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# FUNDING OF GROUP LIFE INSURANCE 

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## INTRODUCTION

Traditionally the group life insurance benefit has been financed on a yearly renewable term basis. Other methods of funding can be and have been employed, but the one year term method is the simplest and most natural approach so long as the life insurance benefit is promised during active employment only.

During the past decade, however, there has been a growing demand for continuation of some or all of the group life insurance benefit after the employee's retirement from active employment. It is logical that funding of any postretirement benefit be at least partially completed by retirement. The retired employee feels more secure with funded benefits; the prudent employer wishes to budget his costs for employee welfare benefits over the period of the employee's active service. The high and sharply increasing term premium at ages above 65 bolsters both viewpoints.

There are other reasons why the various forms of group permanent ${ }^{1}$ insurance, which employ different methods of funding than the traditional one year term, have been developed. One is that the availability of insurance reserves makes possible (but in no way makes necessary) a termina-tion-of-employment benefit, distributable either as cash or as paid-up insurance. Such a benefit is helpful in encouraging an employee to join a contributory plan or to contribute more than he would otherwise consider reasonable; and it brings the premium on conversions down within reach.

It is the general purpose of this paper to look into the various methods appropriate for funding the typical group life benefit. Part I introduces certain concepts, assumptions, and notation necessary for the algebraic development. Part II describes the funding methods commonly employed to fund a simple group life insurance benefit. The algebra is developed for each method (under the rigid conditions of an initially stationary population), and the relationships between ultimate contributions and ultimate reserves are studied by means of an "Equation of Maturity." Part III looks into the characteristics of the various methods under less ideal conditions. Part IV presents certain analogies with the funding of pension

[^0]plans, and Part V states the present situation with respect to employee taxability under the various funding methods.

I. FUNDAMENTAI. CONCEPTS, ASSUMPTIONS, NOTATION

## Equation of Maturity

Under the common actuarial concept that any group of sufficient size can be assumed (for want of better information) to approach a mature or stationary condition eventually, the ultimate population can be represented by the $l_{x}^{t}$ column of an underlying "service" table. If amounts of insurance are equal for all members of the group, it is apparent that the sum of the $d_{x}^{x}$ column of the service table is representative of the eventual yearly claims. If amounts of insurance vary (e.g., by salary), it is possible to construct a different $l_{x}^{*}$ column ${ }^{2}$ representative of amounts of insurance, instead of number of lives, and again the eventual level of claims is represented by $\Sigma d_{x}^{d}$.

It is characteristic of all of the funding methods here under consideration that, at some time coincident with or after the time the population becomes stationary, the yearly contribution to the plan also becomes stationary. Similarly the reserve becomes stationary. It is thus apparent that in the mature state ultimate benefits ( $B$ ), ultimate contributions ( $C$ ), and ultimate reserves $(F)$ are related by the equation $C(1+i)+i F=$ $B$, assuming the contribution is payable at the beginning of the year, deaths occur at the end of the year, and $F$ is defined as the reserves on hand at the end of the previous year, after payment of benefits for that year but before receipt of the year's contribution then falling due. This equation will be found to hold for all the funding methods, and is hereafter referred to as the Equation of Maturity.

Life insurance premiums are almost invariably calculated on a single decrement table. Since the premiums are calculated without a "discount for turnover," a withdrawal benefit arises under most of the permanent forms. This withdrawal benefit may go to the terminating employee, or it may revert to the employer as a credit against future premiums. In any event it can be viewed as a part of $B$ in the Equation of Maturity above.

## Assumptions

The actuarial analysis of a particular funding method is materially simplified if a mature population is assumed, not after many years, but right from the inauguration of the plan. The analysis of the initially ma-

[^1]ture population is undertaken first, but observations and illustrations as to the more realistic situation follow in Part III.

Unless otherwise indicated the algebraic statements found in this paper are based on the following:

Assume a population, stationary from the moment the plan is established, such that the number attaining age $x$ in a given year is $l_{x}^{s}$. Assume two decrements, death and withdrawal, represented by $d_{x}^{s}$ and $w_{x}$ respectively, such that $l_{x}^{s}-d_{x}^{s}-w_{x}=l_{x+1}^{s}$. Both decrements are assumed to occur at the end of the year.

Assume further that $w_{z}$ is zero for all ages above a retirement age $r$, and that the plan provides a unit of death benefit for each of the $l_{x}^{s}$ lives both before and after retirement age $r$. Funds earn interest at a rate $i$.

Imagine also a column of $l_{x}^{\prime}$ from a single decrement mortality table such that $l_{x+1}^{\prime} / l_{x}^{\prime}=1-d_{x}^{s} / l_{x}^{s}$. Assume that premiums are charged and reserves held on this mortality table with rate of interest $i$. This latter assumption has the effect of ignoring expenses and eliminating gain or loss from mortality or interest and is employed, not because the assumption is realistic, but because this paper attempts to study funding methods only.
Notation
Let $a$ be youngest age in the service table, so that the stationary population is supported by $l_{a}^{*}$ new entrants yearly. Let $\omega$ be the limiting age of the service table. Let $C_{t}$ represent the $t$ th annual contribution to the insurance plañ, payable annually in advance, and $F_{t}$ the fund (or reserve) built up after $t$ years (after benefits but before contributions then due); superscripts to the left indicate the funding method under consideration. Let a prime on any of the usual actuarial functions indicate that it is calculated on the table $l_{x}^{\prime}$ and rate of interest $i$.

## iI. DESCRIPTION OF fUNDING METHODS

## One Year Term

As previously stated, the simplest and most common funding arrangement employs the principle of one year term insurance. No contributions are made to the plan beyond those necessary to meet benefit payments expected to fall due within the year for which the contribution is made. Under the conditions assumed, contributions, ${ }^{T} C_{l}$, with a year's interest, are exactly equal to benefits for all values of $t$, and ${ }^{\mathrm{T}} F_{\text {t }}$ is zero for all values of $t$. There are no withdrawal benefits.

Since the initially mature population previously described produces constant deaths, One Year Term funding for such a group produces level
contributions equal to

$$
{ }^{\mathrm{r}} C_{t}=v \sum_{a}^{\omega} d_{x}^{*} .
$$

## Retirement Funding

If One Year Term funding is employed with respect to all active lives, but if the present value of future death benefits is contributed for each life as it reaches retirement, we have what might be called "retirement" funding.

When Retirement funding is applied to an initially mature population, all contributions except the first are equal and can be expressed as

$$
{ }^{\mathbb{R}} C_{t>1}=v \sum_{a}^{r-1} d_{x}^{s}+l_{r}^{e} \mathrm{~A}_{r}^{\prime} .
$$

The principle of full funding for retired lives requires, however, that the first contribution be considerably greater to fund the benefits of those already beyond retirement age at the time the plan is inaugurated. The initial contribution is in fact

$$
{ }^{\mathrm{x}} \mathrm{C}_{1}=v \sum_{a}^{r-1} d_{x}^{s}+\sum_{r}^{\omega} l_{x}^{v} \mathrm{~A}_{x}^{\prime} .
$$

For all values of $t$,

$$
{ }^{\mathrm{a}} F_{t}=\sum_{r+1}^{\omega} l_{x}^{s} A_{x}^{\prime},
$$

and the Equation of Maturity can be checked out by the identity

$$
\left[v \sum_{a}^{r-1} d_{x}^{s}+l_{r}^{s} \mathrm{~A}_{r}^{\prime}\right](1+i)+i \sum_{r+1}^{\omega} l_{x}^{s} \mathrm{~A}_{x}^{\prime} \equiv \sum_{a}^{\omega} d_{x}^{d} .
$$

Note that no withdrawal benefits are included on the right side of the equation because One Year Term funding is employed for the period during which withdrawals other than by death are assumed to take place.

## Ordinary Life Funding

Suppose an ordinary life net premium $\mathrm{P}_{x}^{\prime}$ is computed for ages $a$ and above on the single decrement mortality table $l_{x}^{\prime}$. Under Ordinary Life funding a premium $\mathrm{P}_{z}^{\prime}$ is paid with respect to each of the $l_{x}^{s}$ people at attained age $x$, where $z$ is the age the insurance became effective. It is apparent that the initial premium is

$$
{ }^{\mathrm{o} I} C_{1}=\sum_{a}^{\omega} l_{x}^{s} \mathrm{P}_{x}^{\prime}
$$

the premium in the th year is

$$
{ }^{\circ L} C_{t}=\sum_{a+t}^{\omega} l_{x}^{t} \mathrm{P}_{x}^{\prime}-t+1+\sum_{a}^{a+t-1} l_{x}^{\prime} \mathrm{P}_{a}^{\prime},
$$

and after $\omega-a$ years the ultimate premium,

$$
{ }^{0 L} C_{\infty}=\sum_{a}^{\omega} l_{x}^{e} P_{a}^{\prime},
$$

is reached.
At the end of each year, however, an ordinary life reserve is released with respect to each of the $\sum_{a}^{r-1} w_{x}$ withdrawals. In the ultimate situation the amount of such release can be expressed by

$$
\sum_{a}^{r-1} w_{x} \cdot{ }_{x-a+1} \mathbf{V}_{a}^{\prime} .
$$

This release is best viewed as an additional benefit, or as a reduction to premium, depending on whether "vested" in the employee by the terms of the plan.
${ }^{02} F_{\infty}$ is seen to be

$$
\sum_{a+1}^{\infty} l_{x}^{s} \cdot{ }_{x-a} V_{a}^{\prime},
$$

and the Equation of Maturity is expressed by the identity

$$
\sum_{a}^{\omega} l_{x}^{a} \mathrm{P}_{a}^{\prime}(1+i)+i \sum_{a+1}^{\omega} l_{x}^{*}{ }^{\prime}{ }_{x-a} \mathrm{~V}_{a}^{\prime} \equiv \sum_{a}^{\omega} d_{x}^{a}+\sum_{a}^{r-1} w_{x} \cdot{ }_{x-a+1} V_{a}^{\prime} .
$$

It is apparent that under this method of funding, for the mature population assumed, the initial contribution is high; but the contribution decreases steadily as the ordinary life average premium age decreases. If withdrawal benefits are nonvested, the premium credits also increase, further reducing the net contribution. The ultimate net contribution for identical benefits is certainly lower than under One Year Term, and also generally lower than under Retirement funding. ${ }^{3}$

Under variations of this method the ordinary life premium is paid to retirement only, with the ordinary life reserve providing a portion of the insurance after retirement on a paid-up basis while the remainder is filled out (1) by one year term, or (2) by purchase of additional paid-up life.
${ }^{3}$ The truth of this last statement in a specific case depends on the aggregate ordinary life reserve for all employees exceeding the single premium life reserve for retired employees.

These "to retirement only" variations of ordinary life funding are of theoretical, rather than practical, interest.

## Life Paid-Up at Retirement

Another method of funding, in many ways similar to Ordinary Life funding, employs a level "life paid-up at age $r$ " premium in lieu of ordinary life. Analysis similar to that for ordinary life shows that the ultimate contribution under this method is

$$
{ }^{L R} C_{\infty}=\sum_{a}^{r-1} l_{x}^{s} \cdot{ }^{r-a} \mathrm{P}_{a}^{\prime}
$$

and the ultimate reserve

$$
{ }^{\mathrm{LR}} F_{\infty}=\sum_{a+1}^{r} l_{x}^{s} \cdot{ }_{x-a}^{r-a} \mathrm{~V}_{a}^{\prime}+\sum_{r+1}^{\infty} l_{x}^{s} \mathrm{~A}_{x}^{\prime} .
$$

The Equation of Maturity then becomes

$$
\begin{aligned}
\sum_{a}^{r-1} l_{x}^{s} \cdot{ }^{r-a} \mathbf{P}_{a}^{\prime}(1+i)+i\left[\sum_{a+1}^{r} l_{x}^{s} \cdot{ }_{x-a}^{r-a} V_{a}^{\prime}\right. & \left.+\sum_{r+1}^{\infty} l_{x}^{s} A_{x}^{\prime}\right] \\
& \equiv \sum_{a}^{\infty} d_{x}^{s}+\sum_{a}^{r-1} w_{x} \cdot{ }_{x-a+1}^{r-a} V_{a}^{\prime}
\end{aligned}
$$

This method produces higher initial contributions, higher ultimate reserves, and lower ultimate contributions, than other methods previously discussed.

A common modification of the method employs ten payment life for those initially beyond or within ten years of retirement, in order to reduce the otherwise high contributions for the very early years. After ten years this modification produces identical results, under the conditions assumed, as the unmodified form of Life Paid-Up at Retirement funding.

## Unit Paid-Up

Still another arrangement is that sometimes known as the "unit paidup" plan. Here a level "paid-up" premium is established, usually constant for each thousand of insurance, and payable from the time the insurance becomes effective until retirement. Each payment of the established "paid-up" premium is applied at the attained age to purchase as much paid-up life insurance as it will. Insurance in excess of the accumulated paid-up insurance purchased with respect to a given active employee is provided on a one year term basis. If, as assumed here, insurance is to be continued after retirement, term insurance after retirement would likewise be equal to face less accumulated paid-up.

The actuarial analysis of this plan is somewhat more complicated than those previously considered.

It is apparent that the "paid-up" portion of the aggregate premium is

$$
p \sum_{a}^{\tau-1} l_{x}^{*}
$$

where $p$ is the constant premium per life (or per thousand of insurance).
If the amount of paid-up insurance in force for an individual age $x, t$ years after the plan commences, be represented by ${ }_{i} \Pi_{x}$, it is evident that under the rigorous conditions assumed

$$
\begin{array}{rlrl}
{ }_{t} \Pi_{x} & =p \sum_{a}^{x-1} \frac{1}{\mathrm{~A}_{z}^{\prime}} & & \begin{array}{l}
x<a+t \\
x<r
\end{array} \\
& =p \sum_{x=t}^{x-1} \frac{1}{\mathrm{~A}_{z}^{\prime}} & & a+t \leq x \leq r \\
& =p \sum_{x=1}^{r-1} \frac{1}{\mathrm{~A}_{z}^{\prime}} & & \begin{array}{l}
x>r \\
x>a+t \\
\end{array} \\
=p \sum_{a}^{r-1} \frac{1}{\mathrm{~A}_{z}^{\prime}} & & a+t>x>r
\end{array}
$$

After $t$ exceeds $r-a$, the second and third forms become inappropriate and the expression for ${ }_{t} \Pi_{x}$ becomes

$$
\begin{array}{rlr}
\infty \Pi_{x} & =p \sum_{a}^{x-1} \frac{1}{\mathrm{~A}_{z}^{\prime}} & \\
& x<r \\
& =p \sum_{a}^{r-1} \frac{1}{\mathrm{~A}_{z}^{\prime}} & x \geq r
\end{array}
$$

If $p$ is so established that ${ }_{\infty} \Pi_{x}(x \geq r)$ is not greater than unity, ${ }^{4}$ the term premium for year $t$ is simply

$$
v \sum_{a}^{\omega} d_{x}^{\prime}\left(1-\Pi_{x+1}\right)
$$

${ }^{4}$ This assumption is desirable for the purposes of this paper, since the goal is to compare different funding methods as they apply to identical benefits. As a practical matter $p$ can be set high enough so that paid-up insurance exceeds face sometime prior to retirement.
and eventually

$$
v \sum_{a}^{\omega} d_{x}^{*}\left(1-\infty \Pi_{x+1}\right)
$$

The aggregate reserve after $t$ becomes greater than $r-a$ is

$$
\sum_{a+1}^{\omega} l_{x}^{s} \cdot \infty \Pi_{x} \cdot \mathrm{~A}_{x}^{\prime}
$$

and the reserve released by terminations

$$
\sum_{a}^{r-1} w_{x} \cdot \infty \Pi_{x+1} \cdot A_{x+1}^{\prime}
$$

It is an interesting exercise to prove out the identity expressing the Equation of Maturity,

$$
\begin{aligned}
& {\left[p \sum_{a}^{r-1} l_{x}^{s}+v \sum_{a}^{\omega} d_{x}^{s}\left(1-\infty \Pi_{x+1}\right)\right](1+i)+i \sum_{a+1}^{\omega} l_{x}^{\prime} \cdot \infty \Pi_{x} \cdot A_{x}^{\prime} \equiv \sum_{a}^{\infty} d_{x}^{s} } \\
&+\sum_{a}^{r-1} w_{x} \cdot \infty \Pi_{x+1} A_{x+1}^{\prime}
\end{aligned}
$$

The ultimate level of contributions (and ultimate level of reserve) under Unit Paid-Up funding will depend considerably on the value of $p$.

If $p$ is established such that

$$
p \sum_{a}^{r-1} \frac{1}{\mathrm{~A}_{z}}=1
$$

the ultimate situation will be very similar (but not identical) to that under Life Paid-Up at Retirement funding.

If $p$ is zero, Unit Paid-Up funding is, of course, identical to One Year Term.

If

$$
p \sum_{a}^{r-1} \frac{1}{A_{z}^{\prime}}=\frac{r-a V_{a}^{\prime}}{A_{r}^{\prime}}
$$

we should expect ultimate reserves and premiums to approximate rather closely the first of the "to retirement only" modifications of Ordinary Life funding.

## One Year Term and Pension Funding

At least from a theoretical point of view it is feasible to use One Year Term funding for active lives and any one of several funding methods
commonly employed for pension plans to fund the after retirement death benefit. Practically, this would involve the maintenance of a self-administered or deposit administration fund from which purchase of single premium life insurance is contemplated at retirement, or from which one year term premiums after retirement would be paid. This method is then, in effect, an advance funded form of what we have chosen to call Retirement funding, or it may be thought of as an advance funded form of the after retirement portion of One Year Term funding.

Pension funding methods have been adequately described elsewhere and no further description is needed here. It might be well to point out that pension funding methods, in contrast to insurance funding methods, often employ discount for turnover.

The following is the appropriate Equation of Maturity assuming that the "entry age normal" method of funding, with discount for turnover, is the funding method employed as to the after retirement portion:

$$
\left.\left.\begin{array}{l}
{\left[v \sum_{a}^{r-1} d_{x}^{s}+\sum_{a}^{r-1} l_{x}^{s} \frac{r-a \mid}{\frac{A_{a}^{s}}{(d)}}\right](1+i)+i\left[\sum_{r=1}^{\omega} l_{a: r-a \mid}^{s} \mathrm{~A}_{x}^{(d)}\right.} \\
\\
+\sum_{a+1}^{r} l_{x}^{s}\left(r-x \left\lvert\, \mathrm{A}_{x}^{(d)}-\frac{r-a \mid}{\ddot{a}_{a}^{s}: \overline{r-a} \mid} \mathrm{A}_{a}^{(d)}\right.\right. \\
a_{x: \overline{r-x}}^{s}
\end{array}\right)\right] \equiv \sum_{a}^{\omega} d_{x}^{s} .
$$

In this equation, deferred assurances and temporary annuities are calculated on the service table $l_{x}^{x}$, rather than the single decrement table $l_{x}^{\prime}$.

## Illustration of Initially Mature Situation

To illustrate the foregoing discussion of various funding methods under the assumption of an initially mature population, a numerical calculation has been made, the results of which are shown in Table 1.

The $l_{x}^{*}$ column of the hypothetical stationary population is made up of 1,000 active lives (ages $30-64$ ) and 150 retired lives ( 65 and above), maintained by 100 entrants each year all at age 30 . Each year 10 active lives retire, 5 die, and 85 withdraw. The 10 new retired lives each year exactly replace 10 others who die. Death rates are not far different from current group life mortality prior to retirement and are approximately equivalent to the Standard Annuity $q_{x}$ 's at retired ages. The average group term premium age is just under 45 for active lives, and about 58 when retired lives are included.

Table 1 illustrates the yearly contribution and build-up of funds under each of the several funding methods, assuming $2 \frac{1}{2} \%$ interest and a death benefit of $\$ 1,000$ for each employee (active or retired). For Life Paid-Up at Retirement funding the 10 payment life modification has been em-

TABLE 1-Initially Mature

| Year | One Year Term |  |  | $\begin{aligned} & \text { Retire- } \\ & \text { MENT } \end{aligned}$ | Ordinary Ltre* |  | Life Paid-Up ar Retire ment-10 Payment Modification* |  | Untt Paid-Up* |  | Entey Age Normal $\dagger$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Active | Retired | Total |  | Premium | Withdrawal Values | Premium | Withdrawal Values | Premium | Withdrawal Values |  |
|  | contributions and withorawal values |  |  |  |  |  |  |  |  |  |  |
|  | \$4, ${ }_{4}^{878}$ | \$0,756 | \$14.634 | \$122,773 $\begin{array}{r}11,931 \\ 4\end{array}$ | 338,475 36,079 34,402 | $\$ 1,281$ 2,345 3,182 | 351,964 49,048 46,899 | 31,759 3,192 4,320 | $\$ 28,841$ 28,711 28,566 | $\$ 1,245$ 2,280 3,140 | \$ 24.014 |
| 5. | * | $\cdots$ | " | * | 31,646 | 4,409 | 43,249 | 5,921 | 28,276 | 4,429 | * |
| 10. | * | * | * | * | 26,763 25,954 | 6,057 6,272 | 36,478 21,497 | 7,854 8,080 | 27,422 | 6,359 6,647 | * |
| 15. | ${ }^{*}$ | * | * | * | 23,155 | 6,909 | 19,214 | 8,704 | 26,425 | 7,611 | * |
| 20. | * | * | * | * | 20,675 20,285 | 7,386 7,447 | 17,578 17,359 | 9,112 9,158 | 26,246 24,992 | 8,522 8,660 | $8,644$ |
| 30. | * | « | * | * | 17,820 | 7,642 | 16,336 | 9,287 | 22,413 | 9,185 | * |
|  | * | " | * | * | 16,526 | 7,642 | 16,219 | 9,287 | 19,531 | 9,185 | * |
|  | * | * | ، | * | 16,047 | 7,642 | 16,219 | 9,287 | 17,357 | 9,185 | * |
| Limit. | * | * | * | * | 15,926 | 7,642 | 16,219 | 9,287 | 16,136 | 9, 185 | 8,644 |
|  | peserves at end of year |  |  |  |  |  |  |  |  |  |  |
| 1. |  |  |  | \$110,842 |  | 156 |  | ,504 |  |  |  |
| 2. |  |  |  |  |  | 371 |  | ,499 |  | 799 | - 19.469 |
| 3 |  |  |  | * |  | 536 |  | ,988 |  | 584 | 29,570 |
| 5. |  |  |  | " |  | 944 |  | ,995 |  | 577 | 50,537 |
| 10. |  |  |  | * |  | 737 |  | ,220 |  | 537 | 107,714 |
| 11. |  |  |  | * |  | 862 |  | ,906 |  | 527 | 120,022 |
| 15. |  |  |  | * |  | 333 |  | , 011 |  | 144 | 172,406 |
| 20. |  |  |  | * |  | 7942 |  | ${ }_{2}^{123}$ |  | 886 541 | 245,600 |
| 30 |  |  |  | * |  | 370 |  | ,371 |  | 689 | * |
| 40. |  |  |  | * |  | 718 |  | , 506 |  | 833 | * |
| 50. |  |  |  | * |  | 320 |  | ,506 |  | 099 | " |
| Limit. |  |  |  | * |  | 349 |  | , 506 |  | 839 | 245,600 |

* To get net contribution for nonvested benefits subtract withdrawal value from $\dagger$ Term portion of contribution same as active co
$\ddagger$ Paid-Up Premium $\$ 14,344-$ remainder Term.
ployed. For Unit Paid-Up funding $p$ has been chosen to be $\$ 14.34$ (not quite $\$ 1.20$ per month), which will produce $\$ 1,000$ of paid-up insurance at retirement for those entering at age 30 . For One Year Term and Pension funding, entry age normal (with discount for turnover) with 20-year payment of the accrued liability has been illustrated.

A true comparison of yearly outlay for identical benefits is obtained by treating the withdrawal values shown as a credit against the next premium.

## III. MODIFICATIONS FOR INITIALLY IMMATURE FUND

The foregoing analysis of the initially mature situation is helpful for a thorough understanding, but the initially immature situation is much more important from a practical viewpoint. Surely very few employee groups have reached anything approximating a mature condition, especially in regard to the number of retired lives.

We now abandon the condition that the group is initially mature, although we retain the concepts that the population will eventually approach a stationary condition and that actuarial assumptions will be realized. The equations previously set out still hold, except that (1) the $l_{x}^{s}$ 's of the service table are replaced by the $l_{x}^{\prime \prime}$ 's of the immature population, with a corresponding modification of the $d_{x}^{s}$ 's and $w_{x}$ 's, and (2) the identities expressing the Equation of Maturity do not hold after this substitution until such time as the $l_{x}^{\prime \prime \prime}$ 's approach the $l_{x}^{l}$ 's.

If the initial group is immature it follows that One Year Term funding will produce contributions which are initially low, but which increase and eventually level off when maturity of the group is attained. An increase in the one year term premium is not uncommon in plans providing no insurance after retirement. It is much more likely and much steeper if after retirement insurance is included, both because of a rise in the average premium rate and because of an increase in total insurance provided.

Retirement funding is also likely to produce generally increasing contributions under conditions of an initially immature group, although the increase should not be as marked as under one year term. Moreover, contributions are likely to fluctuate rather widely as the number of retirements varies from year to year.

In general the decreasing cost tendencies of Ordinary Life and Life Paid-Up at Retirement funding are realized even if the population matures.

Under Unit Paid-Up funding, two tendencies work in opposite directions. The average premium age per thousand of term insurance may well rise as average age increases, and total insurance increases also--but the
portion of the insurance on a term basis continually decreases. For reasonably large values of $p$, the over-all term premium is likely to increase rather slowly if it doesn't actually fall.

Table 2 is a numerical representation for an initially immature population. The assumed initial population contains 1,000 active lives with average term premium age just over 40 , with no retired lives. If this group experiences death and withdrawal exactly in accordance with the service table underlying Table 1, and if sufficient new entrants come in at age 30 each year to keep the active staff up to 1,000 , the initially immature population will slowly approach that shown by the service table. Once again a comparison of outlay for equal benefits can be obtained by comparing premiums less withdrawal values. The withdrawal value column in itself is an indication of the cost of "vesting."

## IV. ANALOGIES WITH PENSION FUNDING

The methods of funding previously touched upon under the heading "One Year Term plus Pension Funding" point out some of the differences and similarities between the funding of the group insurance benefit and the funding of a typical pension benefit.

Both group insurance and pension plans provide an important benefit after retirement. One is of the form $\mathrm{A}_{x}$, the other $\ddot{a}_{x}$, but in many ways the funding of a death benefit of $\$ 1,000$ after retirement is identical to that of funding an annual pension benefit of $1,000 \mathrm{~A}_{r} / \ddot{a}_{r}$. One would expect to find similar methods employed-as in fact they are.

More pronounced differences arise before retirement. The group insurance plan provides a currently enjoyed death benefit prior to retirement; a pension plan, while it may carry supplemental death or withdrawal benefits, in its typical form offers nothing until retirement. Secondly, discount for turnover is not commonly employed in group insurance plans.

Because of the differences pointed out above, group insurance funding methods and those employed in pension funding cannot be identical. In many ways they are, however, analogous. A fuller understanding of both pension and insurance funding will be obtained if the student appreciates the similarities between some of the methods discussed in this paper and those previously described for pensions. ${ }^{5}$

If we adopt a pension terminology to point out the rather obvious analogies, One Year Term funding is essentially pay-as-you-go. Retirement funding is analogous to what is commonly known among pension actuaries as terminal funding. Life Paid-Up at Retirement is very similar to individual level premium funding. Unit Paid-Up with its One Year

[^2]TABLE 2-Initially Immature

| Yesm | One Year Term |  |  | $\begin{gathered} \text { Retize- } \\ \text { MENT } \dagger \end{gathered}$ | Ordinary Life* |  | Life Paid-Up at Rettre- <br> ment- 10 Payhent Modification* |  | Unit Paid-Up* |  | Entry Age Normali |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Active | Retired | Total |  | Premium | Withdrawal Values | Premium | Withdrawal Values | Premium $\ddagger$ | Withdrawal Values |  |
|  | CONTRIBUTIONS AND WITHDRAWAL VALUES |  |  |  |  |  |  |  |  |  |  |
| 1 | \$3,832 | \$ 0 | \$ 3,832 | - 3,832 | \$20,205 | \$1,420 | \$28,011 | \$1,886 | \$18,062 | \$1,416 | \$ 13,349 |
| 2 | 3,996 | 10 | -3,996 | 5,407 | 19,885 | 2,555 | 27,419 | -3,283 | 18, 111 | 2,553 | -13,513 |
| 3 | 4,119 | 195 | 4,314 | 6,234 | 19,622 | 3,423 | 26,918 | 4,528 | 18,305 | 3,458 | 13,636 |
| 5 | 4,320 | 355 | 4,675 | 7,476 | 19,206 | 4,597 | 26,093 | 6,057 | 18,412 | 4,704 | 13,837 |
| 10 | 4,604 | 1,500 | 6,104 | 10,029 | 18,605 | 6,092 | 24,748 | 7,836 | 19,070 | 6,455 | 14,121 |
| 11 | 4,640 | 1,674 | 6,314 | 10,181 | 18,525 | 6,293 | 20,838 | 8,054 | 19,104 | 6,721 | 14,157 |
| 15 | 4,678 | 2,859 | 7,627 | 10,645 | 18,252 | 6,999 | 18,962 | 8,796 | 19,643 | 7,735 | 14,195 |
| 20. | 4,898 | 4,415 | 9,313 | 11,031 | 17,923 | 7,633 | 17,521 | 9,428 | 20,200 | 8,809 | 14,415 |
| 21. | 4,921 | 4,723 | 9,644 | 11,092 | 17,851 | 7,702 | 17,323 | 9,487 | 20,281 | 8,957 | 8,687 |
| 30. | 5,133 | 7,442 | 12,575 | 12,891 | 17,107 | 7,570 | 16,361 | 9,203 | 20,503 | 9,104 | 8,899 |
| 40. | 4,807 | 9,512 | 14,319 | 11,807 | 16,559 | 7,611 | 16,219 | 9,245 | 19,115 | 9,141 | 8,573 |
| 50. | 4,850 | 10,084 | 14,934 | 11,691 | 16,088 | 7,644 | 16,219 | 9,287 | 17,343 | 9,185 | 8,616 |
| Limit. . . | 4,878 | 9,756 | 14,634 | 11,931 | 15,926 | 7,642 | 16,219 | 9,287 | 16,136 | 9,185 | 8,644 |
|  | reserves at end of year |  |  |  |  |  |  |  |  |  |  |
| 1. |  |  |  | - 0 |  | 361 |  | 897 |  | 170 | \$ 9,755 |
| 2 |  |  |  | 1,446 |  | 476 |  | 095 |  | 414 | -19,753 |
| 3 |  |  |  | 3,451 |  |  |  |  |  | 932 | 29,802 |
| 5. |  |  |  | 8,935 |  | 379 |  | 510 |  | 479 | 50,393 |
| 10. |  |  |  | 29,236 |  | 473 |  | 968 |  | 305 | 102,770 |
| 11. |  |  |  | 33,931 |  | 633 |  | 402 |  | 376 | 113,378 |
| 15. |  |  |  | 51.730 |  | 599 |  | 325 |  | 412 | 155,408 |
| 20. | . . . | .... | .. | 70.918 |  | 664 |  | 262 |  | 227 | 206,962 |
| 21. |  |  |  | 74,176 |  |  |  |  |  |  | 211,156 |
| 30. |  |  |  | 95,574 |  | 978 |  | 283 |  | 558 | 237,580 |
| 40. |  |  |  | 115,208 |  | 817 |  | 999 |  | 585 | 247,464 |
| 50. |  |  |  | 113,330 |  | 288 |  | 397 |  | 704 | 247,647 |
| Limit. |  |  |  | 110,842 |  | 749 |  | 506 |  | 839 | 245,600 |
| * To get net contribution for nonvested benefits subtract withdrawal values from next succeeding premium. |  |  |  |  |  | $\dagger$ Term portion of contribution same as active column of One Year Term funding <br> $\ddagger$ Paid-Up Premium $\$ 14,344$-remainder Term. |  |  |  |  |  |

Term supplement is not very different from money purchase pension funding supplemented by pay-as-you-go.

## V. TAX SITUATION AS TO THE VARIOUS FUNDING METHODS

It is apparent from previous discussion that a wide variety of funding methods is available to fund the group life benefit, just as there are a multitude of methods for funding the typical pension benefit. For both benefits the simplest type of funding-One Year Term funding for insurance, pay-as-you-go for pension-has an increasing cost tendency in an initially immature situation, and under some circumstances at least this characteristic is unsatisfactory. One would expect, therefore, a degree of development in alternate methods for funding group insurance somewhat similar to that which has taken place in pension funding.

As a matter of fact the development of the advance funded forms of group life insurance has lagged far behind its pension counterpart. Part of the explanation lies in the tendency to discontinue group life insurance after retirement. Under these circumstances the simple one year term arrangement is generally quite satisfactory (note from Table 2 how much of the increase in contributions is due to after retirement insurance).

Another, and perhaps equally important, explanation lies in the current income tax situation as it applies to the insured employee.

The premium paid under the traditional one year term arrangement is nontaxable to the employee, not by specific provision of law, but as a result of Law Opinion 1014, issued in 1920. Ever since 1920 the Federal Tax Regulations ${ }^{6}$ have included the statement "Premiums paid by an employer on policies of group life insurance covering the lives of his employees, the beneficiaries of which are designated by the employees, are not income to the employees." Congress has given stature to this old Law Opinion by permitting it to remain effective over a period of 35 years. It is to be noted that this Law Opinion 1014 and the current Regulations make no distinction between term and permanent insurance, and the tax exempt treatment would appear to apply to group life insurance generally.

Mimeograph 6477 issued in 1950 indicates, however, that the Treasury position as to the permanent forms of group insurance is quite different. This Mimeograph declares the entire premium for any permanent form taxable to the employee, except to the extent it is contributed by the employee himself, and except as provided in paragraph 5 of Mimeograph 6477 quoted below:
5. Where a premium for insurance would be required to be included in the income of an employee in accordance with the foregoing rules but the right of

[^3]the employee to permanent insurance or equivalent benefits, other than current term insurance, provided by the premiums is forfeitable in case of subsequent separation from service, the insurance is not considered a permanent form when the premium is paid and nothing is required to be included in the employee's income on account of the premium payment. The value of any permanent insurance or equivalent benefits in which the right of an employee changes from forfeitable to nonforfeitable in case of subsequent separation from service is considered a premium paid for a permanent form of insurance at the time of the change and such value, less any employee contributions on account of the change, is required to be included in his income for the year in which the change occurs.

This paragraph seems to imply taxability to the employee at retirement, if not before, unless some unusual provisions regarding forfeitability after retirement are incorporated.

It should be noted that Mimeograph 6477 does not apply to group permanent insurance written in conjunction with an approved pension plan.

That Mimeograph 6477 has slowed, if not stopped, the development of group insurance funding on other than a one year term basis is unquestioned. In conjunction with the treatment of one year term, Mimeograph 6477 has the peculiar effect of making the employee's tax status as to an identical economic benefit (i.e., a death benefit payable in event of death either before or after retirement) dependent on the employer's choice as to how such benefit shall be paid for. Law Opinion 1014 and Mimeograph 6477 seem in this respect to be basically inconsistent.

As a result of Mimeograph 6477 a "green light" taxwise seems to be given to only two, or possibly three, of the methods of funding discussed in this paper. The first is the traditional One Year Term. The second is Unit Paid-Up, if and only if the permanent premium $p$ is paid entirely by the employee. The favorable tax treatment of this contributory form of Unit Paid-Up is a result of its premium falling naturally into two parts, (1) a group term premium carrying the tax advantage enjoyed by all one year term, and (2) a permanent premium nontaxable to the employee because it is contributed by the employee himself.

There is feeling in some quarters that advance funding methods can be employed without incurring employee taxability through the route discussed under the heading "One Year Term and Pension Funding." If the funds accumulated are used to pay the periodic group term premiums after retirement, rather than the simpler and actuarially similar procedure of a single premium purchase at retirement, some basis for this viewpoint is seen to exist. Whether this route might involve nondeductibility of employer contributions to the "side fund," the author, for one, does not pro-
fess to know. In any event if employee taxability can be legitimately avoided through this route, as some believe, then this possibility serves to point up once again the inconsistency between Law Opinion 1014 (carried down to the present in Regulations 118) and Mimeograph 6477. Future development of other group insurance forms seems to hinge to a large extent on future change in the tax situation.


[^0]:    ${ }^{1}$ It should be noted that group permanent for pension purposes is outside the scope of the paper.

[^1]:    ${ }^{2}$ This table may involve an increment as well as the usual death and withdrawal decrements.

[^2]:    5 "Fundamentals of Pension Funding," TSA IV, 17.

[^3]:    - Most recently Reg. 118 39.22(a)-3.

