TRANSACTIONS OF SOCIETY OF ACTUARIES
1950 REPORTS VOL. 2 NO. 4

THE PROGRESSIVE ANNUITY MORTALITY TABLE—A GOMPERTZ ADAPTATION OF THE ANNUITY TABLE FOR 1949 (WITH PROJECTION)

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I. INTRODUCTION

LAST fall in their paper entitled, "A New Mortality Basis for Annuities,"[1] Messrs. W. A. Jenkins and E. A. Lew presented to the Society a thorough analysis of past, present, and probable future mortality among annuitants. In their paper they furnished us "with a more satisfactory basis for annuity premiums and reserves."[2] This more satisfactory basis is called The Annuity Table for 1949 (with Projection), and consists of:

a) A set of values of q_x describing male annuitant mortality during the year of life commencing on January 1, 1950,[3] called The Male Annuity Table for 1949 (without Projection),
b) A set of values of q_x describing female annuitant mortality during the year of life commencing on January 1, 1950,[3] called The Female Annuity Table for 1949 (without Projection), and
c) A set of geometric projection factors, called Projection Scale B, which describes the secular trend in the annuitant mortality for each attained age. With these factors, which vary by age, the projected values of q_x for any subsequent calendar year of exposure may be determined.

Using these data a family of sex-year-of-birth mortality tables can be derived. This might be done by listing in one column the 1950 values of q_x and then applying the projection factors to produce in adjacent columns values of q_x for subsequent calendar years. Each diagonal in the resulting grid represents a sex-year-of-birth mortality table. This family of sex-year-of-birth tables forms the Jenkins-Lew basis for the evaluation of annuity benefits. In Charts 1 and 2 the mortality rates for persons born in 1875, 1900, 1925, and 1950 have been plotted together with The Annuity Table for 1949 (without Projection).

There does not seem to be any simple relationship between the actuarial values for the several Jenkins-Lew sex-year-of-birth tables. Consequently it would appear to be a rather cumbersome matter to use these tables in

[1] *TSA* I, 369–466.
[2] *TSA* I, 370.
[3] *TSA* I, 424 (last paragraph).

CHART 1

MORTALITY RATES FOR THE ANNUITY TABLE FOR 1949—MALES

——— The Annuity Table for 1949 (without Projection)

- - - - Year-of-Birth Tables from the Annuity Table for 1949 (with Projection)

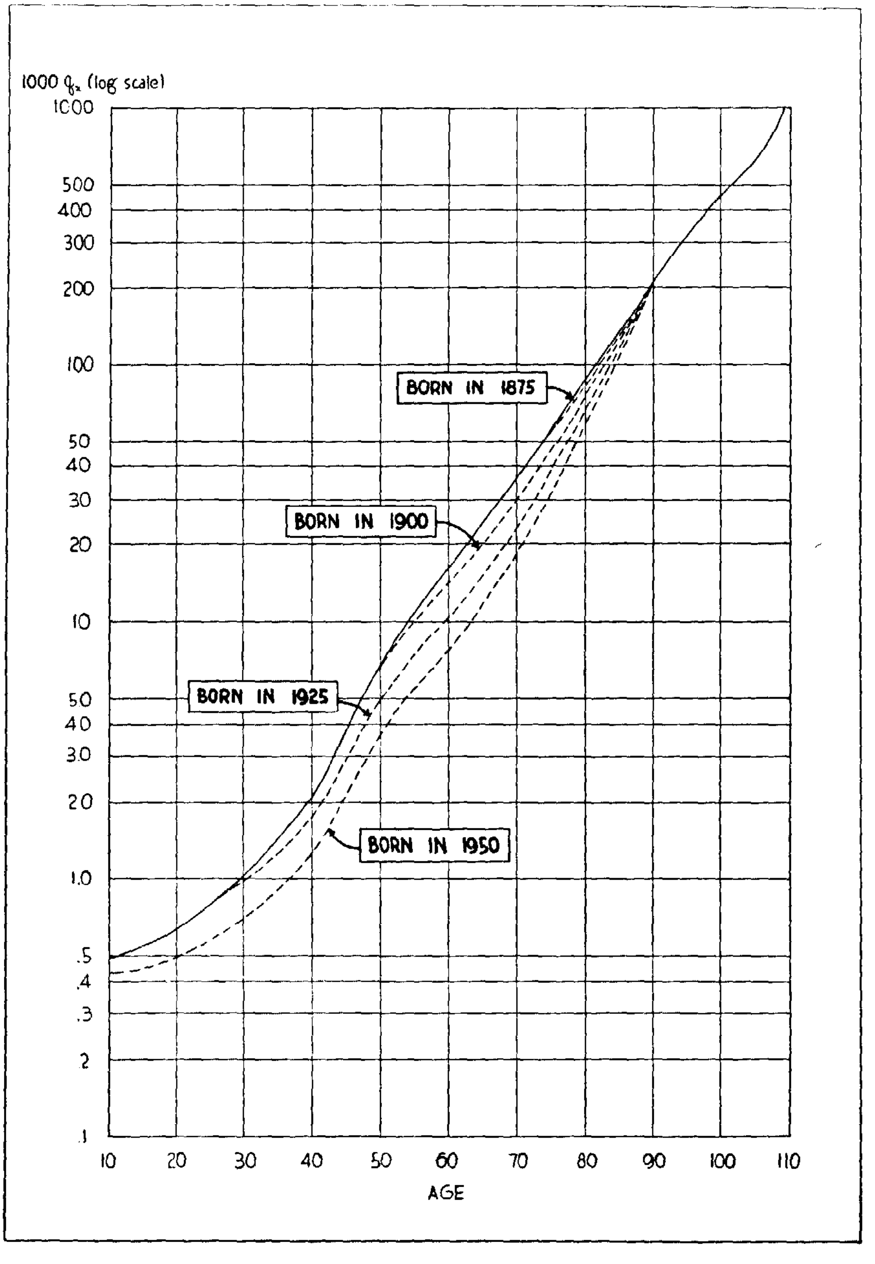
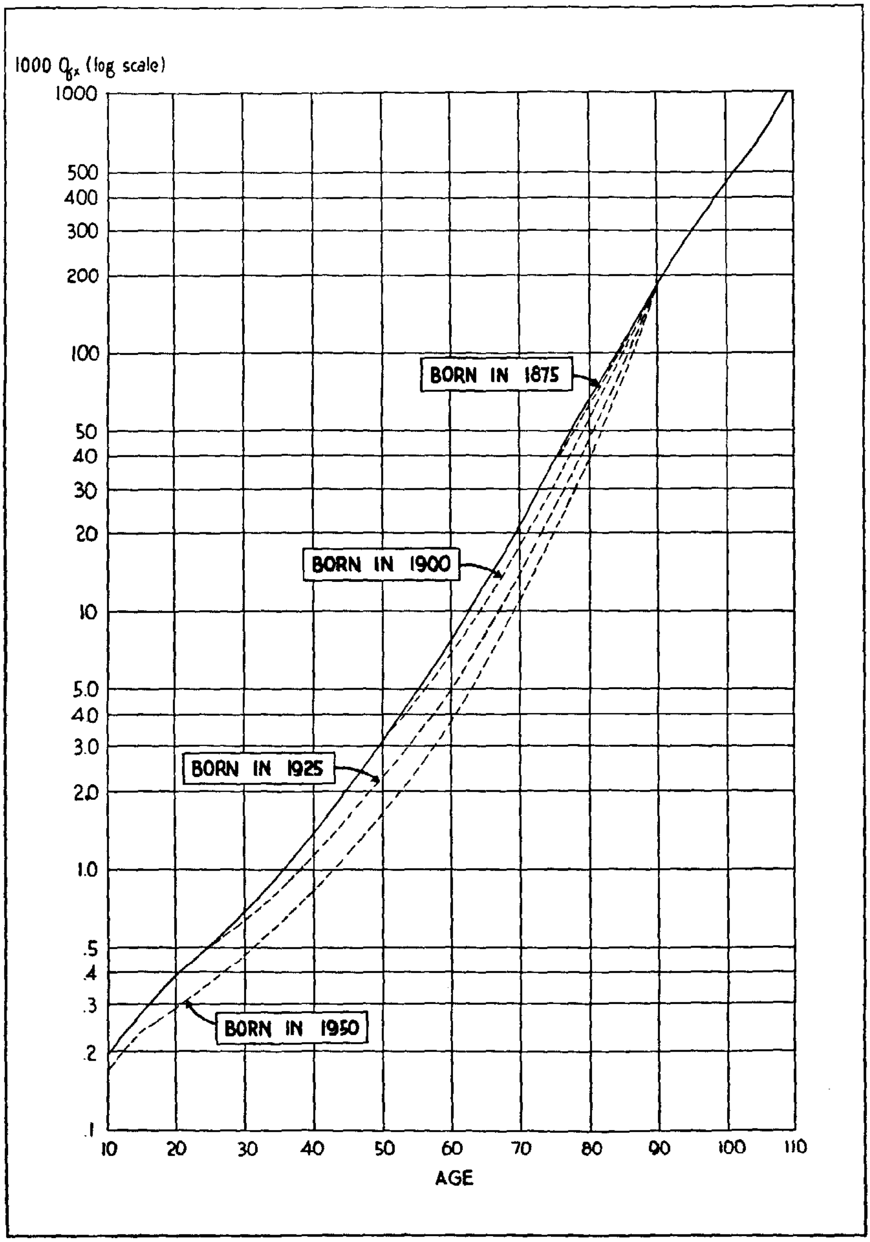


CHART 2

MORTALITY RATES FOR THE ANNUITY TABLE FOR 1949—FEMALES

——— The Annuity Table for 1949 (without Projection)

- - - - - Year-of-Birth Tables from the Annuity Table for 1949 (with Projection)



the routine year-end valuation. Messrs. Jenkins and Lew, in a very descriptive metaphor, invited modification of their new mortality basis. They stated in their paper that they were offering "a bolt of cloth, shears, needles, etc., with which the actuary can fashion a suit designed to satisfy his requirements."[4]

The purpose of the present paper is to propose a family of sex-year-of-birth tables which may be used in place of The Annuity Table for 1949 (with Projection). In this respect the paper merely carries out the suggestion made last fall that, "It would appear possible for the entire (Jenkins-Lew) family of curves to be expressed by . . . a simple master curve, the distinction being through fractional rating of the ages up or down according to birth before or after 1900."[5] This suggestion was a development of the procedure described by Mr. W. A. Jenkins in his paper entitled, "Annuity Premiums and Reserves Based on an Assumption of Decreasing Mortality,"[6] which in turn was a development of the ideas advanced by Mr. Duncan C. Fraser in his 1924 paper entitled, "Notes on Recent Reports on the Mortality of Annuitants."[7]

The new mortality table here proposed makes allowance for progressively improving annuitant mortality at all attained ages and is referred to as "The Progressive Annuity Mortality Table" or more concisely as "The Progressive Table." The values of l_x and d_x, and the derived commutation functions for this Table, are listed in Tables 10, 11, and 12, on the basis of the 1900 year-of-birth group. By a linear transformation of the age, these values may readily be used for other year-of-birth groups. The same table with age adjustment is applicable to male lives and to female lives.

A comparison of The Annuity Table for 1949 (with Projection) and The Progressive Annuity Mortality Table is presented in Charts 3 and 4. This has been done by plotting the single premiums for non-refund immediate annuities assuming interest at 2 percent. Values are shown for persons entering upon the annuities in 1950 and also 1970. Each point on these curves represents a different sex-year-of-birth group. Because of the multiplicity of such groups, this is a concise way in which to make a comprehensive comparison of the two systems. In each chart values are also shown for The 1937 Standard Annuity Mortality Table (set back one year).

This comparison of the non-refund immediate annuities reveals that The Progressive Annuity Mortality Table reproduces with reasonable

[4] *TSA* I, 373.
[5] *TSA* I, 485.
[6] *TASA* XLVII, 265–285.
[7] *JIA* LV, 160.

closeness the single premiums derived according to The Annuity Table for 1949 (with Projection). The principal area in which the two systems produce different results is that for male lives below age 60 who enter during the period 1950–1960. In this area, The Progressive Annuity Mortality Table produces conservative values.

This conservatism arises in part from the fact that Jenkins and Lew did not assume any improvement in mortality at the old ages. The Progressive Table does allow for such improvement. This is illustrated in Charts 5 and 6, where the values of q_x are plotted for the 1900 and 1950 year-of-birth groups. It will be noted from these charts that, since The Progressive Table was designed for simplicity in operation, it does not reproduce all of the fluctuations which are characteristic of recent annuitant mortality. It tends to assume conservative values for q_x at the lower ages.

The Progressive Annuity Mortality Table was originally prepared for the valuation of individual annuities and life income settlements. It produces satisfactory results for this purpose with relative ease. One way that this can be done is to provide on each detailed punch card for two special fields showing the Valuation Year of Birth (Male Basis) and, in the case of the deferred annuities, the Valuation Age (Male Basis) at which annuity payments commence. These fields would be computed in such a way that the valuation could be made without further allowance for sex or year-of-birth variations.

A test was made to measure the effect of using The Progressive Table in the determination of aggregate reserves. As of December 31, 1949, the reserve set up by The Northwestern Mutual Life Insurance Company for its individual immediate annuity business was $49.6 millions. This reserve had been calculated on The 1937 Standard Annuity Mortality Table at 2% (set back one year). According to The Progressive Table at 2% this reserve would have been $49.8 millions.

The Progressive Table allows for the secular trend in annuitant mortality. In order to get some idea of the effect of this progressively improving mortality on the aggregate liability the reserve factors for December 31, 1959 were applied against this same distribution of annuity business. The resulting aggregate reserve according to The Progressive Table at 2% was $50.5 millions or an increase of about 1.4% between December 31, 1949 and December 31, 1959.

One of the properties of The Progressive Annuity Mortality Table is that an advance of twenty-five years in the year-of-birth is handled by a one year adjustment in the age of the annuitant. This property will assist in a solution of the dilemma that we face with regard to guaranteed

CHART 3

SINGLE PREMIUMS FOR NON-REFUND IMMEDIATE ANNUITIES—MALES
(Interest at 2%)

——— The Progressive Annuity Mortality Table
— - — The Annuity Table for 1949 (with Projection)
- - - - - The 1937 Standard Annuity Table (1 year setback)

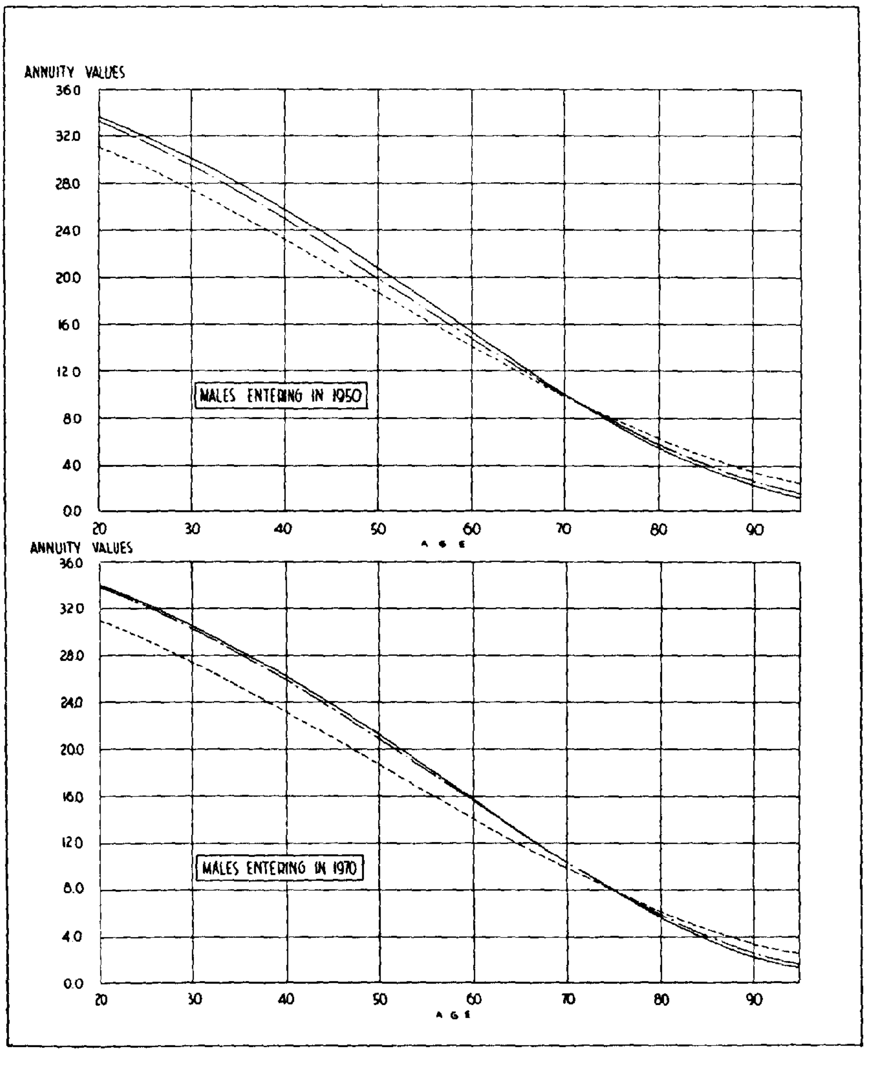


CHART 4

SINGLE PREMIUMS FOR NON-REFUND IMMEDIATE ANNUITIES—FEMALES
(Interest at 2%)

——— The Progressive Annuity Mortality Table
— - — The Annuity Table for 1949 (with Projection)
- - - - - The 1937 Standard Annuity Table (1 year setback)

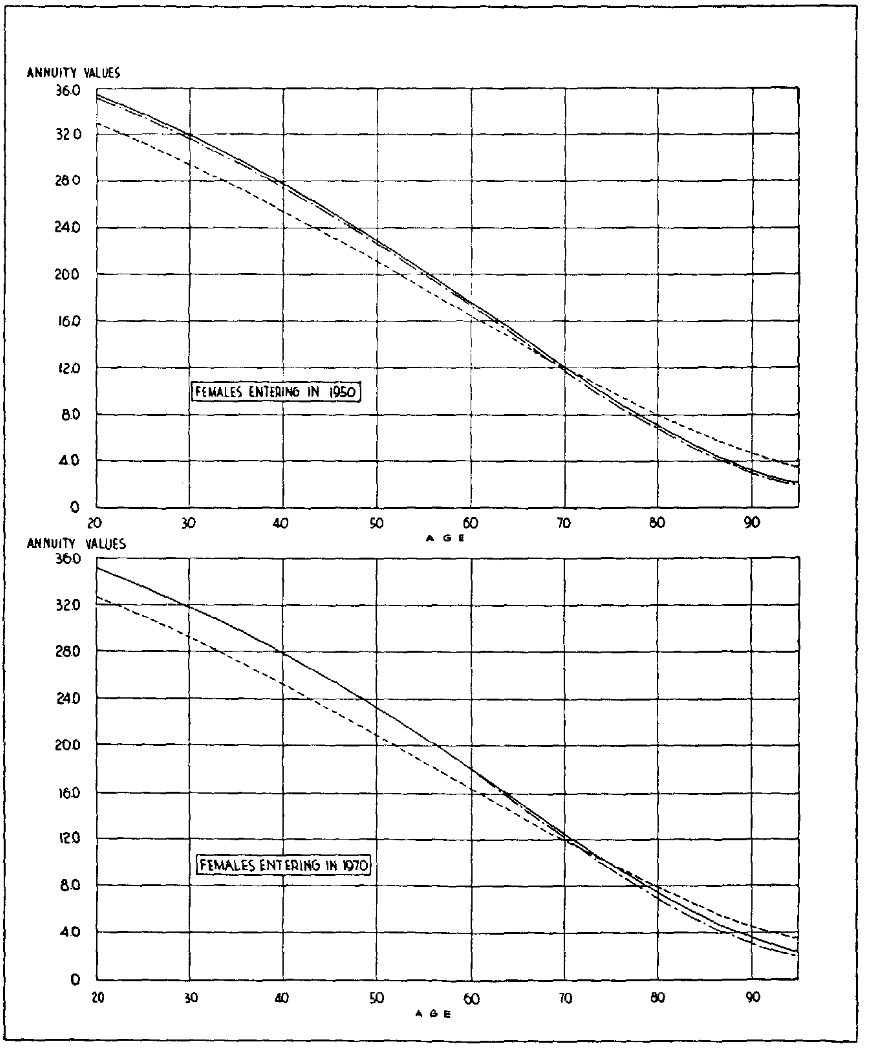


CHART 5

MORTALITY RATES FOR THE 1900 AND 1950 YEAR-OF-BIRTH GROUPS—MALES

——— The Progressive Annuity Mortality Table

- - - - - The Annuity Table for 1949 (with Projection)

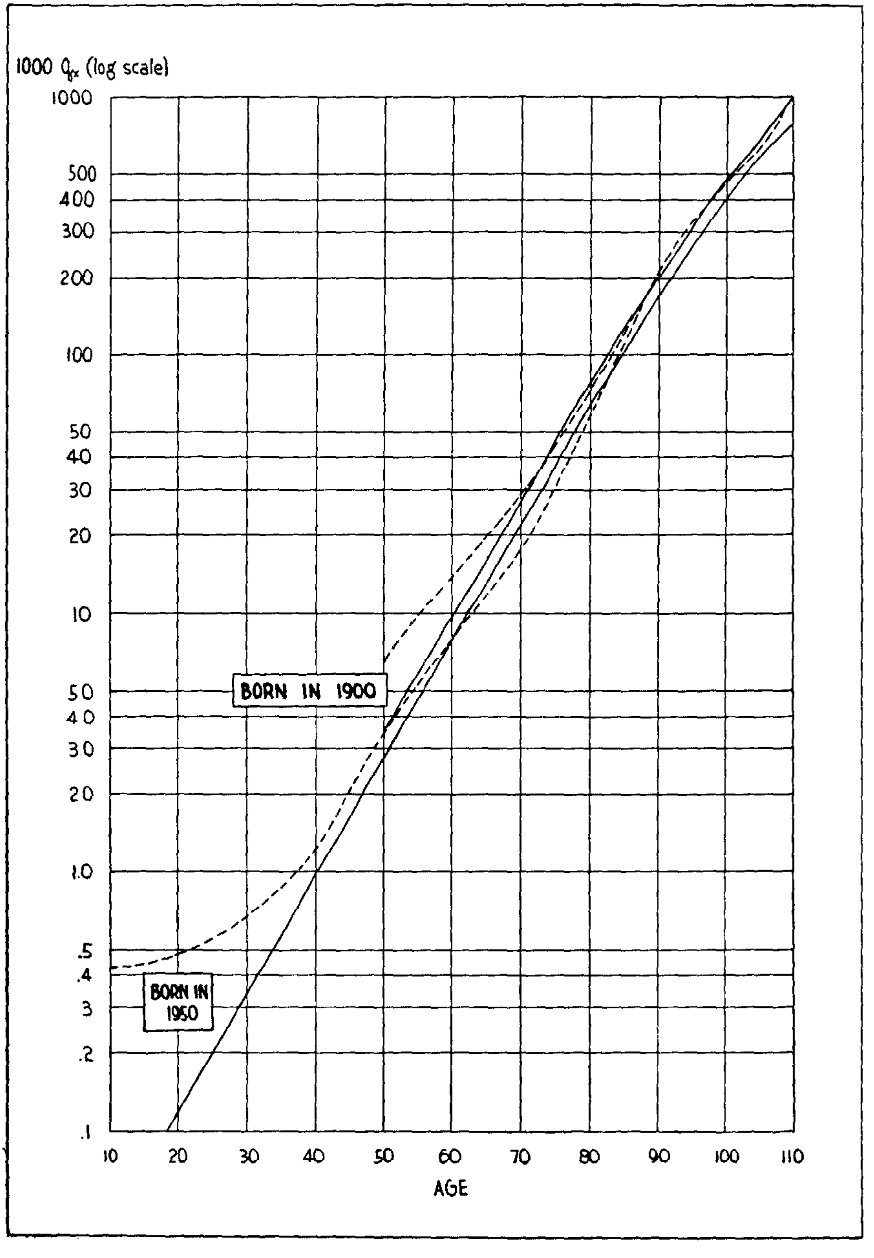
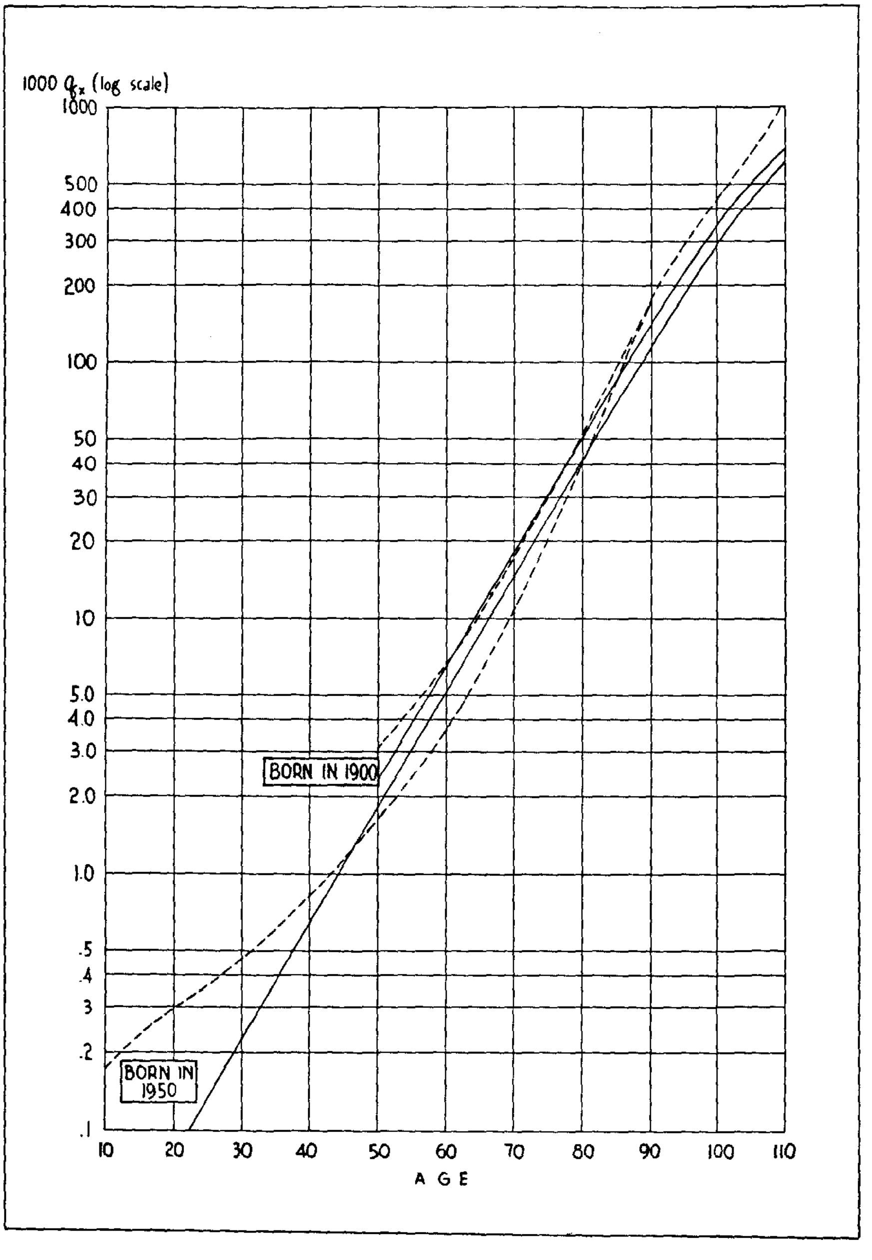


CHART 6

MORTALITY RATES FOR THE 1900 AND 1950 YEAR-OF-BIRTH GROUPS—FEMALES

——— The Progressive Annuity Mortality Table

- - - - - The Annuity Table for 1949 (with Projection)



settlement options. As illustrated in Table 1, a slight modification in the age column of the settlement option table will provide for improving beneficiary mortality and will avoid the present situation in which some of us promise to settle with beneficiaries yet unborn on the same basis as with current beneficiaries.

TABLE 1

LIFE INCOME WITH INSTALLMENTS CERTAIN
MONTHLY INSTALLMENTS FOR EACH $1,000 OF NET PROCEEDS

Beneficiary's Age at Settlement Where Beneficiary's Year of Birth Is:										Payments Certain (Assuming Interest at 2%)			
Prior to 1900		1900–1924		1925–1949		1950–1974		1975 and Subsequent		10 Years	15 Years	20 Years	Installment Refund
Male	Female	Male	Female	Male	Female	Male	Female	Male	Female				
....	.....	....	.....	....	.....	....	.....	....	.....	......	......	......	......
....	.....	....	.....	....	.....	....	.....	....	.....	......	......	......	......
....	.....	....	.....	....	.....	....	.....	....	.....	......	......	......	......
60	64	61	65	62	66	63	67	64	68	5.06	4.81	4.46	4.53
61	65	62	66	63	67	64	68	65	69	5.21	4.92	4.52	4.65
62	66	63	67	64	68	65	69	66	70	5.37	5.03	4.59	4.77
....	.....	....	.....	....	.....	....	.....	....	.....	......	......	......	......
....	.....	....	.....	....	.....	....	.....	....	.....	......	......	......	......
....	.....	....	.....	....	.....	....	.....	....	.....	......	......	......	......

Along the same lines, the maturity values of policies providing income settlement, such as retirement annuities or retirement endowments, might be determined by taking into account the year of birth of the annuitant.

II. THE PROGRESSIVE ANNUITY MORTALITY TABLE

The Progressive Annuity Mortality Table is a Gompertz Table with the constant "c" equal to 1.110 and the value of the constant "β" dependent upon the sex-year-of-birth group. There is a sex variation of four years and a year-of-birth variation of one twenty-fifth of a year.

Actuarial values for The Progressive Annuity Mortality Table are listed in Tables 10, 11, and 12. These were derived on the basis of the 1900 year-of-birth group, as follows:

$$\operatorname{colog}_e p_x^{1900(m)} = \beta^{1900(m)} \cdot c^x \text{ for male lives, and}$$

$$\operatorname{colog}_e p_x^{1900(f)} = \beta^{1900(f)} \cdot c^x \text{ for female lives}$$

where the constants take the values:

$\log_{10} \beta^{1900(m)}$	5.2740390−10
$\log_{10} \beta^{1900(f)}$	5.0927470−10
c	1.110
$\log_{10} c$	.0453230

We defined the radix as $l_6^{1900(m)} = 1{,}000{,}000$, and then derived the remaining values of $l_x^{1900(m)}$ using the expression

$$l_{x+1}^{1900(m)} = p_x^{1900(m)} \cdot l_x^{1900(m)} \, .$$

All the published actuarial functions are based on these $l_x^{1900(m)}$ values. It will be observed that

$$\log_{10} \beta^{1900(f)} = \log_{10} \beta^{1900(m)} - 4 \log_{10} c \, .$$

That is,

$$\beta^{1900(f)} = \beta^{1900(m)} \cdot c^{-4.00} \, .$$

Consequently,

$$\operatorname{colog}_e p_x^{1900(f)} = \operatorname{colog}_e p_{x-4.00}^{1900(m)} \, .$$

If we set

$$l_{10}^{1900(f)} = 1{,}000{,}000 \, ,$$

then

$$l_x^{1900(f)} = l_{x-4.00}^{1900(m)} \text{ for all values of } x \, .$$

The foregoing β relationship is a general one. That is, for any year-of-birth Z

$$\beta^{Z(f)} = \beta^{Z(m)} \cdot c^{-4.00} \, .$$

Furthermore, for The Progressive Table, the male year-of-birth values of β are related as follows:

$$\beta^{Z_1(m)} \cdot c^{0.04Z_1} = \beta^{Z_2(m)} \cdot c^{0.04Z_2} \, .$$

A similar relation may be written for female lives. If we use the 1900 year-of-birth group as a base, then we can write

$$\beta^{Z(m)} = \beta^{1900(m)} \cdot c^{0.04(1900-Z)} \text{ and}$$

$$\operatorname{colog}_e p_x^{Z(m)} = \operatorname{colog}_e p_{x+0.04(1900-Z)}^{1900(m)} \, .$$

If, by definition for one value of x,

$$l_x^{Z(m)} = l_{x+0.04(1900-Z)}^{1900(m)}$$

then this relation exists for all values of x. It follows that all the commutation functions are similarly related.

The following examples will serve to illustrate how The Progressive Table may be used in practice.

Example No. 1

Using The Progressive Annuity Mortality Table, derive the rate of mortality during the year of life commencing in 1975 for a male annuitant born in 1925, that is, for $x = 50$ and $Z = 1925$.

$$p_x^{Z(m)} = p_{x+0.04(1900-Z)}^{1900(m)}$$

$$\therefore \quad q_x^{Z(m)} = q_{x+0.04(1900-Z)}^{1900(m)}$$

$$\therefore \quad q_{50}^{1925(m)} = q_{50+0.04(1900-1925)}^{1900(m)}$$

$$= q_{49}^{1900(m)}$$

$$= 3.120 \text{ per M (as read from Table 10)}.$$

For further examples of this type refer to Table 9.

Example No. 2

Using The Progressive Annuity Mortality Table and 2 percent interest, derive the value in 1950 of a non-refund immediate annuity of one per annum to a female annuitant born in 1900.

$$a_{50}^{1900(f)} = 22.810 \text{ (as read from Table 10)}.$$

Example No. 3

Using The Progressive Annuity Mortality Table and 2 percent interest, derive the value in 1955 of a non-refund immediate annuity of one per annum to a male annuitant born in 1878.

$$a_{77}^{1878(m)} = a_{77+.04(1900-1878)}^{1900(m)}$$

$$= a_{77.88}^{1900(m)}$$

$$= 6.776.$$

For further examples of this type, see Tables 5, 6, 7, and 8.

Example No. 4

Express $a_{x_1:x_2}^{Z_1(m):Z_2(f)}$ in terms of a single life annuity for a male life born in 1900, where this annuity symbol represents the single premium for a joint life annuity entered upon in the calendar year in which the two annuitants attain ages x_1 and x_2 respectively. The first annuitant is male and was born in calendar year Z_1; the second annuitant is female and was born in calendar year Z_2.

$$a_{x_1:x_2}^{Z_1(m):Z_2(f)} = \sum_{t=1}^{t=\omega} v^t \cdot {}_tp_{x_1}^{Z_1(m)} \cdot {}_tp_{x_2}^{Z_2(f)}$$

$$= \sum_{t=1}^{t=\omega} v^t \cdot {}_tp_{x_1+.04(1900-Z_1)}^{1900(m)} \cdot {}_tp_{x_2-4.00+.04(1900-Z_2)}^{1900(m)}$$

$$= a_{y_1:y_2}^{1900(m):1900(m)}$$

where

$$y_1 = x_1 + .04\,(1900 - Z_1)$$

$$y_2 = x_2 - 4.00 + .04\,(1900 - Z_2)\,.$$

If

$$c^w = c^{y_1} + c^{y_2}\,,$$

then

$$a_{x_1:x_2}^{Z_1(m):Z_2(f)} = a_w^{1900(m)}\,.$$

Table 13 has been prepared to facilitate the computation of the equivalent single age for two joint lives. Where x and y are the joint lives ($x \geqq y$) and w is the equivalent single age, the relation used in deriving Table 13 was

$$w - x = \frac{\log_{10}(1 + c^{y-x})}{\log_{10} c}\,.$$

In using Table 13 it is necessary that both ages be on the same sex-year-of-birth basis.

Example No. 5

Using The Progressive Annuity Mortality Table, and 2 percent interest, derive the value in 1963 of a joint and survivor non-refund immediate annuity of one per annum where the lives at risk include a male annuitant born in 1890 and a female annuitant born in 1903.

$$a_{\overline{73:60}}^{1890(m):1903(f)} = a_{73}^{1890(m)} + a_{60}^{1903(f)} - a_{73:60}^{1890(m):1903(f)}$$

$$= a_{73.40}^{1900(m)} + a_{59.88}^{1900(f)} - a_{73.40:55.88}^{1900(m):1900(m)}$$

$$= a_{73.40}^{1900(m)} + a_{59.88}^{1900(f)} - a_{73.40+1.43}^{1900(m)}$$

$$= 8.759 + 17.768 - 8.102$$

$$= 18.425\,.$$

Mr. Duncan C. Fraser, in Par. 38–43 of his 1924 paper "Notes on Recent Reports of the Mortality of Annuitants,"[8] considered the rela-

[8] *JIA* LV, 173.

tionship between a succession of calendar year curves which he calls the (*yq*) curves, and a succession of derived curves to be experienced by annuitants of successive years of birth which he calls the (*aq*) curves. He shows how the (*aq*) curves are distinct from the (*yq*) curves, forming with them a diamond pattern.

He goes on to discuss the curves if according to the Gompertz Law and if the calendar year mortality changes continuously at a uniform rate, and shows that the several (*aq*) curves calculated for annuitants of successive attained ages in a given year, say 1925, give the means of dealing with annuity values in any subsequent year. He states that "the annuity-values being settled for a particular epoch, the table remains unchanged in future years, all that is required being a periodical shift of ages." The Progressive Table herein is such an (*aq*) curve, the rule for age setback being one twenty-fifth of a year for each yearly differential in the years of birth. This means, for example, that $a_{50}^{1900(m)}$ is equal to $a_{51}^{1925(m)}$. It will be noted that the annuitant born in 1900 attains age 50 in 1950, while the annuitant born in 1925 attains age 51 in 1976. Consequently, according to The Progressive Table, the shift of ages occurs every twenty-six calendar years.

Mr. Fraser's illustrative rule for age setback was one-tenth year per calendar year. It is of interest to note from *JIA* LXXIV, 131, that twenty years later the experience in Great Britain indicated a lower setback of about one-thirteenth year for female and one-twentieth year for male lives, for use with the British annuity tables based on 1900–1920 with forecast.

III. COMPARISON OF MORTALITY TABLES

The single premiums for non-refund immediate annuities according to three different mortality bases were compared in Charts 3 and 4. In Tables 5, 6, 7, and 8, additional single premium comparisons are made. Table 5 deals with non-refund immediate annuities; Table 6, with ten year deferred annuities; Table 7, with ten year certain annuities.

Table 8 will be of particular interest to those working with group annuities. The single premiums for annuities deferred to age 65 are tabulated there.

A comparison of the values of q_x for persons born in 1875, 1900, and 1925, is presented in Table 9. A study of these values reveals that for The Progressive Table the annual rate of decrease in the mortality rate is approximately 0.4% at all attained ages. The comparable figures for Projection Scale B range from 1.25% at ages 20–50 to 0.00% above age 89.

It should be noted that in dealing with annuities we are more interested in the improvement in the probability of survival than in the decrease in the probability of death. A large percentage decrease in the value of q_x at a young age where p_x approaches unity is of much less importance than a small percentage decrease in q_x at a higher age. Last fall[9] Mr. Sternhell published a table showing, for Projection Scale A and Projection Scale B, the increase in the values of p_x between 1950 and 1970. In Table 2, his figures are repeated together with comparable values for

TABLE 2

INCREASE IN THE VALUE OF p_x FROM 1950 TO 1970

Age	Male Lives			Female Lives		
	Projection Scale A	Projection Scale B	Progressive Table	Projection Scale A	Projection Scale B	Progressive Table
20	.03%	.01%	.00%	.02%	.01%	.00%
30	.04%	.02%	.00%	.03%	.02%	.00%
40	.07%	.05%	.01%	.05%	.03%	.01%
50	.18%	.15%	.03%	.09%	.07%	.02%
60	.34%	.34%	.08%	.16%	.16%	.05%
70	.54%	.63%	.25%	.32%	.37%	.16%
80	.72%	.89%	.72%	.51%	.62%	.47%
90	.00%	.00%	2.17%	.00%	.00%	1.42%
100	.00%	.00%	6.50%	.00%	.00%	4.23%

The Progressive Annuity Mortality Table. As already noted, Jenkins-Lew did not assume any improvement in mortality after age 89. Perhaps it would have been more conservative to do so. The Progressive Table, *sui generis*, does make provision for such improvement.

IV. PREPARATION OF THE PROGRESSIVE ANNUITY MORTALITY TABLE

The first step in the preparation of The Progressive Annuity Mortality Table was to derive, using Projection Scale B, a complete set of values of $q_x^{Z(m)}$ and $q_x^{Z(f)}$ for The Annuity Table for 1949 (with Projection). The Z in these expressions represents the year of birth. From the resulting values, twelve sex-year-of-birth tables were chosen. These were the tables for which Z = 1871, 1881, 1891, 1901, 1911, and 1921.

For each of these twelve tables, the value of the constant "c" in the Gompertz formula was derived by equating first and second moments. The values so derived are listed in Table 3.

After reviewing these values, it was decided to proceed using $c = 1.110$ for all of the sex-year-of-birth tables. It was recognized at the time that this choice would tend to understate the value of $a_x^{Z(f)}$ for the younger

[9] *TSA* I, 488.

ages in each age range. However, it was felt that a test should be made to see if reasonable results might be obtained keeping the same value of "c" throughout and making some empirical adjustments in the value of $\log_{10}\beta$. Proceeding on this basis the value of $\log_{10}\beta$ was derived for each of the twelve sex-year-of-birth tables by equating first moments.

TABLE 3

Sex-Year-of-Birth Group	Age Range	Value of "c"
1871(m)	79–99	1.100
1881(m)	69–99	1.100
1891(m)	59–99	1.100
1901(m)	49–99	1.100
1911(m)	39–99	1.102
1921(m)	29–99	1.105
1871(f)	79–99	1.119
1881(f)	69–99	1.119
1891(f)	59–99	1.120
1901(f)	49–99	1.120
1911(f)	39–99	1.121
1921(f)	29–99	1.122

TABLE 4

Sex-Year-of-Birth Group (1)	Age Range (2)	Crude Value of $\log_{10}\beta$ (3)	Age Relation for Col. (3) (4)	Final Values of $\log_{10}\beta$ (5)	Age Relation for Col. (5) (6)
1871(m)	79–99	5.296 9364–10	+0.5	5.326 6137–10	+1.16
1881(m)	69–99	5.302 3074–10	+0.6	5.308 4845–10	+0.76
1891(m)	59–99	5.299 6670–10	+0.6	5.290 3553–10	+0.36
1901(m)	49–99	5.289 6557–10	+0.3	5.272 2261–10	−0.04
1911(m)	39–99	5.271 2601–10	−0.1	5.254 0969–10	−0.44
1921(m)	29–99	5.251 0915–10	−0.5	5.235 9677–10	−0.84
1871(f)	79–99	5.193 2339–10	−1.8	5.145 3217–10	−2.84
1881(f)	69–99	5.173 6961–10	−2.2	5.127 1925–10	−3.24
1891(f)	59–99	5.155 2563–10	−2.6	5.109 0633–10	−3.64
1901(f)	49–99	5.138 0700–10	−3.0	5.090 9341–10	−4.04
1911(f)	39–99	5.121 1702–10	−3.4	5.072 8049–10	−4.44
1921(f)	29–99	5.104 1098–10	−3.7	5.054 6757–10	−4.84

The values of a_x for several trial mortality tables were then prepared and these values were compared with similar values for The Annuity Table for 1949 (with Projection). These comparisons suggested that two principal adjustments be made. For the female tables, the values of $\log_{10}\beta$ were reduced by subtracting $\log_{10} c$ from the crude values. This adjustment tended to correct, in some measure, the understatement that was introduced when the value of "c" had been fixed. With regard to the male tables, the principal empirical adjustment was to modify the values of $\log_{10} \beta$ so that the one twenty-fifth relationship which already obtained for the female tables would also apply to the male year-of-birth tables.

The effect of these adjustments may be followed by studying Table 4. Column 3 shows the values originally derived for $\log_{10} \beta$, while Column 5

TABLE 5

SINGLE PREMIUMS FOR NON-REFUND IMMEDIATE LIFE ANNUITIES

a_x with Interest at 2%

Attained Age	1937 Standard Annuity Table (1 Year Setback)	Entered in 1950		Entered in 1970	
		1949 Table (with Projection)	Progressive Table	1949 Table (with Projection)	Progressive Table
Male lives:					
30	27.41	29.56	30.14	30.29	30.44
40	23.22	25.04	25.81	25.90	26.17
50	18.72	19.93	20.82	20.87	21.22
60	14.14	14.80	15.37	15.65	15.80
70	9.85	9.86	10.00	10.44	10.39
Female lives:					
30	29.31	31.41	31.61	31.91	31.90
40	25.37	27.32	27.56	27.91	27.90
50	21.00	22.54	22.81	23.20	23.20
60	16.42	17.18	17.49	17.82	17.92
70	11.94	11.63	12.01	12.11	12.43

shows the final values adopted for The Progressive Annuity Mortality Table. Column 4 shows the age relation existing among the crude values of $\log_{10} \beta$, while Column 6 shows the final age relation existing among the listed year-of-birth tables. The figures in Columns 4 and 6 were derived using the final value of $\log_{10} \beta^{1900(m)}$ as the base. Each of these figures therefore represents the excess of the stated value over the final value for $\log_{10} \beta^{1900(m)}$ divided by $\log_{10} c$.

TABLE 6

SINGLE PREMIUMS FOR NON-REFUND IMMEDIATE LIFE ANNUITIES DEFERRED FOR TEN YEARS

${}_{10|}a_x$ with Interest at 2%

Attained Age	1937 Standard Annuity Table (1 Year Setback)	Entered in 1950		Entered in 1970	
		1949 Table (with Projection)	Progressive Table	1949 Table (with Projection)	Progressive Table
Male lives:					
30	18.54	20.64	21.18	21.35	21.49
40	14.48	16.19	16.91	17.02	17.26
50	10.24	11.34	12.07	12.20	12.45
60	6.19	6.69	7.05	7.37	7.42
70	2.89	2.77	2.77	3.09	3.04
Female lives:					
30	20.41	22.47	22.65	22.96	22.93
40	16.55	18.42	18.63	18.99	18.96
50	12.37	13.74	13.98	14.36	14.36
60	8.16	8.66	8.95	9.21	9.34
70	4.41	3.91	4.25	4.22	4.57

TABLE 7

SINGLE PREMIUMS FOR IMMEDIATE LIFE ANNUITIES WITH PAYMENTS CERTAIN FOR TEN YEARS

$a_{\overline{10}|} + {}_{10|}a_x$ with Interest at 2%

Attained Age	1937 Standard Annuity Table (1 Year Setback)	Entered in 1950		Entered in 1970	
		1949 Table (with Projection)	Progressive Table	1949 Table (with Projection)	Progressive Table
Male lives:					
30	27.52	29.62	30.17	30.33	30.47
40	23.47	25.17	25.89	26.00	26.24
50	19.22	20.33	21.05	21.18	21.44
60	15.17	15.67	16.03	16.35	16.41
70	11.87	11.75	11.76	12.08	12.03
Female lives:					
30	29.39	31.45	31.63	31.94	31.91
40	25.53	27.40	27.61	27.97	27.95
50	21.35	22.73	22.96	23.34	23.34
60	17.14	17.64	17.94	18.19	18.32
70	13.39	12.89	13.23	13.20	13.55

TABLE 8

SINGLE PREMIUMS FOR NON-REFUND LIFE ANNUITIES DEFERRED TO AGE 65

${}_{64-x|}a_x$ with Interest at 2%

Attained Age	1937 Standard Annuity Table (1 Year Setback)	Entered in 1950		Entered in 1970	
		1949 Table (with Projection)	Progressive Table	1949 Table (with Projection)	Progressive Table
Male lives:					
30	4.70	6.05	6.28	6.57	6.53
40	5.89	7.14	7.55	7.79	7.85
50	7.61	8.61	9.20	9.39	9.57
60	10.51	11.15	11.68	11.97	12.09
Female lives:					
30	6.08	7.47	7.54	7.85	7.79
40	7.56	8.94	9.08	9.41	9.38
50	9.59	10.81	11.03	11.38	11.39
60	12.73	13.45	13.76	14.07	14.18

TABLE 9

MORTALITY RATES ($1000q_x$) FOR THE 1875, 1900, AND 1925 YEAR-OF-BIRTH GROUPS

Attained Age	1937 Standard Annuity Table (1 Year Setback)	Born 1875		Born 1900		Born 1925	
		1949 Table	Progressive Table	1949 Table	Progressive Table	1949 Table	Progressive Table
Male lives:							
30	1.936	....	....	....	....	.943	.388
40	4.037	....	....	....	....	1.678	1.100
50	8.613	....	....	6.557	3.463	4.788	3.120
60	18.321	....	....	13.880	9.801	10.262	8.834
70	38.763	....	....	28.994	27.580	22.839	24.881
80	81.050	83.387	84.372	73.560	76.339	64.897	69.042
90	165.320	208.485	221.420	208.485	201.865	208.485	183.835
100	331.840	463.415	508.680	463.415	472.832	463.415	438.299
Female lives:							
30	1.496	....	....	....	....	.637	.255
40	2.763	....	....	....	....	1.122	.725
50	5.898	....	....	3.109	2.282	2.271	2.056
60	12.566	....	....	6.649	6.467	4.917	5.828
70	26.675	....	....	17.320	18.254	13.643	16.460
80	56.167	59.895	56.411	52.840	50.966	46.616	46.033
90	116.257	176.161	151.995	176.161	138.028	176.161	125.245
100	232.198	449.400	373.830	449.400	344.095	449.400	316.102

TABLE 10

THE PROGRESSIVE ANNUITY MORTALITY TABLE

Elementary Functions and Annuity Values for the 1900 Year-of-Birth Groups

Age x		l_x	d_x	$1000q_x$	a_x At 2%	a_x At 2½%
Male	Female					
6	10	1000000	35	.035	37.374	32.729
7	11	999965	39	.039	37.123	32.549
8	12	999926	43	.043	36.867	32.364
9	13	999883	48	.048	36.606	32.174
10	14	999835	53	.053	36.340	31.980
11	15	999782	59	.059	36.069	31.781
12	16	999723	66	.066	35.792	31.578
13	17	999657	73	.073	35.510	31.370
14	18	999584	81	.081	35.223	31.156
15	19	999503	90	.090	34.931	30.938
16	20	999413	100	.100	34.632	30.714
17	21	999313	111	.111	34.329	30.485
18	22	999202	123	.123	34.019	30.250
19	23	999079	136	.136	33.704	30.011
20	24	998943	151	.151	33.382	29.765
21	25	998792	168	.168	33.055	29.514
22	26	998624	187	.187	32.722	29.257
23	27	998437	207	.207	32.383	28.994
24	28	998230	230	.230	32.037	28.725
25	29	998000	254	.255	31.686	28.450
26	30	997746	282	.283	31.327	28.168
27	31	997464	313	.314	30.963	27.881
28	32	997151	348	.349	30.592	27.587
29	33	996803	387	.388	30.215	27.286
30	34	996416	428	.430	29.831	26.979
31	35	995988	475	.477	29.441	26.666
32	36	995513	528	.530	29.044	26.345
33	37	994985	585	.588	28.641	26.018
34	38	994400	649	.653	28.231	25.684
35	39	993751	720	.725	27.814	25.344
36	40	993031	798	.804	27.391	24.996
37	41	992233	886	.893	26.961	24.642
38	42	991347	982	.991	26.525	24.280
39	43	990365	1089	1.100	26.082	23.912
40	44	989276	1208	1.221	25.633	23.537
41	45	988068	1339	1.355	25.178	23.155
42	46	986729	1484	1.504	24.716	22.766
43	47	985245	1644	1.669	24.249	22.370
44	48	983601	1823	1.853	23.775	21.968
45	49	981778	2019	2.056	23.295	21.559
46	50	979759	2236	2.282	22.810	21.143
47	51	977523	2476	2.533	22.320	20.721
48	52	975047	2742	2.812	21.824	20.293
49	53	972305	3034	3.120	21.323	19.859
50	54	969271	3357	3.463	20.818	19.419

TABLE 10—*Continued*

Age x Male	Age x Female	l_x	d_x	$1000q_x$	a_x At 2%	a_x At 2½%
51	55	965914	3712	3.843	20.308	18.974
52	56	962202	4104	4.265	19.794	18.523
53	57	958098	4535	4.733	19.276	18.068
54	58	953563	5008	5.252	18.755	17.608
55	59	948555	5528	5.828	18.232	17.143
56	60	943027	6099	6.467	17.705	16.675
57	61	936928	6723	7.176	17.177	16.203
58	62	930205	7406	7.962	16.647	15.728
59	63	922799	8152	8.834	16.116	15.250
60	64	914647	8964	9.801	15.585	14.771
61	65	905683	9848	10.874	15.054	14.290
62	66	895835	10806	12.062	14.524	13.808
63	67	885029	11842	13.380	13.995	13.326
64	68	873187	12959	14.841	13.469	12.845
65	69	860228	14159	16.460	12.945	12.364
66	70	846069	15444	18.254	12.425	11.885
67	71	830625	16814	20.243	11.909	11.409
68	72	813811	18264	22.443	11.398	10.936
69	73	795547	19794	24.881	10.893	10.467
70	74	775753	21395	27.580	10.395	10.002
71	75	754358	23058	30.566	9.903	9.543
72	76	731300	24771	33.873	9.420	9.090
73	77	706529	26514	37.527	8.945	8.644
74	78	680015	28267	41.568	8.480	8.205
75	79	651748	30002	46.033	8.024	7.775
76	80	621746	31688	50.966	7.580	7.354
77	81	590058	33286	56.411	7.147	6.943
78	82	556772	34753	62.419	6.725	6.542
79	83	522019	36041	69.042	6.317	6.152
80	84	485978	37099	76.339	5.921	5.773
81	85	448879	37873	84.372	5.538	5.407
82	86	411006	38309	93.208	5.170	5.053
83	87	372697	38356	102.915	4.815	4.711
84	88	334341	37970	113.567	4.475	4.383
85	89	296371	37119	125.245	4.149	4.068
86	90	259252	35784	138.028	3.838	3.767
87	91	223468	33966	151.995	3.541	3.479
88	92	189502	31692	167.238	3.260	3.206
89	93	157810	29011	183.835	2.993	2.946
90	94	128799	26000	201.865	2.740	2.699
91	95	102799	22761.8	221.420	2.502	2.467
92	96	80037.2	19414.1	242.563	2.278	2.247
93	97	60623.1	16086.9	265.359	2.067	2.041
94	98	44536.2	12909.4	289.863	1.870	1.848
95	99	31626.8	9997.3	316.102	1.686	1.667
96	100	21629.5	7442.6	344.095	1.515	1.499
97	101	14186.9	5303.49	373.830	1.356	1.342
98	102	8883.41	3600.08	405.259	1.208	1.197
99	103	5283.33	2315.68	438.299	1.072	1.062
100	104	2967.65	1403.20	472.832	.947	.939

TABLE 10—*Continued*

Age x Male	Age x Female	l_x	d_x	$1000q_x$	a_x At 2%	a_x At 2½%
101	105	1564.45	795.804	508.680	.832	.825
102	106	768.646	419.392	545.624	.726	.721
103	107	349.254	203.751	583.389	.630	.626
104	108	145.503	90.4511	621.644	.543	.539
105	109	55.0519	36.3347	660.008	.464	.461
106	110	18.7172	13.0656	698.053	.393	.391
107	111	5.65160	4.15573	735.319	.329	.327
108	112	1.49587	1.153799	771.323	.267	.265
109	113	.342071	.275566	805.581	.191	.190
110	114	.066505	.066505	1000.000	....	....

TABLE 11

THE PROGRESSIVE ANNUITY MORTALITY TABLE
COMMUTATION COLUMNS AT 2% FOR THE 1900 YEAR-OF-BIRTH GROUPS

Age x		D_x	N_x	C_x	M_x	R_x
Male	Female					
6	10	820348.300	31480211.580	28.14921	203089.24936	13957116.63024
7	11	804234.890	30659863.280	30.75123	203061.10015	13754027.38088
8	12	788434.827	29855628.390	33.24040	203030.34892	13550966.28073
9	13	772942.080	29067193.563	36.37800	202997.10852	13347935.93181
10	14	757749.975	28294251.483	39.37978	202960.73052	13144938.82329
11	15	742852.753	27536501.508	42.97830	202921.35074	12941978.09277
12	16	728244.034	26793648.755	47.13473	202878.37244	12739056.74203
13	17	713917.605	26065404.721	51.11163	202831.23771	12536178.36959
14	18	699868.109	25351487.116	55.60089	202780.12608	12333347.13188
15	19	686089.604	24651619.007	60.56742	202724.52519	12130567.00580
16	20	672576.299	23965529.403	65.97758	202663.95777	11927842.48061
17	21	659322.551	23292953.104	71.79913	202597.98019	11725178.52284
18	22	646322.859	22633630.553	78.00118	202526.18106	11522580.54265
19	23	633571.860	21987307.694	84.55412	202448.17988	11320054.36159
20	24	621064.328	21353735.834	92.03916	202363.62576	11117606.18171
21	25	608794.557	20732671.506	100.39332	202271.58660	10915242.55595
22	26	596757.016	20123876.949	109.55620	202171.19328	10712970.96935
23	27	584946.342	19527119.933	118.89553	202061.63708	10510799.77607
24	28	573357.910	18942173.591	129.51583	201942.74155	10308738.13899
25	29	561986.082	18368815.681	140.22601	201813.22572	10106795.39744
26	30	550826.521	17806829.599	152.63136	201672.99971	9904982.17172
27	31	539873.370	17256003.078	166.08822	201520.36835	9703309.17201
28	32	529121.529	16716129.708	181.03960	201354.28013	9501788.80366
29	33	518565.558	16187008.179	197.38090	201173.24053	9300434.52353
30	34	508200.225	15668442.621	214.01182	200975.85963	9099261.28300
31	35	498021.503	15160242.396	232.85600	200761.84781	8898285.42337
32	36	488023.519	14662220.893	253.76257	200528.99181	8697523.57556
33	37	478200.668	14174197.374	275.64450	200275.22924	8496994.58375
34	38	468548.540	13695996.706	299.80440	199999.58474	8296719.35451
35	39	459061.509	13227448.166	326.08110	199699.78034	8096719.76977
36	40	449734.222	12768386.657	354.32015	199373.69924	7897019.98943
37	41	440561.584	12318652.435	385.67946	199019.37909	7697646.29019
38	42	431537.442	11878090.851	419.08692	198633.69963	7498626.91110
39	43	422656.836	11446553.409	455.63840	198214.61271	7299993.21147
40	44	413913.809	11023896.573	495.51774	197758.97431	7101778.59876
41	45	405302.334	10609982.764	538.48384	197263.45657	6904019.62445
42	46	396816.746	10204680.430	585.09425	196724.97273	6706756.16788
43	47	388450.931	9807863.684	635.46783	196139.87848	6510031.19515
44	48	380198.778	9419412.753	690.84124	195504.41065	6313891.31667
45	49	372053.059	9039213.975	750.11479	194813.56941	6118386.90602
46	50	364007.786	8667160.916	814.44740	194063.45462	5923573.33661
47	51	356055.931	8303153.130	884.18208	193249.00722	5729509.88199
48	52	348190.260	7947097.199	959.97152	192364.82514	5536260.87477
49	53	340403.029	7598906.939	1041.37297	191404.85362	5343896.04963
50	54	332687.087	7258503.910	1129.64477	190363.48065	5152491.19601
51	55	325034.166	6925816.823	1224.61154	189233.83588	4962127.71536
52	56	317436.335	6600782.657	1327.38701	188009.22434	4772893.87948
53	57	309884.706	6283346.322	1438.02796	186681.83733	4584884.65514
54	58	302370.508	5973461.616	1556.87658	185243.80937	4398202.81781
55	59	294884.798	5671091.108	1684.83637	183686.93279	4212959.00844
56	60	287417.906	5376206.310	1822.41867	182002.09642	4029272.07565
57	61	279959.843	5088788.404	1969.48402	180179.67775	3847269.97923
58	62	272500.950	4808828.561	2127.02643	178210.19373	3667090.30148
59	63	265030.768	4536327.611	2295.37252	176083.16730	3488880.10775
60	64	257538.713	4271296.843	2474.51838	173787.79478	3312796.94045
61	65	250014.416	4013758.130	2665.24234	171313.27640	3139009.14567
62	66	242446.930	3763743.714	2867.17007	168648.03406	2967695.86927
63	67	234825.899	3521296.784	3080.44442	165780.86399	2799047.83521
64	68	227141.025	3286470.885	3304.91001	162700.41957	2633266.97122
65	69	219382.370	3059329.860	3540.14099	159395.50956	2470566.55165

TABLE 11—*Continued*

Age x Male	Age x Female	D_x	N_x	C_x	M_x	R_x
66	70	211540.613	2839947.490	3785.71223	155855.36857	2311171.04209
67	71	203607.046	2628406.877	4040.71925	152069.65634	2155315.67352
68	72	195574.032	2424799.831	4303.11903	148028.93709	2003246.01718
69	73	187436.128	2229225.799	4572.15406	143725.81806	1855217.08009
70	74	179188.756	2041789.671	4845.06278	139153.66400	1711491.26203
71	75	170830.188	1862600.915	5119.27643	134308.60122	1572337.59803
72	76	162361.300	1691770.727	5391.75703	129189.32479	1438028.99681
73	77	153785.988	1529409.427	5657.98582	123797.56776	1308839.67202
74	78	145112.591	1375623.439	5913.79342	118139.58194	1185042.10426
75	79	136353.452	1230510.848	6153.70207	112225.78852	1066902.52232
76	80	127526.153	1094157.396	6372.07556	106072.08645	954676.73380
77	81	118653.565	966631.243	6562.17071	99700.01089	848604.64735
78	82	109764.853	847977.678	6717.04170	93137.84018	748904.63646
79	83	100895.560	738212.825	6829.39771	86420.79848	655766.79628
80	84	92087.8177	637317.2650	6892.03705	79591.40077	569345.99780
81	85	83390.1372	545229.4473	6897.86891	72699.36372	489754.59703
82	86	74857.1676	461839.3101	6840.46889	65801.49481	417055.23331
83	87	66548.9111	386982.1425	6714.56983	58961.02592	351253.73850
84	88	58529.4606	320433.2314	6516.66371	52246.45609	292292.71258
85	89	50865.1604	261903.7708	6245.69553	45729.79238	240046.25649
86	90	43622.1088	211038.6104	5903.00641	39484.09685	194316.46411
87	91	36863.7669	167416.5016	5493.24037	33581.09044	154832.36726
88	92	30647.7076	130552.7347	5024.97232	28087.85007	121251.27682
89	93	25021.7999	99905.0271	4509.68859	23062.87775	93163.42675
90	94	20021.4877	74883.2272	3962.38828	18553.18916	70100.54900
91	95	15666.5213	54861.7395	3400.87065	14590.80088	51547.35984
92	96	11958.4639	39195.2182	2843.81016	11189.93023	36956.55896
93	97	8880.17409	27236.75431	2310.23165	8346.12007	25766.62873
94	98	6395.82137	18356.58022	1817.56124	6035.88842	17420.50866
95	99	4452.85187	11960.75885	1379.95698	4218.32718	11384.62024
96	100	2985.58406	7507.90698	1007.18055	2838.37020	7166.29306
97	101	1919.86265	4522.32292	703.629817	1831.189652	4327.922856
98	102	1178.58847	2602.46027	468.267978	1127.559835	2496.733204
99	103	687.210911	1423.871795	295.298120	659.291857	1369.173369
100	104	378.438068	736.660884	175.429062	363.993737	709.881512
101	105	195.588652	358.222816	97.5411589	188.5646749	345.8877750
102	106	94.212421	162.634164	50.3966610	91.0235160	157.3231001
103	107	41.968458	68.421743	24.0038663	40.6268550	66.2995841
104	108	17.141681	26.453285	10.4470851	16.6229887	25.6727291
105	109	6.358484	9.311604	4.11436375	6.17590363	9.04974043
106	110	2.119444	2.953120	1.45047515	2.06153988	2.87383680
107	111	.627411	.833676	.45230156	.61106473	.81229692
108	112	.162808	.206265	.12311494	.15876317	.20123219
109	113	.036500	.043457	.02882744	.03564823	.04246902
110	114	.006957	.006957	.00682079	.00682079	.00682079

TABLE 12

THE PROGRESSIVE ANNUITY MORTALITY TABLE
COMMUTATION COLUMNS AT 2½% FOR THE 1900 YEAR-OF-BIRTH GROUPS

Age x		D_x	N_x	C_x	M_x	R_x
Male	Female					
6	10	781198.402	26349317.529	26.67507	138532.12054	9381860.69410
7	11	762118.107	25568119.127	28.99868	138505.44547	9243328.57356
8	12	743500.862	24806001.020	31.19308	138476.44679	9104823.12809
9	13	725335.502	24062500.158	33.97091	138445.25371	8966346.68130
10	14	707610.421	23337164.656	36.59467	138411.28280	8827901.42759
11	15	690315.035	22629554.235	39.74387	138374.68813	8689490.14479
12	16	673438.339	21939239.200	43.37487	138334.94426	8551115.45666
13	17	656969.639	21265800.861	46.80511	138291.56939	8412780.51240
14	18	640899.184	20608831.222	50.66775	138244.76428	8274488.94301
15	19	625216.829	19967932.038	54.92438	138194.09653	8136244.17873
16	20	609912.714	19342715.209	59.53863	138139.17215	7998050.08220
17	21	594977.255	18732802.495	64.47598	138079.63352	7859910.91005
18	22	580401.139	18137825.240	69.70376	138015.15754	7721831.27653
19	23	566175.310	17557424.101	75.19105	137945.45378	7583816.11899
20	24	552290.965	16991248.791	81.44798	137870.26273	7445870.66521
21	25	538739.006	16438957.826	88.40743	137788.81475	7308000.40248
22	26	525510.622	15900218.820	96.00575	137700.40732	7170211.58773
23	27	512597.284	15374708.198	103.68170	137604.40157	7032511.18041
24	28	499991.230	14862110.914	112.39209	137500.71987	6894906.77884
25	29	487683.930	14362119.684	121.09264	137388.32778	6757406.05897
26	30	475668.107	13874435.754	131.16238	137267.23514	6620017.73119
27	31	463935.284	13398767.647	142.03018	137136.07276	6482750.49605
28	32	452477.759	12934832.363	154.06064	136994.04258	6345614.42329
29	33	441287.655	12482354.604	167.14737	136839.98194	6208620.38071
30	34	430357.394	12041066.949	180.34682	136672.83457	6071780.39877
31	35	419680.526	11610709.555	195.26952	136492.48775	5935107.56420
32	36	409249.146	11191029.029	211.76340	136297.21823	5798615.07645
33	37	399055.696	10781779.883	228.90168	136085.45483	5662317.85822
34	38	389093.728	10382724.187	247.75016	135856.55315	5526232.40339
35	39	379355.887	9993630.459	268.15005	135608.80299	5390375.85024
36	40	369835.155	9614274.572	289.95087	135340.65294	5254767.04725
37	41	360524.834	9244439.417	314.07356	135050.70207	5119426.39431
38	42	351417.472	8883914.583	339.61376	134736.62851	4984375.69224
39	43	342506.700	8532497.111	367.43270	134397.01475	4849639.06373
40	44	333785.446	8189990.411	397.64268	134029.58205	4715242.04898
41	45	325246.695	7856204.965	430.01417	133631.93937	4581212.46693
42	46	316883.834	7530958.270	464.95640	133201.92520	4447580.52756
43	47	308690.004	7214074.436	502.52338	132736.96880	4314378.60236
44	48	300658.456	6905384.432	543.64734	132234.44542	4181641.63356
45	49	292781.676	6604725.976	587.41232	131690.79808	4049407.18814
46	50	285053.247	6311944.300	634.67978	131103.38576	3917716.39006
47	51	277466.049	6026891.053	685.66130	130468.70598	3786613.00430
48	52	270012.923	5749425.004	740.80275	129783.04468	3656144.29832
49	53	262686.439	5479412.081	799.69954	129042.24193	3526361.25364
50	54	255479.753	5216725.642	863.25429	128242.54239	3397319.01171
51	55	248385.285	4961245.889	931.26119	127379.28810	3269076.46932
52	56	241395.847	4712860.604	1004.49325	126448.02691	3141697.18122
53	57	234503.650	4471464.757	1082.91183	125443.53366	3015249.15431
54	58	227701.137	4236961.107	1166.69211	124360.62183	2889805.62065
55	59	220980.758	4009259.970	1256.42367	123193.92972	2765444.99882
56	60	214334.560	3788279.212	1352.39278	121937.50605	2642251.06910
57	61	207754.495	3573944.652	1454.39863	120585.11327	2520313.56305
58	62	201232.914	3366190.157	1563.07634	119130.71464	2399728.44978
59	63	194761.717	3164957.243	1678.55968	117567.63830	2280597.73514
60	64	188332.872	2970195.526	1800.73828	115889.07862	2163030.09684
61	65	181938.649	2781862.654	1930.06939	114088.34034	2047141.01822
62	66	175571.052	2599924.005	2066.16967	112158.27095	1933052.67788
63	67	169222.661	2424352.953	2209.03303	110092.10128	1820894.40693
64	68	162886.246	2255130.292	2358.43971	107883.06825	1710802.30565
65	69	156554.971	2092244.046	2513.98108	105524.62854	1602919.23740

TABLE 12—*Continued*

Age x Male	Age x Female	D_x	N_x	C_x	M_x	R_x
66	70	150222.576	1935689.075	2675.25602	103010.64746	1497394.60886
67	71	143883.355	1785466.499	2841.53320	100335.39144	1394383.96140
68	72	137532.471	1641583.144	3011.29793	97493.85824	1294048.56996
69	73	131166.723	1504050.673	3183.95944	94482.56031	1196554.71172
70	74	124783.575	1372883.950	3357.54921	91298.60087	1102072.15141
71	75	118382.524	1248100.375	3530.26954	87941.05166	1010773.55054
72	76	111964.876	1129717.851	3700.03561	84410.78212	922832.49888
73	77	105533.990	1017752.975	3863.79211	80710.74651	838421.71676
74	78	99096.1979	912218.9851	4018.78112	76846.95440	757710.97025
75	79	92660.4364	813122.7872	4161.41449	72828.17328	680864.01585
76	80	86239.0113	720462.3508	4288.06868	68666.75879	608035.84257
77	81	79847.5521	634223.3395	4394.45121	64378.69011	539369.08378
78	82	73505.5996	554375.7874	4476.22051	59984.23890	474990.39367
79	83	67236.5596	480870.1878	4528.89383	55508.01839	415006.15477
80	84	61067.7496	413633.6282	4548.13812	50979.12456	359498.13638
81	85	55030.1542	352565.8786	4529.78182	46430.98644	308519.01182
82	86	49158.1735	297535.7244	4470.17501	41901.20462	262088.02538
83	87	43489.0186	248377.5509	4366.49689	37431.02961	220186.82076
84	88	38061.8140	204888.5323	4217.12600	33064.53272	182755.79115
85	89	32916.3511	166826.7183	4022.05849	28847.40672	149691.25843
86	90	28091.4548	133910.3672	3782.83270	24825.34823	120843.85171
87	91	23623.4646	105818.9124	3503.06979	21042.51553	96018.50348
88	92	19544.2128	82195.4478	3188.82118	17539.44574	74975.98795
89	93	15878.7035	62651.2350	2847.86470	14350.62456	57436.54221
90	94	12643.5533	46772.5315	2490.03892	11502.75986	43085.91765
91	95	9845.13505	34128.97819	2126.74551	9012.72094	31583.15779
92	96	7478.26429	24283.84314	1769.71087	6885.97543	22570.43685
93	97	5526.15672	16805.57885	1430.65051	5116.26456	15684.46142
94	98	3960.72190	11279.42213	1120.06541	3685.61405	10568.19686
95	99	2744.05353	7318.70023	846.245136	2565.548640	6882.582813
96	100	1830.88025	4574.64670	614.630735	1719.303504	4317.034173
97	101	1171.59390	2743.76645	427.294678	1104.672769	2597.730669
98	102	715.723763	1572.172553	282.978875	677.378091	1493.057900
99	103	415.288211	856.448790	177.581019	394.399216	815.679809
100	104	227.578211	441.160579	104.981726	216.818197	421.280593
101	105	117.045797	213.582368	58.0866575	111.8364706	204.4623956
102	106	56.104364	96.536571	29.8652769	53.7498131	92.6259250
103	107	24.870688	40.432207	14.1554046	23.8845362	38.8761119
104	108	10.108681	15.561519	6.13073473	9.72913162	14.99157567
105	109	3.731393	5.452838	2.40268250	3.59839689	5.26244405
106	110	1.237701	1.721445	.84290824	1.19571439	1.66404716
107	111	.364605	.483744	.26156185	.35280615	.46833277
108	112	.094150	.119139	.07084894	.09124430	.11552662
109	113	.021005	.024989	.01650840	.02039536	02428232
110	114	.003984	.003984	.00388696	.00388696	.00388696

TABLE 13

THE PROGRESSIVE ANNUITY MORTALITY TABLE
Age of Single Life Corresponding to Two Joint Lives on the Same Sex-Year-of-Birth Basis

Difference of Ages (Years)	Addition to Older Age (Years)	Difference of Ages (Years)	Addition to Older Age (Years)	Difference of Ages (Years)	Addition to Older Age (Years)
0	6.64	20	1.12	40	.15
1	6.15	21	1.02	41	.13
2	5.69	22	.92	42	.12
3	5.26	23	.83	43	.11
4	4.85	24	.75	44	.10
5	4.46	25	.68	45	.09
6	4.10	26	.62	46	.08
7	3.77	27	.56	47	.07
8	3.45	28	.50	48	.06
9	3.16	29	.45	49	.06
10	2.89	30	.41	50	.05
11	2.64	31	.37	51	.05
12	2.41	32	.33	52	.04
13	2.20	33	.30	53	.04
14	2.00	34	.27	54	.03
15	1.82	35	.25	55	.03
16	1.65	36	.22	56	.03
17	1.50	37	.20	57–61	.02
18	1.36	38	.18	62 and over	.01
19	1.24	39	.16		

DISCUSSION OF PRECEDING PAPER

WALTER G. BOWERMAN:

The authors state that their Progressive Table was prepared for valuation purposes and "does not reproduce all of the fluctuations which are characteristic of recent annuitant mortality. It tends to assume conservative values for q_x at the lower ages." It would be of interest to know why they chose a four year sex variation, instead of the five years which has been in vogue for more than a decade. In their table the male death rate at ages 10 to 90 hovers close to 152% of the corresponding female death rate. Thus it omits the "peaks" in this ratio at ages 20 and 60 and also the "valley" at ages 30–35, which have been found not only in "recent annuity mortality" but also in both insured life and population death rates covering experience during the last decade or more. After age 90 in their table this ratio gradually declines to 122% at age 109; which compares with 100% in other recent annuity tables and in the general population. The rise to 143% at the final male age, 110, is due to their insertion of unity as a death rate. If they had not done that, this ratio would have been more intelligible and the elementary and derived functions would have been smoother at the advanced ages and longer durations. This is frequently of considerable importance, and more especially in the use of approximate summation formulas.

The Progressive Table extends down to age 6 male and 10 female where the death rate per 1,000 is .035. This represents a one year term rate of three and one-half cents for each thousand dollars of coverage—surely, a world's record for low mortality! In most mortality tables the lowest death rate for *each* sex is at one of the ages from 9 to 12. But in this table age 6 male has that honor. One wonders what will be done when the inevitable extension of the table to age zero occurs.

The low figure just quoted was taken from the authors' Table 10 which applies to persons born in the year 1900. Thus this death rate of .035 per 1,000 at age 6 would be one supposed to have been experienced in the year 1906. The American Men table covered experience during the years 1900–1915, centering not far from 1906. It is of interest, therefore, to observe that in both Hooker's extension (*Actuarial Study No. 1*, p. 86, at 3.08 per 1,000) and in mine (*TASA* XXXVII, 17, at 3.38 per 1,000) the death rate at age 6 is just about one hundred times as large as in the Progressive Table! This is progress indeed; but it is a nice question just how far the mechanical conveniences of a table should carry us away from biological reality.

While the death rates in the Progressive Table are *very* low at the young ages, they are very high at ages above 95 or 100, when compared with population and other recent tables. Thus a male aged 100 in the year 2000 would have a death rate of 473 per 1,000, although the U.S. Whites 1939–41 table showed 389 per 1,000 at a date two generations earlier in time. Similar comparisons are available at other advanced ages for each sex. In the case of annuitants this seems a serious situation in which the authors find themselves. Incidentally, it seems better to look at the table itself rather than the charts, for the latter are on the logarithmic scale, and this tends to minimize differences as observed visually.

Messrs. Fassel and Noback have done a very interesting piece of work, for which I would express my personal appreciation. While one needs to scrutinize their product with care, it may turn out to be a useful tool and suggestive of other modifications of practical benefit to the actuarial profession and the business of insuring lives and writing annuity contracts.

DONALD D. CODY:

The authors have produced a table which they feel reasonably reflects secular mortality trends and mortality differences between sexes and which they suggest be used especially for valuation and possibly also for rates. They have designed their table with simplicity of operation primarily in mind. From a technical point of view, the authors are to be congratulated for the highly competent methodology adopted in fitting a Gompertz curve to the myriad Jenkins-Lew mortality curves. We are indebted to them for this paper, which will bring into discussion some of the basic problems inherent in use of projection mortality tables.

I am going to confine my remarks exclusively to a discussion of the practical adaptability of the results as compared with using the Jenkins-Lew-Sternhell approach. Any general mortality table system must be satisfactory for both immediate and deferred annuities and for settlement options with respect to ratemaking, policy forms, valuation, dividends, and statistical standards. Fidelity is a prime criterion as to nonparticipating ratemaking, dividends, and statistical standards. Practicality of application and an acceptable over-all reserve level are important to valuation. Simplicity of presentation is of great moment in policy forms. Competitive needs and adequacy are serious and opposing requirements in ratemaking. Naturally, the scope of the arithmetical calculations must be kept within reasonable bounds also. It is to be realized that new tables will have to be considered every few decades and perhaps more often.

The kernel of the authors' ideas is the use of an age setback principle because of obvious mechanical advantages in valuation, ratemaking, and policy drafting. The Gompertz Law is of course not essential to the age setback principle. In the last two decades, we have all experimented with age setbacks in connection with the American Annuitants and Standard Annuity mortality tables. It has been apparent that a fixed age differential is an inadequate method of treating the mortality difference by sex and a uniform age setback over all ages is an inadequate method of reflecting secular trends in mortality. There must be very compelling reasons therefore for deciding to adopt such a system in lieu of the Jenkins-Lew projection tables, which are recognized generally as representing by and large the best possible estimate of present and future mortality on individual deferred and immediate life annuities.

Inasmuch as the Society of Actuaries has under consideration the preparation of tables on the Jenkins-Lew projection basis along with the well-conceived Sternhell auxiliary commutation functions, it does not appear that simplicity of calculation need be emphasized to the detriment of reasonably accurate mortality representation.

As the authors have indicated, the use of age setbacks precludes the down-grading of the provision for secular mortality improvement at the high ages. Thus any single table like the Fassel-Noback Table which will furnish reasonably representative results over all ages and calendar years must necessarily have somewhat high mortality and low annuity values at high ages. Actually this situation appears to exist only in the Fassel-Noback male annuities. Although actuaries have not been generally satisfied with the Standard Annuity Table, we have always had the assurance that as a closed block of annuities grew older our valuation reserves (if computed with a suitable age setback for secular mortality improvement) became more conservative because of the greater margins in the mortality rates at high ages. With the Fassel-Noback Table or any similar table with vanishing high age margins we would not have the same assurance and I think this is a possible objection to such a table for valuation purposes as compared with the more accurate Jenkins-Lew Tables.

Of course, aggregate reserves can be adjusted in arbitrary ways or tested over a larger block of issues so as to balance deficiencies with redundancies, but the use of accurate projection tables will assure proper reserves without such general testing procedures. I would conclude that the Fassel-Noback Table is a reasonable basis for valuation at least in the near future but that it has no essential practical advantage over the Jenkins-Lew Tables and is not as theoretically acceptable. I presume that a company using the Jenkins-Lew Tables would probably use, say, Projection B with entry in 1955 for valuation during the 1950–1959 decade and then Projection B with entry in 1965 for valuation during the 1960–1969 decade.

For purposes of comparison, I am showing in Table A the aggregate reserves for nonrefund life annuities distributed like the Equitable's current in-force for all years of issue and also for years of issue prior to 1935 (a) on the basis of the Fassel-Noback Table and (b) on the basis of the Jenkins-Lew Projection B Table on the assumptions that the valuation is made in 1950 and in 1975. The greater conservatism of the Jenkins-Lew Tables on a closed block of male immediate nonrefund annuities is evident from these figures.

The "Special" table is introduced merely as an example of the sort of table which will result from introducing a particular set of age setback

TABLE A

2⅓% AGGREGATE RESERVES—NONREFUND IMMEDIATE ANNUITIES
(Per $1,000,000 of Reserves on Jenkins-Lew B Basis in Each Category)

	YEAR OF VALUATION							
	1950				1975			
	Mortality Basis				Mortality Basis			
	Jenkins-Lew Projection B	Fassel-Noback	Special*	Standard Annuity	Jenkins-Lew Projection B	Fassel-Noback	Special*	Standard Annuity
	All Issues							
Male...	$1,000,000	$1,005,076	$1,000,000	$ 979,075	$1,000,000	$ 990,764	$1,014,938	$ 920,343
Female.	1,000,000	1,035,588	1,025,081	1,019,100	1,000,000	1,032,514	1,048,996	974,087
Both sexes..	1,000,000	1,028,188	1,018,998	1,009,394	1,000,000	1,022,261	1,040,632	960,888
	Issues at Least 15 Years Old							
Male...	$1,000,000	$ 998,664	$1,000,000	$ 990,495	$1,000,000	$ 989,424	$1,021,536	$ 932,968
Female.	1,000,000	1,044,805	1,048,248	1,068,949	1,000,000	1,052,174	1,086,855	1,025,401
Both sexes..	1,000,000	1,031,820	1,034,670	1,046,871	1,000,000	1,034,283	1,068,231	999,046

* Special basis is Jenkins-Lew Male Projection B entered in 1950 with female age set back 4 years in all calendar years and with 1⅔ year age setback in 1975 for secular mortality trend in male and female mortality.

assumptions directly into the Jenkins-Lew system. It is the Jenkins-Lew 1949 Male Table with Projection B assuming entry in 1950 with age set back two-thirds of a year for each secular decade after 1950 and with female ages taken as 4 years younger. The "Special" table does not have decreasing mortality margins at higher ages on closed blocks of business, although at lower ages for females and at lower ages for calendar years after 1950 annuity values are lower than on the Jenkins-Lew projection.

A paramount criterion, I believe, of whether an age setback approach in a mortality table for general use should be seriously considered, is whether such an approach provides a much more practicable handling of settlement option guarantees and deferred annuity and retirement income settlements in policy forms. The authors have shown a simplified presentation of settlement option figures in their Table 1, which appears at first blush to be very attractive. However, income rate guarantees which are stepped down at intervals of 25 years, as integral age setbacks in the Fassel-Noback Table require, must suffer competitively with equally conservative income rate guarantees which are stepped down at shorter

TABLE B

MONTHLY INCOME PER $1,000 OF NET PROCEEDS

Life Annuity—10 Years Certain (2½%)

AGE	MORTALITY BASIS							
	Fassel-Noback Table Entered in Year				Jenkins-Lew 1949 Table—Projection B Entered in Year			
	1955 *	1965 *	1975 *	1985 *	1955 (1950–1959)†	1965 (1960–1969)†	1975 (1970–1979)†	1985 (1980 and Later)†
	Male							
50.....	$4.09	$4.09	$4.01	$4.01	$4.23	$4.15	$4.08	$4.01
60.....	5.33	5.19	5.19	5.05	5.37	5.27	5.17	5.08
65.....	6.13	5.96	5.96	5.96	6.16	6.05	5.95	5.85
70.....	7.07	7.07	6.87	6.87	7.08	6.98	6.89	6.81
80.....	8.80	8.80	8.80	8.67	8.78	8.76	8.74	8.72
	Female							
50.....	$3.79	$3.79	$3.72	$3.72	$3.83	$3.78	$3.74	$3.70
60.....	4.80	4.68	4.68	4.57	4.82	4.75	4.69	4.63
65.....	5.48	5.33	5.33	5.33	5.56	5.48	5.41	5.34
70.....	6.31	6.31	6.13	6.13	6.48	6.40	6.33	6.26
80.....	8.20	8.20	8.20	8.02	8.49	8.47	8.45	8.43

* Guarantees (as in authors' Table 1) are based on assumption of year of birth in 1887 for actual years of birth prior to 1900, in 1912 for those in 1900–1924, in 1937 for those in 1925–1949, etc.

† Years in parentheses refer to interval during which settlements are made at rates indicated.

intervals. On the other hand, if fractional age setbacks are used, the sort of policy table suggested by the authors would not be possible and the decision to use Fassel-Noback mortality would have to be made on other grounds.

I am showing above as Table B what might be called equally conservative tables of life annuity—10 years certain incomes on a 2½% interest basis using (1) Fassel-Noback figures from a policy form set up like the authors' Table 1 and (2) Jenkins-Lew Projection B figures, assuming guarantees in the policy are changed in successive steps each decade. Table B indicates the advantages of shifting the level of guarantees at policy year intervals of less than 25 years. It is evident also that the Jenkins-Lew basis has a competitive advantage over nearly the whole range of female ages; this is the result of forcing the female mortality into the four-year age setback mold.

For the reasons outlined, I do not feel that the Fassel-Noback Table is as acceptable as the Jenkins-Lew Tables for general use as a basis of mortality projection. The problem of handling secular trends and sex differentials in mortality is a very complicated one and I would argue strongly that it be kept within the structure of the Jenkins-Lew Projection B system. It is my personal feeling that age setbacks should be used only for minor adjustments within that system, such as, for instance, distinction between refund and nonrefund annuities, payee and nonpayee elections, deliberate mortality margins in participating annuity rates, etc.

EDWARD H. WELLS:

I will skip the usual compliments to this paper. Unless I thought the paper had considerable merit, I would not be up here—unless, of course, I thought it had no merit at all.

Fassel and Noback add another to our pairs in these papers. It is a little bit difficult to remember the Fassel-Noback and the Jenkins-Lew combinations, but we will probably have to do that from now on.

It seems to me that one extremely valuable thing learned from this paper is the superiority of the cohort table idea for life income options over current tables, suitable for settlements beginning in given calendar years. I think it is so superior mainly from a public relations point of view. It is much better to have a frame of reference for any form of guarantees attached to the individual rather than to his environment. In order to make this a little bit clearer, the Mutual Life is one of the companies that has a three-year setback differential between insured elections of settlement options and beneficiary elections. That is a frame of reference attached to the environment. I do not know that I like it in practice as much as something attached to an individual, such as, in the present instance, being born in a given year or a given quarter century. That is a whole lot easier to get across than a frame of reference attached to the year in which the election has been made.

Any family of cohort tables can, of course, be transformed into a family of current tables. For that reason, it has always seemed strange to me that there is so much debate among demographers on whether the cohort theory is correct or not. I do not think this is pertinent. The tables can be transformed from one situation to the other.

You could use the Jenkins and Lew values and abandon the Fassel and Noback tables and still preserve the same principle of having cohort tables in your policy if you want to take account of the secular improvement of the mortality. In some respects, that might be a proper solution because, after all, Jenkins and Lew spent a great deal of time and did a monumental piece of work. Their standards may conceivably get imbedded into state department rulings on minimum valuation, and things of that sort. It might be preferable to retain them.

There is, however, a very practical problem in regard to the settlement options that I do not think has been brought out too well up to this point. For that purpose, I want to take just our own Mutual Life policy form to show what is involved. We have four settlement options of a life income character. We have the ten year certain basis, the twenty year certain basis, the refund basis and the joint two-thirds survivorship basis. That is more than we used to have before the CSO policy came out. A couple of options were introduced by the clamoring of one of our agents. When half of the agency force wants something, an actuary can stand up against them and combat them successfully if it is not actuarially sound or practical; but, if we get a one-man clamor, an actuary is absolutely defenseless.

Our present tables of options fill up an area of about $69\frac{3}{8}$ square inches. That is about 3 pages in the *Reader's Digest*. If we adopted the Fassel-Noback idea of extending the columns of ages to take account of quarter century setbacks (incidently a male, as you can see, is merely a female born a century ago), I figure it would take up the equivalent of 7 pages in the *Reader's Digest*. If we didn't make use of the graduation of Fassel and Noback at all, but used the Jenkins and Lew figures, pure and simple, where we do not have the setback principle but actually require the values all the way through, it would take up about 150 pages of the *Reader's Digest*. On this basis we would seriously have to think of publishing a supplement to our policy which would be about the size of an average issue of the *Reader's Digest*. I do not know how practical this would be. I would not recommend it to any company but otherwise it would appear that we have four practical alternatives if we want to introduce this secular improvement into our policy guarantees. One is to accept the Fassel-Noback suggestion in its entirety, using the seven-page idea I referred to. The second alternative is to cut down on the extent of the settlement options in the policies—not have quite so many life income options. With us that would be such a difficult feat, and having had the one-man clamor, it is hard to reverse the situation. The third is to abandon the projection principles in favor of single conservative tables for males and females. That is just going back to what we have now, using perhaps somewhat more up-to-date tables. The fourth is to get the laws changed so complete tables for specified options need not be included in the policy. I do not know how helpful that alternative is unless the State of Massachusetts, for instance, cares to reverse itself. We really do have a serious problem here and I do not know what ultimate solution will be adopted by the various companies.

EDWARD A. LEW:

Messrs. Fassel and Noback are to be congratulated on having produced a single mortality table in the very convenient Gompertz form which can be used to calculate annuity values that include conservative margins for future improvement in mortality. For contracts entered in 1950 the annuity values on The Progressive Annuity Mortality Table are generally several percent higher than those on the Annuity Table for 1949 with Projection Scale B; for contracts entered in 1970 the differences between the annuity values on these two mortality bases are less pronounced but the values on The Progressive Annuity Mortality Table are in most cases slightly higher. It is important, however, to realize that a comparison of the annuity values does not bring out the size of the margins for future improvement in mortality included in the respective mortality bases. Actually, in the case of contracts entered in 1950, the margins for future improvement in mortality implicit in The Progressive Annuity Mortality Table, when measured from the Annuity Table for 1949 without projection, are very much greater than those resulting from the application of Projection Scale B; in the case of contracts entered in 1970, the corresponding margins for future improvement in mortality implicit in The Progressive Annuity Mortality Table are in most cases only somewhat higher than those produced by Projection Scale B. This is shown in the following Table A.

These figures indicate that in the case of single premium immediate nonrefund and 10 years certain life annuities (at 2% interest) entered in 1950 the margins for future improvement in mortality included in The Progressive Annuity Mortality Table are roughly double those produced by Projection Scale B for males at ages 40 to 70; for females the margins included in The Progressive Annuity Mortality Table range from 130% of those produced by Projection Scale B at age 40 to 350% at age 70. In the case of single premium deferred to age 65 nonrefund life annuities (at 2% interest) entered in 1950, the margins for future improvement in mortality included in The Progressive Annuity Mortality Table range from about 120% of those by Projection Scale B at age 30 to about 200% at age 60, for both males and females.

In the case of life annuities (at 2% interest) entered in 1970, whether

TABLE A

Comparison of Margins for Future Improvement in Mortality as Measured from the Annuity Table for 1949

MARGINS AS A PERCENTAGE OF SINGLE PREMIUM IMMEDIATE NONREFUND LIFE ANNUITIES AT 2% INTEREST

Age	Annuities Entered in 1950			Annuities Entered in 1970		
	Projection Scale B (1)	Progressive Table (2)	Ratio (2)/(1) (3)	Projection Scale B (4)	Progressive Table (5)	Ratio (5)/(4) (6)
	Males					
40	4.2%	7.4%	176%	7.7%	8.9%	116%
50	3.7	8.3	224	8.6	10.4	121
60	2.8	6.7	239	8.7	9.7	111
70	1.6	3.1	194	7.6	7.1	93
	Females					
40	2.9%	3.8%	131%	5.1%	5.1%	100%
50	2.7	3.9	144	5.7	5.7	100
60	2.2	4.0	182	6.0	6.6	110
70	1.4	4.7	336	5.6	8.1	145

MARGINS AS A PERCENTAGE OF SINGLE PREMIUM IMMEDIATE LIFE ANNUITIES WITH PAYMENTS CERTAIN FOR 10 YEARS—AT 2% INTEREST

Age	Annuities Entered in 1950			Annuities Entered in 1970		
	Projection Scale B (1)	Progressive Table (2)	Ratio (2)/(1) (3)	Projection Scale B (4)	Progressive Table (5)	Ratio (5)/(4) (6)
	Males					
40	4.1%	7.1%	173%	7.5%	8.5%	113%
50	3.6	7.2	200	7.9	9.2	116
60	2.4	4.8	200	6.9	7.3	106
70	1.0	1.1	110	3.9	3.4	87
	Females					
40	2.9%	3.7%	128%	5.0%	5.0%	100%
50	2.7	3.7	137	5.4	5.4	100
60	2.0	3.8	190	5.2	6.0	115
70	.9	3.6	400	3.4	6.1	179

TABLE A—*Continued*

MARGINS AS A PERCENTAGE OF SINGLE PREMIUM NONREFUND LIFE ANNUITIES DEFERRED TO AGE 65—AT 2% INTEREST

Age	Annuities Entered in 1950			Annuities Entered in 1970		
	Projection Scale B (1)	Progressive Table (2)	Ratio (2)/(1) (3)	Projection Scale B (4)	Progressive Table (5)	Ratio (5)/(4) (6)
	Males					
30	18.9%	23.4%	124%	29.1%	28.3%	97%
40	13.5	20.0	148	23.8	24.8	104
50	8.3	15.7	189	18.1	20.4	113
60	3.7	8.7	235	11.3	12.5	111
	Females					
30	11.7%	12.7%	109%	17.3%	16.4%	95%
40	8.6	10.3	120	14.3	14.0	98
50	5.6	7.7	138	11.1	11.2	101
60	2.8	5.2	186	7.6	8.4	111

immediate nonrefund and 10 years certain or deferred to age 65 nonrefund, the margins included in The Progressive Annuity Mortality Table are generally from 10% to 20% higher than those produced by Projection Scale B for males and the same or slightly higher for females.

These figures indicate that if we measure the margins for future improvement in mortality from the Annuity Table for 1949 without projection, then the margins implicit in The Progressive Annuity Mortality Table decrease with the passage of time. It is worth noting, however, that the margins included in The Progressive Annuity Mortality Table for contracts entered in 1970 will nevertheless on the whole be as conservative as those produced by Projection Scale B.

If we measure future improvement in mortality from the Annuity Table for 1949 without projection, then it can also be shown that there is a very wide difference between The Progressive Annuity Mortality Table and the Annuity Table for 1949 with Projection B with respect to the rates of improvement in mortality assumed for the future. Specifically, it can be demonstrated that the use of The Progressive Annuity Mortality Table is on this basis equivalent to assuming an annual rate of mortality improvement that decreases with the passage of time for each attained age, whereas Projection Scale B assumes an annual rate of mortality improvement that remains constant for each attained age. Table B below compares the annual rates of improvement in mortality assumed in Projection Scale B and in the use of The Progressive Annuity Mortality Table when the improvement is measured from the Annuity Table for 1949 without projection.

These rates of decrease in mortality must, of course, be distinguished from the rate of decrease "of approximately 0.4% at all attained ages" cited for The Progressive Annuity Mortality Table by Messrs. Fassel and Noback in the section entitled "Comparison of Mortality Rates." The rates cited by Messrs. Fassel and Noback represent the an-

TABLE B

COMPARISON OF ANNUAL RATES OF IMPROVEMENT IN MORTALITY INVOLVED IN PROJECTION SCALE B AND THE PROGRESSIVE ANNUITY MORTALITY TABLE

As Measured from the Annuity Table for 1949 without Projection

Attained Age	Projection Scale B	The Progressive Annuity Mortality Table			
		Next 5 Years	Next 10 Years	Next 20 Years	Next 40 Years
20	1.25%	26.8%	14.7%	7.8%	4.2%
40	1.25	10.7	5.7	3.1	1.8
60	1.20	8.6	4.6	2.5	1.5
80	.50	.3	.3	.4	.4

nual rates of improvement in mortality measured from The Progressive Annuity Mortality Table death rates for 1950, whereas the rates of decrease in mortality indicated in Table B above are measured from the Annuity Table for 1949 without projection. The rates of improvement in mortality shown in Table B above for The Progressive Annuity Mortality Table explain why that table produces much larger margins for future improvement in mortality than does Projection Scale B, and also why the margins in The Progressive Annuity Mortality Table decrease with the passage of time.

The substantially greater margins for future improvement in mortality and the strikingly higher annual rates of improvement in mortality involved in the use of The Progressive Annuity Mortality Table for annuity contracts to be issued in the near future underline my main point, which is that the use of The Progressive Annuity Mortality Table implies materially different assumptions regarding future improvement in mortality than does the Annuity Table for 1949 with Projection Scale B.

(AUTHORS' REVIEW OF DISCUSSION)

ELGIN G. FASSEL AND JOSEPH C. NOBACK:

The authors want to take this opportunity to thank each of the members who have contributed to the discussion of this paper. The various opinions expressed with regard to gross premiums, liability valuation and settlement options will assist each of us in solving our own problems.

In establishing gross premiums, there can be no substitute for the use of the best estimate of future annuitant mortality. For this purpose the Jenkins-Lew Tables are indispensable, at least as a test. The gross premiums charged may then be based directly on those tables with a simple loading formula, or they may be established in proper relation to those tables by use of other net premiums with a somewhat modified loading formula.

In determining the valuation liability for outstanding annuity contracts, however, individual equity is not a prime consideration. Simplicity of office procedure is desirable. Indeed, as the business with which most of us are connected continues to grow, we must continually strive for simplification of system. Therein lies our hope of postponing the operation of the law of diminishing returns.

The Progressive Table provides the actuary with a convenient means of obtaining aggregate annuity reserves on a basis that makes provision for the secular trend in annuitant mortality. It substitutes one family of Gompertz mortality curves for the two families of Jenkins-Lew sex-year-of-birth tables. The authors feel that The Progressive Table is a practical one to use for valuation purposes at this time and for an indefinite period in the future. It would seem that as additional annuitant mortality experience accumulates the valuation basis will again be subject to review as it has in the past.

As described in the paper, The Progressive Table was derived from the Jenkins-Lew Tables by a series of Gompertz graduations. This procedure made possible the simplicity of The Progressive Table. It also made it inevitable that the annuity values and the margins for future mortality improvement in the two systems of tables would differ.

In his discussion, Mr. Lew has developed this point very lucidly. He has demonstrated that at the present time The Progressive Table tends to produce more conservative annuity values than The Annuity Table for 1949 (with Projection Scale B). He has also demonstrated that the margin between these tables narrows gradually over the next twenty years. He shows further that in 1970 The Progressive Table will still tend to provide conservative aggregate reserves when measured by the Annuity Table for 1949 (with Projection Scale B).

Mr. Cody has focused his attention on the valuation of a closed block of male annuities. He finds that for these male annuities The Progressive Table is not as conservative as the 1937 Standard Annuity Table. It might be pointed out, in reply, that the same absence of conservatism is inherent in the use of the Annuity Table for 1949 (with Projection Scale B) in place of the 1937 Standard Annuity Table.

It is true that the Progressive Table reserves for male annuities at advanced ages tend to be slightly less than those of the Annuity Table for 1949 (with Projection Scale B). However, this problem is not a vital one in our Company because we have had a reasonable volume of annuities on female lives and there does not seem to be any compelling reason for keeping a separate accounting by sex. Under these conditions it would seem that The Progressive Table would give satisfactorily conservative reserves for the annuity business in the aggregate, and indeed for any block of years of issue.

Mr. Wells has devoted his discussion to the suggestion made in Table 1 of the paper that year-of-birth groupings be used to introduce the projection principle into the settlement option tables of our policies. It would seem that he favors this cohort approach for he has mustered several forceful arguments in its behalf. Under the cohort system the guaranteed settlement rates for a given individual are found in one column defined by the beneficiary's year of birth. These guaranteed rates change only on the date the beneficiary's age advances. There are no rate setbacks introduced at any time. Incidentally, as stated by Mr. Wells, this year-of-birth system may be adopted in connection with any cohort type table. Its use is not limited to The Progressive Table.

Mr. Cody prefers the calendar year-of-election arrangement wherein a new column of the table becomes effective on the first day of each decade. Under this system the guaranteed rates change not only on the beneficiary's age-change dates, but also on the first day of each decade. Consequently, his year-of-election arrangement employs the rate setback, while the cohort system does not.

In his Table B, Mr. Cody compares the Fassel-Noback values derived using quarter-century year-of-birth groups with Jenkins-Lew values derived on a ten-year year-of-election basis. In this table, the Fassel-Noback values take on a peculiar trend which may be somewhat misleading. The trend shown is not a characteristic of The Progressive Table but is due rather to Mr. Cody's adoption of a shorter interval of reference for his values.

In his discussion Mr. Bowerman has placed emphasis upon the individual values of q_x. He has implied that someone may use these rates to determine inadequate term insurance premiums. In an annuity table, the concern is rather with p_x in which the percentage variation is little affected by changes in the mortality rate where such rate is small.

Mr. Bowerman has asked for an explanation of our choice of "a four year sex variation instead of the five years which has been in vogue for more than a decade." Other members of the society have informally asked about our choice of a year-of-birth variation of one in twenty-five years. Actually, the explanation for each of these choices is given in Section IV of the paper. However, that text is very concise, and may lead to the impression that the final values chosen for c and $\log_{10}\beta$ resulted immediately from twelve simple Gompertz graduations.

The process was not as straightforward as that. It involved a number of tests. Comparisons were made between the annuity values for several trial tables and those of the Jenkins-Lew Table. From these tests the final table gradually evolved.

At the outset our objective was defined as the production of a single family of Gompertz sex-year-of-birth tables which would reproduce with reasonable closeness the immediate annuity single premiums of the Annuity Table for 1949 (with Projection Scale B). As described in Section IV, twelve Jenkins-Lew sex-year-of-birth tables were chosen for graduation. A number of test values of c were then derived for each of these tables by equating first and second moments, over several age ranges. On the basis of these tests c was fixed as 1.110. Some of the values of c derived in these graduations appear in Table 3 of the paper. It may be of interest to record that all these test graduations were carried out using punch card equipment. As a result, the process was a fairly rapid one.

Having fixed upon a value of c the next step was to determine for each of the twelve Jenkins-Lew tables the corresponding value of $\log_{10}\beta$. This was done by equating first moments. The resulting values are to be found in Column (3) of Table 4 of the paper.

At this point the tentative decision was made to proceed using the 1901(*f*) Table as the basic one and to define the other sex-year-of-birth tables using a sex variation of three years and a year-of-birth variation of one year in twenty-five. The reason for these decisions may be discerned by studying Column (4) of Table 4. It will be observed, for example, that there is a 3.3 year age relationship between the 1901(*f*) Table and the

1901(m) Table. Furthermore, it will be observed that the following relationships obtain between the various year-of-birth tables:

Sex-Year-of-Birth Tables	Age Relations	25 Year Basis at Same Rate
1871(m) and 1881(m)	−0.1 Years	−0.25
1881(m) and 1891(m)	0.0 Years	0.00
1891(m) and 1901(m)	0.3 Years	0.75
1901(m) and 1911(m)	0.4 Years	1.00
1911(m) and 1921(m)	0.4 Years	1.00
1871(f) and 1881(f)	0.4 Years	1.00
1881(f) and 1891(f)	0.4 Years	1.00
1891(f) and 1901(f)	0.4 Years	1.00
1901(f) and 1911(f)	0.4 Years	1.00
1911(f) and 1921(f)	0.3 Years	0.75

Proceeding on the basis of this tentative decision, annuity single premiums were determined. These were compared with the Jenkins-Lew values and it was discovered that, while the male values were satisfactory, the female values tended to be too small. These tests indicated that the year-of-birth variation was satisfactory. However, the sex variation was not. We experimented further and found that, if the male tables were left unchanged and the sex variation was increased to four years, both the male and female values of a_x would be satisfactory.

This, then, is a brief description of the derivation of The Progressive Table. In reading this description a question may arise concerning our choice of decennial Jenkins-Lew Tables centering about 1901, rather than 1900. This arose because most of our work was done on the assumption that The Annuity Table for 1949 (without Projection) described the mortality during 1949. It was not until we were well advanced that the Jenkins-Lew definition, given in the last paragraph on page 424 of *TSA* I, was called to our attention and we discovered that the table defined in our work as The 1900 Table was in reality The 1901 Table. This little side light also explains why the last digit in Column (6) of Table 4 is 4 or 6 rather than zero.

It may be helpful to those who are considering The Progressive Table for reserve computations to have a practical application of its use. For this reason the following procedure for valuing the deferred portion of Refund Life Annuities is presented. All other annuity benefits may be handled by similar means. Of course, in Joint and Survivor cases, three detail cards would be required.

Suppose that the Annuity detail card now includes the following data:

1. Year of inception of contract
2. Age of Annuitant at that time
3. Office Year-of-Birth (Item 1 minus Item 2)
4. Sex
5. Year of last certain payment
6. Total annual payment.

The first step is to determine and punch on the detail card to the nearest integer the Valuation Year-of-Birth on a 1900(m) basis.

(a) For male lives,

$$\begin{aligned}[\text{Valuation Year-of-Birth}] &= [\text{Office Year-of-Birth}] - .04 \\ &\quad \times [1900 - (\text{Office Year-of-Birth})] \\ &= 1.04\,[\text{Office Year-of-Birth}] - 76\end{aligned}$$

(b) For female lives,

$$[\text{Valuation Year-of-Birth}] = 1.04\,[\text{Office Year-of-Birth}] - 72\,.$$

The second step is to determine and punch on the detail card the Valuation Age on which the Deferred Annuity commences. This is defined as $x + n$.

$x + n =$ (Year of last certain payment) minus (Valuation Year-of-Birth).

The Valuation Year-of-Birth and the Valuation Deferred Age need only be determined once for each contract. They remain unchanged as long as The Progressive Table is used for valuation purposes. At the end of each year summary cards will be prepared showing the Total Annual Payment for each Valuation Year-of-Birth and Valuation Deferred Age combination. The attained age x will be determined on each summary card by subtracting the Valuation Year-of-Birth from the Year of Valuation.

For each summary card, the mean reserve will then be computed by multiplying the Total Annual Payment by the appropriate reserve factor. This factor, which depends upon the value of x and $x + n$, will be derived for the male 1900 Year-of-Birth group of The Progressive Table and may take the form:

$$N_{x+n+1/2} \div D_{x+1/2}\,.$$