing standard arguments against such an application for an insurer in general, he dealt with questions such as the nonnormality of exchange rates and the distribution and estimation of the returns of the excess of loss treaty.

7.4. Life Insurance Company Investments

The common approach to the analysis of alternative investment opportunities is based on quadratic programming models because the objective function involves the variance of distributions, a quadratic.

Brodt (1983) showed how to develop a linear programming alternative based on the mean absolute deviation of returns, rather than the variance of returns. The objective was to minimize risk, as measured by mean absolute deviation, subject to intra-temporal and inter-temporal constraints. Since the objective function was nonlinear, goal programming was resorted to for an actual solution.

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[3] Chance-constrained programming has also been used to address this problem. However, the use of a chance-constrained programming model to investigate the risks associated with investment and underwriting returns has been criticized by Kahane (1977), who argued that a full quadratic programming model is less restrictive and provides similar results, given the same assumptions.

8. DYNAMIC PROGRAMMING

Dynamic programming extends the single-stage assumption of linear programming to a multistage environment, and is concerned with the overall effectiveness of sequential, interrelated decisions over the planning horizon. Areas of application include life insurance programming, risk management, and no-claim limits in automobile insurance.

8.1. Dynamic Life Insurance Programming

One of the early attempts to bring dynamic programming to bear on the problem of choosing an optimal life insurance program was the study by Belth (1964). The study was naive by current standards, but the interrelationship of the variables is as relevant today as when they were first done.

8.2. Risk Management

Lilly and Gleason (1977) discussed the use of dynamic programming as an aid to the risk manager in equipment replacement decisions where the goal is to minimize potential losses.

The problem is that the risk manager must balance the replacement of the equipment while it still has economic value against potential direct and indirect losses involving business interruption loss, bodily injury and property damage loss, replacement cost of damage to manufacturer, workers' loss, and loss of current boiler.

Backward dynamic programming was used to resolve the problem, where the stages were the years of the planning horizon and the state within the stages were the years of purchase of the boiler.

8.3. Optimal No-Claim Limits

The problem common to all motorists is the optimal choice of when to make a claim. Motorists have an informal set of rules which they follow, under which claims are made if the extent of the accident exceeds some minimal amount, and no claim is made otherwise. This is a standard type of problem in Markov Chains, since it interrelates insurance claims during a given period and insurance premiums during subsequent periods.

Martin-Lof (1973) discussed insurance with a bonus system under which annual premiums depend on the bonus class to which the insured belongs. An optimal decision rule was developed based upon an adapted policy iteration which reduced the size of the system which needed to be solved at each iteration.

Similar problems were resolved by Haehling von Lanzenauer (1974) and Hastings (1976) who dealt with determining whether to file a claim after an accident when there was more than one possible accident during a given policy year.

9. INVENTORY MODEL

Inventory models seek the optimal balance between the cost of holding inventory and the cost of procuring it. Specific insurance applications include the maintenance of cash reserves and the procurement of insurance coverage.

9.1 Insurer Cash Reserves

White and Norman (1965) formulated the control of cash reserves of an insurer as an inventory control problem where storage and penalty costs are well defined. The optimum value for the short-term effective bank balance was resolved using dynamic programming.

9.2. Optimal Insurance Coverage

Smith (1968) envisioned the optimal insurance coverage in the context of an optimal inventory stockage under uncertainty. Specifically, if an insurable loss (demand) is exceeded by the insurance coverage (inventory level), excessive insurance cost (inventory holding cost) is incurred. Conversely, if insurable loss is greater than insurance coverage, unrecoverable losses must be absorbed by the insured. The problem, then, becomes one of choosing optimal insurance (inventory) levels.

10. NETWORK MODELS

The standard applications of network models for the administration and coordination of prospects and organizations were easily transferable to the insurance environment, as were the planning and control techniques of PERT and CPM. Specific examples of application are cash flow management and business interruption downtime.

10.1. Cash Flow Management

Management of a property-casualty insurance company has two primary responsibilities. The first responsibility revolves about the selection of an appropriate composition for the insurance portfolio. Considerations in this area include which lines to insure, the amount of coverage to be provided in each line, and marketing considerations. The other responsibility revolves about the insurer's investment portfolio. Here, considerations include the proper mix of securities, appropriate maturity dates, the limit on investment in any particular security, and so on.

Crum and Nye (1981) applied a network model to the problem of managing the overall cashflow of a medium-size insurer. The thrust was to coordinate the insurance underwriting operations and the investment portfolio pursuits.

10.2. Business Interruption Downtime

Close (1982) discussed the application of PERT to the reduction of downtime due to business interruption. The system discussed was applied to the situation where a fire loss occurred, and a network was given showing the stages between the fire loss and the resumption of the former level of production.

11. QUEUING MODELS

The numerous applications of queuing theory used to resolve waiting line problems of business are directly applicable to the insurance industry. An application, however, which is unique to insurance has to do with the ruin of an insurance enterprise.

11.1. Insurer Ruin as a Queuing Problem

Under a proper scenario, the solvency of an insurer may be viewed as a queuing problem, where the probability of not being ruined by some time t, u(U,t), is essentially the same probability as that involving a customer waiting less than some time U, given that the customer joined the queue t periods after the server was free. A discussion of this aspect is given by Seal (1978, Chapter 2).

12. MARKOV PROCESS

A Markov process provides a dynamic system under which only the immediate past is relevant to the prediction of future behavior. Its application to life tables is an obvious one.

12.1. Working Life Tables

Since working life tables evolve from the dynamics of labor force participation, Hoem (1977) advocated that such tables be produced using the theory of continuous-time Markov chains. The transitions in this case are due to death, accession to the labor force, and separation from the labor force. His discussion of an application of such a model is based on a previous study by him and Fong (1976), which contain the formal details of the model.

Similarly, Braun (1978) emphasized stochastic stable population theory where the forces of fertility and mortality depend on age, parity, and place of residence.

13. SIMULATION

Because of the easy access to computers, the use of simulation within insurance and related industries has become commonplace. Parallelling this application of computers and simulation, has been the need for the development of models which adequately encompass particulars of specific areas within the industry. A number of these models are discussed by Pitacco elsewhere in this volume. Representative examples of areas of application include variable life insurance, merit-rating, and reinsurance.

13.1. Equity Linked Life Insurance Policies

Brennan and Schwartz (1979) used simulation to explore risk-reducing investment strategies associated with equity linked life insurance policies. There are implications for the development of a Bayesian approach for finding estimates of the model's parameters.

13.2. Merit-Rating Plans

Ferreira (1974) modeled a driver's annual premium as a Markov process and used a compound Poisson accident model and Monte Carlo simulation to compute 50 year premium histories in order to investigate various merit-rating plans.

13.3. Insurer and Reinsurer Interaction

Galitz and Brown (1981) discussed the qualitative nature of a simulation model for insurance and reinsurance operations. Although the relationships were not specifically defined, important overall considerations were delineated. The basic components of the model were the surplus, capital, unearned premium reserve, and the loss reserves.

The model used a flow-driven system which fixed the variables at some starting point, and the flow of variables therefrom, in order to determine the state of the initial variables at some later time.

14. OR SYSTEMS AND MODELS

OR authorities view the "systems approach", which coordinates overall relationships and interdisciplinary teams, as the fundamental thrust of OR. Representative applications include the areas of population planning, the insurance industry and workmen's compensation.

14.1. Population Planning

Reinke (1970) discussed the role of model building in population planning in underdeveloped countries. The problem was that there was a conflict between national and family goals, sparce relevant information, and limited resources. Within this framework, the role of OR was to analyze the decision process and organize professional activities in this area.

14.2. The Insurance Industry

Pentikainen (1983) discussed a general-purpose model for studying the solvency condition of a broad range of insurers. The model includes provisions for state variables and parameters, such as portfolio mix, reserves, and assets, exogenous factors, such as business cycles and inflation, and business strategies, such as retention levels, investment policy and sales effort. An important attribute of the paper is that it discussed many facets of the model which require consideration.

14.3. Workmen's Compensation Insurance

Jewell et. al. (1974) discussed the multidisciplinary nature of a comprehensive project involving workmen's compensation insurance. Each phase of the project is discussed, as are the interrelationships of the phases. The study embodies the spirit of model building which OR authorities stress.

15. COMMENT

This article provides only a cursory overview of the literature pertaining to applications of OR techniques in insurance. Nonetheless, it is hoped that it stimulates discussion and provides direction and insight into further research in this area. To the extent it has met these criteria, it will have served its purpose.

16. BIBLIOGRAPHY

Belth, J. M. 1964, 'Dynamic Life Insurance Programming,' Journal of Risk and Insurance, 31 No.4, pp. 539-556.

Borch K. 1960, 'Reciprocal Reinsurance Treaties Seen As A Two-Person Co-operative Game,' Skandinavisk Aktuarietidskrift, 43 No. 1-2, pp. 29-58.

Borch, K. 1968, The Economics of Uncertainty, Princeton University Press.

Bragg, J. M. 1966, 'Prices And Commissions Based On The Theory Of Games,' Journal of Risk and Insurance, 33 No. 2, pp. 169-193.

Braun, H. 1978, 'Stochastic Stable Population Theory In Continuous Time,' Scandinavian Actuarial Journal, No. 4, pp. 185-203.

Brennan, M. J. and E. S. Schwartz. 1979, 'Alternate Investment Strategies For The Issuers Of Equity Linked Life Insurance Policies With An Asset Value Guarantee,' Journal of Business, 52 No.1, pp. 63-93.

Brodt, A. I. 1983, 'Min-Mad Life: A Multi-period Optimization Model For Life Insurance Company Investment Decisions,' Insurance: Mathematics & Economics, 2 pp. 91-102.

Bülhmann, H. 1976, 'Minimax Credibility,' Scandinavian Actuarial Journal, No. 2, pp. 65-78.

Close, D. B. 1982, 'PERT Networks' Use Can Help Reduce BI Downtime,' Risk Management, 29 No. 3, pp. 26-30.

Crum, R. L. and D. J. Nye. 1981, 'A Network Model Of Insurance Company Cash Flow Management,' Mathematical Programming Study, 15 pp. 86-101.

Denenberg, H. S. 1968, 'A Review Article - A Basic Look At Operations Research,' Journal of Risk and Insurance, 35 No. 1, pp. 159-163.

Doherty, N. A. 1977, 'Stochastic Choice In Insurance And Risk Sharing,' Journal of Finance, 32 No. 3, pp. 921-926.

Doherty, N. A. 1980, 'Stochastic Ordering Of Risk/Insurance Exchanges,' Scandinavian Actuarial Journal, No. 4, pp. 203-208.

Drandell, M. 1977, 'A Resource Association Model For Insurance Management Utilizing Goal Programming,' Journal of Risk and Insurance, 44 No. 2, pp. 311-315.

Fabozzi, F. J. and J. Valente. 1976, 'Mathematical Programming in American Companies: A Sample Survey,' Interfaces, 7 No. 1, pp. 93-98.

Ferreira, J., Jr. 1974, 'The Long-term Effects Of Merit-Rating Plans On Individual Motorists,' Operations Research, 22 No. 5, pp. 954-978.

Freifelder, L. R. 1976, A Decision Theoretic Approach to Insurance Ratemaking (Pennsylvania, Pa.: S. S. Huebner Foundation for Insurance Education).

Galitz, L. and M. Brown. 1981, 'ASIR: The Advanced Simulation Model Of Insurance And Reinsurance Operations,' Geneva Papers on Risk and Insurance, 19 pp. 85-96.

Gandhi, D. K., A. Saunders, and E. G. Sugars. 1981, 'Stochastic Dominance: An Application To The Insurance Portfolio Decision,' Geneva Papers on Risk and Insurance, 21 pp. 51-62.

Gerber, H. U. 1979, An Introduction to Mathematical Risk Theory (Pennsylvania, Pa.: S. S. Huebner Foundation for Insurance Education).

Gleason, J. M. and C. C. Lilly. 1977, 'A Goal Programming Model For Insurance Agency Management,' Decision Sciences, 8 No. 1, pp. 180-190.

Haehling von Lanzenauer, C. 1974, 'Optimal Claim Decisions By Policyholders In Automobile Insurance With Merit-Rating Structures,' Operations Research, 22 No. 5, pp. 979-990.

Halmstad, D. G. 1974, 'Actuarial Techniques And Their Relations To Noninsurance Models,' Operations Research, 22 No. 5, pp.942-953.

Hastings, N. A. J. 1976, 'Optimal Claiming On Vehicle Insurance,' Operational Research Quarterly, 27 No. 4, pp. 805-813.

Hillier, F. S. and G. J. Lieberman. Introduction to Operations Research, 3rd (San Francisco: Holden-Day, Inc.).

Hoem, J. M. 1977, 'A Markov Chain Model Of Working Life Tables,' Scandinavian Actuarial Journal, No. 1, pp. 1-20.

Hoem, J. M. and M. S. Fong. 1976, A Markov Chain Model of Working Life Tables. Copenhagen Univ. Lab. of Actuarial Math., Working Paper No. 2.

Hofflander A. E. and M. Drandell. 1969, 'A Linear Programming Model Of Profitability, Capacity, And Regulation In Insurance Management,' Journal of Risk and Insurance, 36 No. 1, pp. 41-54.

Jewell, W. S. 1974, 'Operations Research In The Insurance Industry: I. A Survey Of Applications,' Operations Research, 22 No. 5, pp. 918-928.

Jewell, W. S. 1980, 'Models in Insurance: Paradigms, Puzzles, Communications, and Revolutions,' Transactions of the 21st International Congress of Actuaries, S pp. S87-S141.

Jewell, W. S., T. L. Johnston, and S. S. Leavitt. 1974, 'Operations Research In The Insurance Industry: II. An Application In Claims Operations Of Workman's Compensation Insurance,' Operations Research, 22 No. 5, pp. 929-941.

Kahane, Y. 1977, 'Insurance Exposure And Investment Risks: A Comment On The Use Of Chance Constrained Programming,' Operations Research, 25 No. 2, pp. 330-337.

Klock, D. R. and S. M. Lee. 1974, 'A Note On Decision Models For Insurers,' Journal of Risk and Insurance, 41 No. 3, pp. 537-543.

Lawrance, K. D. and G. R. Reeves. 1982, 'A Zero-one Goal Programming Model For Capital Budgeting,' Computers and Operations Research, 9 No. 4, pp. 303-309.

Ledbetter, W. N. and J. F. Cox. 1977, 'Are OR Techniques Being Used?' Industrial Engineering (February), pp. 19-21.

Lemaire, J. 1980 'A Game Theory Look At Life Insurance Underwriting,' Astin Bulletin, 11 No. 1, pp. 1-16.

Lemaire, J. 1984, 'An Application Of Game Theory: Cost Allocation,' Astin Bulletin, 14 No. 1, pp. 61-81.

Lilly, C. C. and J. M. Gleason. 1977, 'A Dynamic Programming Approach To Equipment Loss Analysis,' Journal of Risk and Insurance, 44 No. 2, pp. 305-310.

Louberge', H. 1983, 'A Portfolio Model Of International Reinsurance Operations,' Journal of Risk and Insurance, 50 No.1, pp. 44-60.

Markle, J. L. and A. E. Hofflander. 1976, 'A Quadratic Programming Model Of The Non-life Insurer,' Journal of Risk and Insurance, 43 No. 1, pp. 99-120.

Markowitz, H. M. 1952, 'Portfolio Selection,' The Journal of Finance, 12 (March), pp. 77-91.

Martin-Lof, A. 1973, 'A Method For Finding The Optimal Decision Rule For A Policy Holder Of An Insurance With A Bonus System,' Skandinavisk Aktuarietidskrift, 56 No. 1, pp. 23-29.

McGuinness, J. S. 1960, 'A Managerial Game For AN Insurance Company,' Operations Research, 8 No. 2, pp. 196-209.

Miller, R. B. 1972, 'Insurance Contracts As Two-Person Games,' Management Science, 18 No. 7, pp. 444-447.

Narragon, E. A. 1980, A Study Manual For Operations Research, (Chicago: University of Chicago).

Neter, J. and C. A. Williams, Jr. 1973, 'Performance Of The Expected Utility Method And Two Other Normative Methods In Insurance Decision Making,' Decision Sciences, 4 pp. 517-532.

Pentikainen, T. 1983, 'On Model Building for Insurance Industry,' European Journal of Operations Research, 13 No. 3, pp. 310-325.

Reinke, W. A. 1970, 'The Role Of Operations Research In Population Planning,' Operations Research, 18 No. 6, pp. 1099-1111.

Schleef, H. 1980, 'Using Linear Programming For Planning Life Insurance Purchases,' Decision Sciences, 11 No. 3, pp. 522-534.

Seal, H. L. 1978, Survival Probabilities, (New York: John Wiley & Sons).

Shapiro, A. F. and J. M. McAdon. 1982, Operations-Research Applications for Pension Consultants and Administrators, (Washington, D. C.: American Society of Pension Actuaries).

Sharpe, W. F. 1963, 'A Simplified Model For Portfolio Analysis,' Management Science, 9 No. 2, pp. 277-293.

Smith, V. L. 1968, 'Optimal Insurance Coverage,' Journal of Political Economy, 76 No. 1, pp. 68-77.

Sutton, R. L. 1965, 'Operational Research On Actuarial Work,' Opsearch, 2 No. 3-4, pp. 17-22.

Taylor, G. C. 1977, 'Upper Bounds On Stop-loss Premiums Under Constraints On Claim Size Distribution,' Scandinavian Actuarial Journal, Vol No. 2, pp. 94-105. No. 2, pp. 94-105.

Turban, E. 1972, 'A Sample Survey of Operations-Research Activities at the Corporate Level,' Operations Research, 20 pp. 708-721.

Wade, R. C., B. C. Rogomentich, and E. Y. Kung. 1970, Operations Research and Insurance Applications: An Annotated Bibliography, (Bryn Mawr, Pa.: McCahan Foundation).

White, D. J. and J. M. Norman. 1965, 'Control Of Cash Reserves,' Operational Research Quarterly, 16 No. 3, pp. 309-328.

Williams, C. A., Jr. 1960, 'Game Theory And Insurance Consumption,' Journal of Risk and Insurance, 27 No. 4, pp. 47-56.

Williams, C. A., Jr. and O. D. Dickerson. 1966, 'Game Theory and Insurance Consumptions: The Worry Factor Revisited,' Journal of Risk and Insurance, 33 No. 3, pp. 411-425.

Zubay, E. A. 1965, 'Feasibility Study Of Operations Research In Insurance,' Journal of Risk and Insurance, 32 No.3, pp. 325-336.

Zubay, E. A. 1969, 'The Role Of Operations Research And Life Insurance,' American Society Of Chartered Life Underwriters, 23 No. 1, pp. 56-61.