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MODELING FLEXIBLE BENEFIT SELECTION

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ABSTRACT

A mathematical framework for benefits and choices must be created, in order to model benefit selection. This paper creates such a framework by defining benefit plans as reimbursement functions. These are then used with a defined choice function to calculate the cost deviation due to selection. Finally, utility functions can be applied to this framework, to predict choice.

I. INTRODUCTION

The problem of selection has been recognized by actuaries since the early days of the profession, and has been a continuing concern since then. Highan [14] in 1851, for example, authored an article in the first volume of the *Journal of the Institute*, entitled "On the Value of Selection as exercised by the Policy-holder against the Company." Similarly, McClintock [19] in 1892, in an early volume of the *Transactions of the Actuarial Society of America*, published an actuarial essay "On the Effect of Selection."

During the early periods, the analysis was primarily descriptive, and concerned with identifying situations conducive to adverse selection and the associated hazards. In recent years, the emphasis has changed towards an attempt to model the selection process and an analysis of the sensitivity of those models. Moreover, while the initial concern was raised by actuaries in the context of insurance, it has come to be recognized as an issue common to a number of commodities, and, as such, has become an important field of study in economics.

A number of issues have emerged. The optimal form of an insurance contract for a risk-adverse insured was studied by Borch [5], Arrow [2], Raviv [22], Bühlmann and Jewell [7] and Blazendo [4]. Models which addressed the difficulty created by asymmetric market information regarding the riskiness of the insured where developed by Akerlof [1]), Rothchild and Stiglitz [23], Wilson [25], Miyazaki [20], and Spence [24]. Still others have studied the role of wealth in this decision process. These have included Gould [13] who concluded that it was not appropriate to consider demand without regard for the wealth position of the individual, Mayers and Smith [18], and Doherty and Schlesinger [11], who showed how assets correlate with the demand for insurance.

This paper extends the analysis by dealing with some of the statistical aspects of choice in benefit plans. Although the techniques presented could be used for any choice in insurance plans the focus will be on group health benefit plans. By group health benefit plan we will mean a system in which the members of a group are eligible to receive insurance benefits for some part of the cost of their (and sometimes their family's) medical care. The insurance benefits may require the payment of premiums. Generally the particular plan of benefits and premiums are unique to each group. The group is usually formed for some other purpose than the insurance coverage. The most common groups are the employees of a single employer.

Most of the remarks will deal with the traditional health insurance indemnity plans in which the group members obtain health care from licensed health care providers and then are reimbursed for a portion of the charges made by these providers. Some benefit plans include a provision for an employee choice between more than one formula for the amount of reimbursement. The employee may be required to contribute different premiums for each option.

Employee choice in group health benefits has only started to become popular in the last 5 or 10 years in the United States. Of course, trivially, most plans have always allowed the choice of rejecting the coverage if the employee is required to pay premiums for the coverage. Thus, there is a choice between the benefit plan and a null plan.

II. REIMBURSEMENT

Before we can write some expressions for the effects of selection or predict it we need to express the whole set of choices and outcomes in a functional and probabilistic setting.

Let the random variable X be the covered charges for an individual during a period, usually one year. Assume that X is a one dimensional positive random variable.

We define the notation: $x^+ = \max\{0,x\} = \begin{cases} 0 & x<0 \\ x & x\geq 0 \end{cases}$.

Let r(X) be the amount of reimbursement in a benefit plan for covered charges equal to X, where r is a function called here a reimbusement function. Note that we are assuming that the amount of reimbursement is determined only by the total of covered charges during the year and not by when the services were performed or by which providers.

Although any function r could be a reimbursement function we note that they generally have the following properties:

I. They are continuous: $\lim_{x\to a} r(x) = r(a)$;

II. They are nondecreasing: $x > y \Rightarrow r(x) \geq r(y)$;

III. $x > y \Rightarrow r(x) - r(y) \leq x-y$; and

IV. $r(0) = 0$

Property I says that the amount reimbursed cannot vary too much for small changes in covered charges. Property II says that as the covered charges increase the reimbursement cannot decrease. Property III says that amount of reimbursement cannot increase faster than covered charges. Property IV says that there is no reimbursement when there are no covered charges.

Example 2.1

The reimbursement function can be the identity function: $r(x) = x$. This is full reimbursement for all covered charges.

Example 2.2

The reimbursement function can be identically equal to zero: $r(x) = 0$ for all x. This is the case of no benefits.

Example 2.3

For a given fixed constant d,

$$r(x) = (x-d)^+ = \begin{cases} 0 & x \le d \\ x-d & x > d \end{cases}.$$

This is called full coverage after a deductible. The constant d is the deductible.

Example 2.4

For a constant c, $0<c<1$, $r(x) = cx$. The constant is called the coinsurance rate.

Example 2.5

We can have both a deductible and coinsurance (a combination of examples 2.3 and 2.4):

$$r(x) = c(x-d)^+ = \begin{cases} 0 & x \le d \\ c(x-d) & x > d \end{cases}.$$

Example 2.6

There can be a limit on the coinsurance of example 2.4. For constant $L>0$ and c, $0<c<1$:[1]

$$r(x) = cx + [(1-c)x-L]^+ = \begin{cases} cx & x < L/(1-c) \\ x-L & x \ge L/(1-c) \end{cases}.$$

Here L is known as the coinsurance limit. Note that L is not the amount of covered charges that has to be reached before full reimbursement but rather is the maximum that is not reimbursed.

Example 2.7

Examples 5 and 6 can be combined to get a plan with deductible, coinsurance, and coinsurance limit.

$$r(x) = c(x-d)^+ + [(1-c)(x-d)-L]^+ = \begin{cases} 0 & x<d \\ c(x-d) & d \le x < L/(1-c)+d \\ x-d-L & L/(1-c)+d \le x \end{cases}.$$

In this case $L+d$ is sometimes called the out-of-pocket limit.

Example 2.8

Often there is an overall individual annual benefit maximum. For a constant M:

$$r(x) = \min\{x,M\} = \begin{cases} x & x<M \\ M & x \ge M \end{cases}.$$

Example 2.9

There can be the combination of examples 2.7 and 2.8. This would be a plan with deductible, coinsurance, coinsurance maximum, and overall annual maximum:

$$r(x) = \min\{c(x-d)^+ + [(1-c)(x-d)-L]^+,\ M\} = \begin{cases} 0 & x<d \\ c(x-d) & d \le x < L/(1-c)+d \\ x-d-L & L/(1-c)+d \le x < M+d+L \\ M & M+d+L \le x \end{cases}.$$

[1]Note that we have deviated from the usual convention of reserving the uppercase for random variables.

For this example we will define the intervals: B = [d,L/(1-c)+d), C = [L/(1-c)+d, M+d+L), and D = [M+d+L,∞). Even though this looks rather complicated, this is often just called a comprehensive major medical plan of benefits. Of course, examples 2.1 through 2.8 can be treated as special cases of this example 2.9. All of the r's in examples 2.1 through 2.9 satisfy the properties I through IV above.

Table 1 illustrates some sample r's. r_1 is a very rich plan. r_2 reimburses less and r_3 is a cheap plan. r_4 is the null or 0 reimbursement of example 2.2 and r_5 is the full reimbursement of example 2.1.

Example 2.10

Assume that the random variable X has the discrete distribution:
$Pr\{X=ks\} = p_k$ for k=0, 1, 2,... and a constant s called the unit or span.[2]

Of course, $\sum_{k=0}^{\infty} P_k = 1$. Using the r's of example 2.9, we can calculate some values:

$$E[r(X)] = \sum_{ks\in B} c(ks-d)\, p_k + \sum_{ks\in C} (ks-d-L)\, p_k + \sum_{ks\in D} M\, p_k \ ,$$

$$E[r^2(X)] = \sum_{ks\in B} c^2(ks-d)^2 p_k + \sum_{ks\in C} (ks-d-L)^2 p_k + \sum_{ks\in D} M^2 p_k \ ,$$

and

$$Var[r(X)] = E[r^2(X)] - E^2[r(X)].$$

Where we have used the notation: $r^2(X) = [r(X)]^2$ or $E^2(X) = [E(X)]^2$.

Table 2 shows an example of such a distribution. This distribution was based on some data obtained from Health Care Service Corp. (Blue Cross Blue Shield of Illinois).

Table 3 shows the expectation and variance of the 5 reimbursements ofexample 2.9 when using this distribution, with s=$1,000.

Example 2.11

Similarly, let X have the mixed distribution where $Pr\{X=0\}=p_0$ and $Pr\{a<x\le b\} = \int_a^b f(t)dt$ for $a\ge 0$ and a density function f such that $\int_0^\infty f(t)dt = 1-p_0$. See Hogg and Klugman [15, page 50] for a discussion of mixed distributions. Again, assuming the ψ of example 2.9, we have the values,

$$E[r(X)] = \int_B c(t-d)f(t)\ dt + \int_C (t-d-L)f(t)\ dt + \int_D Mf(t)\ dt \ ,$$

and

$$E[r^2(X)] = \int_B c^2(t-d)^2 f(t)dt + \int_C (t-d-L)^2 f(t)dt + \int_D M^2 f(t)dt \ .$$

Table 3 also shows a calculation of these values using the Pareto distribution with the same mean and variance as the discrete distribution and $p_0=0$. The Pareto distribution is discussed in [9] and [15]. It is often used for claim size distributions. The Pareto has density: $f(x) = \alpha\lambda^\alpha(\lambda+x)^{-\alpha-1}$ and expectation of $\lambda/(\alpha-1)$.

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[2]This formulation has the advantage of simplicity. An alternate formulation would be that the $Pr\{ks \le X < (k+1)s\} = p_k$.

III. COST DEVIATIONS DUE TO SELECTION

We assume that a group is composed of m individuals, $m \geq 1$. The covered charges for individual i will be denoted with the positive random variable X_i, $1 \leq i \leq m$. Now assume that each individual is given a choice at the beginning of the year between n reimbursement functions: $r_1(x),\ldots,r_n(x)$. In order to avoid long subscripts we will write $r_j(x) = r(j,x)$, $1 \leq j \leq n$. We define the "mean group reimbursement at r_j" as the random variable

$$\Psi(j) = \frac{1}{m}\sum_{i=1}^{m} r(j, X_i) .$$

In the prechoice environment, insurers have been estimating $E(\Psi_j)$ by using relatively complicated manual rating formulas that take into account the characteristics of the group, the individuals in the group, and r_j. The formulas are complicated because they must reflect the deductible, the coinsurance, and so on.[3] Incidentally, insurer's will often use the group's experience to estimate $E(\Psi_j)$.

Assume that the i-th member of the group, $1 \leq i \leq m$, chooses reimbursement level $\chi(i)$, $1 \leq \chi(i) \leq n$. Thus $\chi(i)$ is a function $\chi:\{1,2,\ldots,m\} \to \{1,2,\ldots,n\}$ called the choice function. Also, we define P(j), $1 \leq j \leq n$ as the annual premium payable by an individual for reimbursement j. The total reimbursement to the group $R = \sum_{i=1}^{m} r(\chi(i),X_i)$, the total premiums paid $P = \sum_{i=1}^{m} P(\chi(i))$, and $G = P - R = \sum_{i=1}^{m} [P(\chi(i)) - r(\chi(i),X_i)]$ is the insurer's gain.

EXAMPLE 3.1

We have a set of X_i, $1 \leq i \leq m$, mutually independent and identically distributed as in example 2.10. The set of functions $r_j(x) = r(j,x)$, $1 \leq j \leq n$, are as in example 2.9 where d(j), c(j), L(j) and M(j) correspond to r_j and therefore we have the intervals B(j), C(j) and D(j). For a choice function χ, we can calculate the values:

$$E[r(\chi(i),X_i)] = \sum_{ks \in B(\chi(i))} c(\chi(i))[ks - d(\chi(i))]p_k + \sum_{ks \in C(\chi(i))} [ks - d(\chi(i)) - L(\chi(i))]p_k + \sum_{ks \in D(\chi(i))} M(\chi(i))p_k$$

and

$$E[r^2(\chi(i),X_i)] = \sum_{ks \in B(\chi(i))} c^2(\chi(i))[ks - d(\chi(i))]^2 p_k + \sum_{ks \in C(\chi(i))} [ks - d(\chi(i)) - L(\chi(i))]^2 p_k + \sum_{ks \in D(\chi(i))} M^2(\chi(i))p_k .$$

From these we can then calculate:

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[3] Of course, this is not true for simple reimbursement functions such as in examples 2.1, 2.2, and 2.4, where:

$$E[\Psi(j)] = r_j\left(\frac{1}{m}\sum_{i=1}^{m} E(X_i)\right) .$$

$$E[R] = \sum_{i=1}^{m} E[r(\chi(i),X_i)],$$

$$Var[R] = \sum_{i=1}^{m} Var[r(\chi(i),X_i)],$$

(given a set of P_i's) E[G], and Var[G].

Example 3.2

We can let the X_i have the distribution of example, 2.11. We can also have the reimbursements r_i's and the choice function $\chi(i)$ of example 3.1. Then:

$$E[r(\chi(i),X_i)] = \int_{B(\chi(i))} c(\chi(i))[t-d(\chi(i))]\, f(t)\, dt + \int_{C(\chi(i))} [t-d(\chi(i))-L(\chi(i))]\, f(t)\, dt + \int_{D(\chi(i))} M(\chi(i))\, f(t)\, dt$$

and

$$E[r^2(\chi(i),X_i)] = \int_{B(\chi(i))} c^2(\chi(i))[t-d(\chi(i))]^2 f(t)\, dt + \int_{C(\chi(i))} [t-d(\chi(i))-L(\chi(i))]^2 f(t)\, dt + \int_{D(\chi(i))} M^2(\chi(i))\, f(t)\, dt$$

The expressions for $Var[r(\chi(i),X_i)]$, E[R], Var[R], E[G], and Var[G] are the same as in example 3.1.

Now we define the "cost deviation due to selection", a random variable for a group with m individuals as:

$$A = R - \sum_{i=1}^{m} \Psi[\chi(i)]$$

$$= \sum_{i=1}^{m} r(\chi(i),X_i) - \sum_{i=1}^{m} \left[\frac{1}{m} \sum_{k=1}^{m} r(\chi(i),X_k) \right].$$

This is called the cost deviation due to selection because A is equal to the deviation in the reimbursement due to the choice χ. Since

$$R = A + \sum_{i=1}^{m} \Psi[\chi(i)],$$

and

$$E[R] = E[A] + E\left[\frac{1}{m} \sum_{i=1}^{m} \Psi[\chi(i)] \right] = E[A] + \frac{1}{m} \sum_{i=1}^{m} E[\Psi(\chi(i))].$$

the problem of estimating E[R] is reduced to estimating E[A] and using the traditional rating techniques (e.g. manual rates as discussed above) for $E[\Psi(\chi(i))]$ in the second term.

Here are some of the properties of A (proofs omitted):

I. A is exactly equal to the amount that the actual reimbursement exceeds what the reimbursement would have been if each individual was reimbursed at the mean rate for the group. That is, if we define the mean reimbursement for the group

$$\bar{r}(x) = \frac{1}{m}\sum_{k=1}^{m} r(\chi(k),x),$$

then

$$A = \sum_{i=1}^{m}\left[r(\chi(i),X_i)-\bar{r}(X_i)\right] = \sum_{i=1}^{m} A(i)$$

for

$$A(i) = r(\chi(i),X_i)-\bar{r}(X_i)\ .$$

II. If the X_i are identically distributed then $E(A)=0$.

III. If χ is a constant, $\chi(1) = \chi(2) = \ldots = \chi(m)$, then $A=0$.

IV. Often the insurer sets $P(i) = E[\Psi(i)]$. In which case $E(G) = -E(A)$.

V. If the values of $\chi(i)$ are treated as random variables, that are independent of the X_i, then $E(A)=0$.

Example 3.3

Table 4, presents a hypothetical group with $m=100$. Shown for each individual is $E(X_i)$ and the choice $\chi(i)$. Here $n=4$ and the four choices are #1 through #4 of example 2.9. Table 5 shows the expectations and variances of $\Psi(j)$ $(1 \leq j \leq 4)$, R/m, and A/m. These have been calculated under the two assumptions: 1) Each X_i has the distribution of example 2.10 with $s=E(X_i)/1433.67$, and 2) Each X_i has the distribution of example 2.11 (table 3, Pareto) with $\lambda=E(X_i)(1.15738)$. This value of λ will give a Pareto distribution with the required expectation.

Table 4 also shows for each individual in the group an example outcome of values for X_i, the corresponding values of $r(j,X_i)$ for $j=1,2$, and 3, and the value of A(i). Thus there were covered charges of $153,970 (compare to the expected value of 141,360), reimbursements R of $129,546 and A of $30,007.

The values of $E(X_i)$ can be thought of as the expected covered charges due to known (to the insurer) characteristics of the individuals in the group, such as their ages. In such case E(A) can be thought of as the expected cost deviation due to demographic selection. If the actual value of A greatly exceeds this E(A), then the insurer might wonder if the individuals knew more about their health status and used this knowledge to antiselect. We can approximate the probability that a value of A was realized randomly by using E(A) and Var(A) with the normal approximation.

IV. PRIOR YEAR'S CHARGES

Let us assume that each individual has a, possible unknown, parameter for the distribution of his covered charges. We will call this parameter $y = \{y(i) \mid 1 \leq i \leq m\}$ where y(i) pertains to individual i. Note that the y(i)'s could themselves be treated as realizations of random variables Y(i)'s and may be multidimensional. In any case, if we knew the values of the y(i)'s we could calculate $E[A \mid y]$. Since there is generally a correlation between successive years' charges, we could take a set of y(i)'s to be each individual's prior year's charges.[4]

Example 4.1

Table 6 expands table 4. The values that were previously called X_i are now taken to represent last year's claims and are identified as y(i). Table 6 also shows a value of $E[X_i | Y_i = y(i)]$. Here we have set $E[X_i | y(i)] = .75E[X_i] + .25y(i)$. Table 7 shows the $E[\Psi(j) | y]$, $(1 \leq j \leq 5)$, $Var[\Psi(j) | y]$, $E[R/m | y]$, $Var[R/m | y]$, $E[A/m | y]$ and $Var[A/m | y]$. These are computed using the two assumptions of example 3.3. We have assumed that the X_i always have the same distributions except for a scale change.

Example 4.2

Very often the parameter y would be unknown. If we assume that it is equal to the prior year's charges we could assume that each y(i) has the distribution of X_i. If we set $E[X | Y_i = y(i)] = .75E[X_i] + 25y(i)$, then, we can calculate $E[R] = E[E(R | Y)]$ and $Var[R] = Var[E(R | Y)] + E[Var(R | Y)]$. The calculations involved are long and tedious so no example values have been calculated. A Monte Carlo simulation technique could be used instead.

V. PREDICTING CHOICE

In order to predict employee choice we assume that each of the individuals, i $(1 \leq i \leq m)$ has a utility function $u_i(w)$ for wealth $w \geq 0$.[5] Now we assume that each individual will select the reimbursement that maximizes his expected utility. That is, if each individual's initial wealth is w(i) and there exists a $1 \leq k \leq n$ such that:

$$E\left(u_i[w(i) - X_i + r(k, X_i) - P(k)]\right) \geq E\left(u_i[w(i) - X_i + r(j, X_i) - P(j)]\right)$$

for every j, $1 \leq j \leq n$, then $\chi(i) = k$. Trivially, if there are two (or more) reimbursements for which the expected utility is equal and greater than all of the other reimbursements we will assume an arbitrary selection.

For simplicity we want to use the same form of a utility function for each individual. In order to model the actual situation we will need that each individual has a different aversion to risk. In order to do this we will select a utility function that is decreasingly risk averse. That is, the larger the individual's initial wealth the less risk averse he is. Common measures of risk aversion are the Arrow-Pratt ([2] and [21]) measures of absolute risk aversion and relative risk aversion: $\rho_u(w) = -u''(w)/u'(w)$ and $\delta_u(w) = w\, \rho_u(w)$, respectively.[6]

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[4] Fuhrer [12, p.403], found a correlation of 24.35 percent and Cookson [8, p.1602], reported seeing estimates of 15 to 25 percent.

[5] See [6], Chapter 1, for an introduction to risk averse utility functions. A good reference on utility functions is [16], particularly Chapter 4, which has an excellent section on various types of utility functions.

[6] Kimball [17, p.2] suggests "standard risk aversion" as another alternative. It is characteristic of utility functions associated with constant relative risk aversion.

Example 5.1

We can use the assumptions of example 3.3 with the choice depending on the utility function: $u_i(w) = \ln(w+a(i))$ for a positive constant a(i). This utility function is convenient because the property that almost any level of risk adverseness can be selected based on the size of the parameter a(i).[7] Table 6 shows some sample values of a(i) for our sample group and the resulting choice in column (1) using the discrete distribution to calculate expectations. Note that we have slightly changed the reimbursements to not have a maximum M. The end of Table 6 summarizes the choices and Table 8 shows the calculated values. We have assumed that P(j) = E(Ψ(j)].

Example 5.2

For this example, use the assumptions of example 4.1, with a fixed known parameter set y(i), with the utility based choice of example 5.1. The calculated values are also shown in table 8.

Example 5.3

This is example 5.1, except we use the parameter adjusted discrete distribution of example 4.1 to calculate the expected utilities and determine the choices. The Table 6 shows the choices in column (2) and Table 8 shows the calculated values using the parameter adjusted distributions as in example 5.2. Note that the choices (2) has a larger E[A] then choices (1).

Example 5.4

Here we combine example 4.4 with the utility function of example 5.1. Now that the choice is random, we could calculate, for each i and j, $\Pr\{\chi(i)=j\}$. We define N(j) as the number of individuals for whom $\chi(i)=j$. We could also calculate E[N(j)], $1 \leq j \leq 4$.

Example 5.5

Let $S(j) = \{i:\chi(i)=j\}$. Then let

$$P(j) = \frac{1}{N(j)} \sum_{i \in S(j)} r(y(i))$$

in example 5.2. That is, we set the premiums for a reimbursement equal to the experience those that selected it (using the sample selection). The resulting choice (Table 6, column (3)) is much more heavily weighted towards the cheaper plans. This illustrates the selection spiral that can occur if premium rates are based only on the experience of those that choose a particular reimbursement plan.

VI. CONCLUSION AND AREAS FOR FURTHER RESEARCH

The framework of this paper allows us to predict employee choice and cost deviations due to selection given any arbitrary combination of individual charge distributions, a set of reimbursement plans and their premiums, and a set of utility functions. Using this method various combinations of plans and premiums can be explored until the plan administrator can pick the combination that best fits the group's needs.

The calculations of examples 4.2 and 5.4 could be completed. A few more distributions

[7]For $u(w)=\ln(w)$, the absolute risk aversion is $\rho_u(w) = 1/w$, which is a decreasing function of w, and the relative risk aversion is $\delta_u(w) = 1$.

could be used to calculate the values. A term could be added to each reimbursement's wealth to model affinities that individuals may have for a particular plan. This might be used in the HMO choice, as individuals might prefer the traditional plan over the HMO so that they could continue with their current physicians.

The parameters of the utility function could be estimated from some actual choice data. These could then be used to predict actual past choices and then see how accurate the predictions were.

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Table 1 Some Sample Reimbursements Functions

Reimbursement #:	1	2	3	4	5
d (Deductible):	$100	$500	$1,000		0
c (Coinsurance):	80%	80%	75%	0%	100%
L (coinsurance max):	$400	$1,000	$3,000		
M (Maximum):	1,000,000	1,000,000	500,000		None

Table 2 Sample Discrete Distribution

s=1: Mean = 1.433, Variance = 28.175, Standard Deviation = 5.308

k	p(k)	k	p(k)	k	p(k)	k	p(k)
0	0.600839	43	0.000139	85	0.000023	131	0.000005
1	0.212998	44	0.000126	86	0.000019	132	0.000004
2	0.057230	45	0.000097	87	0.000023	133	0.000005
3	0.033316	46	0.000082	88	0.000015	134	0.000003
4	0.022218	47	0.000136	89	0.000005	135	0.000003
5	0.015504	48	0.000107	90	0.000011	136	0.000008
6	0.011159	49	0.000095	91	0.000017	137	0.000009
7	0.008179	50	0.000048	92	0.000018	138	0.000009
8	0.006329	51	0.000060	93	0.000009	139	0.000003
9	0.004906	52	0.000077	94	0.000004	140	0.000002
10	0.003751	53	0.000098	95	0.000006	142	0.000005
11	0.002734	54	0.000077	96	0.000015	145	0.000001
12	0.002257	55	0.000044	97	0.000007	146	0.000005
13	0.001984	56	0.000050	98	0.000021	147	0.000006
14	0.001629	57	0.000067	99	0.000014	148	0.000005
15	0.001230	58	0.000092	100	0.000005	150	0.000004
16	0.001179	59	0.000066	101	0.000013	151	0.000005
17	0.001041	60	0.000055	102	0.000015	152	0.000004
18	0.000854	61	0.000024	103	0.000015	153	0.000003
19	0.000741	62	0.000033	104	0.000012	158	0.000001
20	0.000633	63	0.000027	105	0.000011	159	0.000016
21	0.000554	64	0.000031	106	0.000003	160	0.000006
22	0.000529	65	0.000041	107	0.000004	169	0.000001
23	0.000528	66	0.000036	108	0.000007	170	0.000004
24	0.000485	67	0.000043	111	0.000002	172	0.000004
25	0.000397	68	0.000041	112	0.000007	173	0.000007
26	0.000387	69	0.000046	113	0.000005	185	0.000003
27	0.000352	70	0.000038	114	0.000007	186	0.000002
28	0.000403	71	0.000010	115	0.000006	197	0.000006
29	0.000333	72	0.000017	116	0.000001	202	0.000003
30	0.000306	73	0.000029	117	0.000009	203	0.000003
31	0.000253	74	0.000033	118	0.000002	204	0.000004
32	0.000258	75	0.000012	119	0.000005	205	0.000001
33	0.000245	76	0.000011	120	0.000004	206	0.000005
34	0.000228	77	0.000014	121	0.000005	245	0.000005
35	0.000204	78	0.000012	122	0.000010	263	0.000006
36	0.000231	79	0.000016	123	0.000004	285	0.000005
37	0.000193	80	0.000007	125	0.000002	292	0.000005
38	0.000172	81	0.000011	126	0.000003	323	0.000002
39	0.000177	82	0.000002	127	0.000005	324	0.000003
40	0.000133	83	0.000021	128	0.000013	519	0.000003
41	0.000121	84	0.000020	130	0.000005	520	0.000002
42	0.000136						

Table 3 Calculation of Values for the Reimbursements

Reimbursement #:	1	2	3	4	5
Discrete Distribution s=$1,000:					
Mean	$1,282.10	$1,091.57	$846.98	$0.00	$1,433.67
Variance	27,313,585	25,789,764	22,912,997	0	28,175,197
Standard Deviation	$5,226.24	$5,078.36	$4,786.75	$0.00	$5,308.03
Pareto Distribution					
Mean	$2,865.45	$2,436.31	$1,955.29	$0.00	$3,207.80
Variance	67,725,832	65,540,408	52,346,277	0	141,052,606
Standard Deviation	$8,229.57	$8,095.70	$7,235.07	$0.00	$11,876.56

TABLE 4 SAMPLE GROUP

i	E[Xi]	Chi(i)	Xi	r(1,Xi)	r(2,Xi)	r(3,Xi)	r[chi(i),Xi]	A(i)
1	$286.73	1	$5	$0	$0	$0	$0	$0
2	286.73	1	3,358	2,858	2,287	1,769	2,858	951
3	286.73	1	4,090	3,590	2,872	2,317	3,590	1,158
4	286.73	2	0	0	0	0	0	0
5	286.73	2	0	0	0	0	0	0
6	286.73	2	0	0	0	0	0	0
7	286.73	3	478	302	0	0	0	(73)
8	286.73	3	0	0	0	0	0	0
9	286.73	3	0	0	0	0	0	0
10	286.73	4	0	0	0	0	0	0
11	286.73	4	0	0	0	0	0	0
12	286.73	4	0	0	0	0	0	0
13	286.73	4	0	0	0	0	0	0
14	645.15	3	1,000	720	400	0	0	(269)
15	645.15	3	1,522	1,138	818	392	392	(226)
16	645.15	3	0	0	0	0	0	0
17	645.15	3	0	0	0	0	0	0
18	645.15	3	0	0	0	0	0	0
19	645.15	3	1,211	889	569	158	158	(252)
20	645.15	3	707	486	166	0	0	(156)
21	645.15	4	102	2	0	0	0	(0)
22	1,146.94	3	512	330	10	0	0	(81)
23	1,146.94	3	0	0	0	0	0	0
24	1,146.94	3	0	0	0	0	0	0
25	1,146.94	3	0	0	0	0	0	0
26	1,146.94	3	0	0	0	0	0	0
27	1,146.94	3	0	0	0	0	0	0
28	1,146.94	4	551	360	40	0	0	(96)
29	1,577.04	1	2,115	1,615	1,292	836	1,615	600
30	1,577.04	2	0	0	0	0	0	0
31	1,577.04	2	0	0	0	0	0	0
32	1,577.04	2	0	0	0	0	0	0
33	1,577.04	3	0	0	0	0	0	0
34	1,577.04	3	1,798	1,359	1,039	599	599	(204)
35	$1,863.78	1	15,396	14,896	13,896	11,396	14,896	3,655
36	1,863.78	2	0	0	0	0	0	0
37	1,863.78	2	0	0	0	0	0	0
38	1,863.78	3	213	90	0	0	0	(22)
39	1,863.78	3	0	0	0	0	0	0
40	1,863.78	3	295	156	0	0	0	(37)
41	2,293.88	1	0	0	0	0	0	0
42	2,293.88	1	0	0	0	0	0	0
43	2,293.88	1	0	0	0	0	0	0
44	2,293.88	2	0	0	0	0	0	0
45	2,293.88	2	0	0	0	0	0	0
46	2,293.88	3	0	0	0	0	0	0
47	3,154.08	1	5,795	5,295	4,295	3,596	5,295	1,627
48	3,154.08	1	6,588	6,088	5,088	4,191	6,088	1,813
49	3,154.08	1	15,649	15,149	14,149	11,649	15,149	3,691
50	3,154.08	1	39,806	39,306	38,306	35,806	39,306	7,073
51	3,154.08	2	0	0	0	0	0	0
52	4,014.29	1	0	0	0	0	0	0
53	4,014.29	1	593	394	74	0	394	282
54	4,014.29	1	4,960	4,460	3,568	2,970	4,460	1,405
55	4,014.29	2	0	0	0	0	0	0

56	573.47	1	0	0	0	0	0	0
57	573.47	2	0	0	0	0	0	0
58	573.47	2	1,084	787	467	63	467	142
59	573.47	2	0	0	0	0	0	0
60	573.47	3	794	555	235	0	0	(190)
61	573.47	3	1,104	803	483	78	78	(260)
62	573.47	3	275	140	0	0	0	(34)
63	573.47	3	0	0	0	0	0	0
64	573.47	3	0	0	0	0	0	0
65	573.47	3	0	0	0	0	0	0
66	573.47	3	0	0	0	0	0	0
67	573.47	4	0	0	0	0	0	0
68	$573.47	4	0	0	0	0	0	0
69	573.47	4	39	0	0	0	0	0
70	573.47	4	0	0	0	0	0	0
71	573.47	4	0	0	0	0	0	0
72	573.47	4	0	0	0	0	0	0
73	1,003.57	1	1,891	1,433	1,113	668	1,433	568
74	1,003.57	2	1,780	1,344	1,024	585	1,024	233
75	1,003.57	3	0	0	0	0	0	0
76	1,003.57	3	965	692	372	0	0	(256)
77	1,003.57	3	2,261	1,761	1,409	946	946	(174)
78	1,003.57	3	0	0	0	0	0	0
79	1,003.57	4	0	0	0	0	0	0
80	1,003.57	4	0	0	0	0	0	0
81	1,146.94	1	5,563	5,063	4,063	3,422	5,063	1,572
82	1,146.94	2	0	0	0	0	0	0
83	1,146.94	2	0	0	0	0	0	0
84	1,146.94	3	0	0	0	0	0	0
85	1,146.94	3	0	0	0	0	0	0
86	1,146.94	3	0	0	0	0	0	0
87	1,146.94	3	0	0	0	0	0	0
88	2,007.14	1	7,311	6,811	5,811	4,733	6,811	1,983
89	2,007.14	2	997	717	397	0	397	130
90	2,007.14	2	1,218	895	575	164	575	160
91	2,007.14	2	4,536	4,036	3,229	2,652	3,229	477
92	2,007.14	2	232	106	0	0	0	(25)
93	2,437.25	1	1,883	1,426	1,106	662	1,426	567
94	2,437.25	1	3,754	3,254	2,603	2,066	3,254	1,063
95	2,437.25	2	0	0	0	0	0	0
96	2,437.25	3	0	0	0	0	0	0
97	2,867.35	1	6,751	6,251	5,251	4,313	6,251	1,851
98	2,867.35	2	0	0	0	0	0	0
99	3,297.45	1	2,708	2,208	1,767	1,281	2,208	767
100	3,584.19	1	2,079	1,583	1,263	809	1,583	593
Tot	$141,360		$153,970	$139,349	$120,037	$98,122	$129,546	$30,007
i	E[Xi]	Ci	Xi	r1(Xi)	r2(Xi)	r3(Xi)	rci(Xi)	Ai

Number Selecting Reimburesments

j	#
1	24
2	24
3	38
4	14
Tot	100

Table 5
EXPECTATION, VARIANCE, & STANDARD DEVIATIONS of Mean Reimbursements, R/m, & A/m
Sample Selection, Distributions based on Unadjusted Expected Values

Reimbursement #:	1	2	3	4	5	R/m	A/m
Number Selecting	24	24	38	14	0		
Discrete Distribution:							
Mean	$1,871	$1,668	$1,411	$0	$2,027	$1,564	178.543
Variance	818,820	796,109	693,181	0	851,073	774,686	33,655
Std Dev	$905	$892	$833	$0	$923	$880	$183
Pareto Distribution							
Mean	$2,764	$2,457	$2,083	$0	$3,021	$2,472	427.442
Variance	1,301,139	1,284,966	1,008,308	0	3,407,162	1,272,618	53,851
Std Dev	$1,141	$1,134	$1,004	$0	$1,846	$1,128	$232

TABLE 6 SAMPLE GROUP

i	E[Xi]	Chi(i)	Xi=y(i)	E[Xi¦y(i)]	---Chi(i)---- (1)	(2)	(3)	a(i)
1	$286.73	1	$5	$216	3	3	3	$6,000
2	286.73	1	3,358	1,055	3	1	2	8,500
3	286.73	1	4,090	1,238	2	1	2	3,600
4	286.73	2	0	215	2	2	2	3,400
5	286.73	2	0	215	3	3	3	6,800
6	286.73	2	0	215	3	3	3	7,600
7	286.73	3	478	335	3	3	3	7,500
8	286.73	3	0	215	2	2	3	4,400
9	286.73	3	0	215	2	2	3	4,900
10	286.73	4	0	215	3	3	3	7,200
11	286.73	4	0	215	3	3	3	6,500
12	286.73	4	0	215	2	2	3	4,100
13	286.73	4	0	215	3	3	3	8,500
14	645.15	3	1,000	734	3	3	2	11,000
15	645.15	3	1,522	864	3	3	2	14,500
16	645.15	3	0	484	3	3	3	9,500
17	645.15	3	0	484	3	3	3	13,100
18	645.15	3	0	484	2	2	3	5,400
19	645.15	3	1,211	787	3	3	2	12,300
20	645.15	3	707	661	3	3	3	9,000
21	645.15	4	102	509	1	2	2	3,800
22	1,146.94	3	512	988	1	2	2	19,800
23	1,146.94	3	0	860	1	3	2	23,500
24	1,146.94	3	0	860	1	2	2	6,500
25	1,146.94	3	0	860	1	3	2	19,600
26	1,146.94	3	0	860	1	3	2	17,500
27	1,146.94	3	0	860	1	3	2	20,900
28	1,146.94	4	551	998	1	2	2	14,400
29	1,577.04	1	2,115	1,712	1	1	2	31,800
30	1,577.04	2	0	1,183	1	1	2	20,100
31	1,577.04	2	0	1,183	1	1	2	30,300
32	1,577.04	2	0	1,183	1	1	2	31,400
33	1,577.04	3	0	1,183	1	1	2	25,200
34	1,577.04	3	1,798	1,632	1	1	2	7,700
35	$1,863.78	1	15,396	$5,247	1	1	2	$11,300
36	1,863.78	2	0	1,398	1	1	2	28,400
37	1,863.78	2	0	1,398	1	1	2	36,000
38	1,863.78	3	213	1,451	1	1	2	26,600
39	1,863.78	3	0	1,398	1	1	2	7,800
40	1,863.78	3	295	1,472	1	1	2	21,700
41	2,293.88	1	0	1,720	1	1	2	35,000
42	2,293.88	1	0	1,720	1	1	2	15,900
43	2,293.88	1	0	1,720	1	1	2	27,600
44	2,293.88	2	0	1,720	1	1	2	20,800
45	2,293.88	2	0	1,720	1	1	2	39,300
46	2,293.88	3	0	1,720	1	1	2	45,200
47	3,154.08	1	5,795	3,814	1	1	2	17,400
48	3,154.08	1	6,588	4,013	1	1	2	10,800
49	3,154.08	1	15,649	6,278	1	1	2	59,900
50	3,154.08	1	39,806	12,317	1	1	2	8,200

51	3,154.08	2	0	2,366	1	1	2	49,800
52	4,014.29	1	0	3,011	1	1	2	6,500
53	4,014.29	1	593	3,159	1	1	2	79,900
54	4,014.29	1	4,960	4,251	1	1	2	10,000
55	4,014.29	2	0	3,011	1	1	2	40,200
56	573.47	1	0	430	3	3	3	9,500
57	573.47	2	0	430	3	3	3	9,900
58	573.47	2	1,084	701	3	3	2	8,300
59	573.47	2	0	430	3	3	3	8,300
60	573.47	3	794	629	3	3	3	14,300
61	573.47	3	1,104	706	3	3	2	7,400
62	573.47	3	275	499	3	3	3	6,900
63	573.47	3	0	430	3	3	3	14,200
64	573.47	3	0	430	3	3	3	6,300
65	573.47	3	0	430	1	2	2	3,800
66	573.47	3	0	430	3	3	3	12,600
67	573.47	4	0	430	3	3	3	9,400
68	$573.47	4	0	$430	2	2	3	$4,700
69	573.47	4	39	440	3	3	3	13,400
70	573.47	4	0	430	3	3	3	8,500
71	573.47	4	0	430	3	3	3	7,600
72	573.47	4	0	430	3	3	3	5,700
73	1,003.57	1	1,891	1,225	2	1	2	14,700
74	1,003.57	2	1,780	1,198	1	1	2	5,900
75	1,003.57	3	0	753	1	3	2	8,700
76	1,003.57	3	965	994	2	2	2	13,900
77	1,003.57	3	2,261	1,318	2	1	2	13,000
78	1,003.57	3	0	753	1	3	2	8,400
79	1,003.57	4	0	753	1	1	2	5,000
80	1,003.57	4	0	753	2	3	2	12,300
81	1,146.94	1	5,563	2,251	1	1	2	12,800
82	1,146.94	2	0	860	1	1	2	4,700
83	1,146.94	2	0	860	1	1	2	4,300
84	1,146.94	3	0	860	1	3	2	16,300
85	1,146.94	3	0	860	1	3	2	16,700
86	1,146.94	3	0	860	1	2	2	6,500
87	1,146.94	3	0	860	1	3	2	23,900
88	2,007.14	1	7,311	3,333	1	1	2	28,900
89	2,007.14	2	997	1,755	1	1	2	4,200
90	2,007.14	2	1,218	1,810	1	1	2	31,600
91	2,007.14	2	4,536	2,639	1	1	2	34,200
92	2,007.14	2	232	1,563	1	1	2	9,000
93	2,437.25	1	1,883	2,299	1	1	2	24,700
94	2,437.25	1	3,754	2,766	1	1	2	14,600
95	2,437.25	2	0	1,828	1	1	2	28,400
96	2,437.25	3	0	1,828	1	1	2	18,200
97	2,867.35	1	6,751	3,838	1	1	2	10,400
98	2,867.35	2	0	2,151	1	1	2	27,700
99	3,297.45	1	2,708	3,150	1	1	2	47,400
100	3,584.19	1	2,079	3,208	1	1	2	45,100
Tot	$141,360		$153,970	$139,349				

Table 7
EXPECTATION, VARIANCE, & STANDARD DEVIATIONS of Mean Reimbursements, R/m, & A/m
Sample Selection, Distributions based on y Conditioned Expected Values

Reimbursement	1	2	3	4	5	R/m	A/m
No. Selecting:	24	24	38	14	0		
Discrete Distribution:							
Mean	$1,919	$1,721	$1,465	$0	$2,072	$1,657	226.304
Variance	1,134,917	1,112,045	902,406	0	1,295,754	1,104,684	48,116
Std Dev	$1,065	$1,055	$950	$0	$1,138	$1,051	$219
Pareto Distribution							
Mean	$4,178	$3,892	$3,366	$0	$4,599	$3,959	742.948
Variance	5,435,419	5,423,544	3,243,718	0	37,442,838	5,421,039	359,771
Std Dev	$2,331	$2,329	$1,801	$0	$6,119	$2,328	$600

Table 8
VALUES FOR R/m & A/m

		Discrete Distribution			Pareto Distribution		
		Mean	Var	Std Dev	Mean	Var	Std Dev
5.1	R/m	$1,779	812,894	$902	$2,715	1,300,147	$1,140
	A/m	63	2,011	45	182	5,907	77
5.2	R/m	1,831	1,129,125	1,063	4,129	5,434,103	2,331
	A/m	65	4,412	66	218	58,528	242
5.3	R/m	1,795	1,126,295	1,061	4,104	5,433,383	2,331
	A/m	74	7,302	85	272	100,422	317
5.5	R/m	1,697	1,110,082	1,054	3,887	5,423,368	2,329
	A/m	48	3,605	60	142	54,459	233