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PAYMENT OF RESERVE IN ADDITION TO FACE AMOUNT

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SEE PAGE 10 OF THIS VOLUME

CECIL J. NESBITT:

There have been several references in our actuarial publications to the individual or classical theory of risk. Among these are "A Statistical Treatment of Actuarial Functions," by W. O. Menge, *RAIA* XXVI, 65–88 and "The Mathematical Risk of Lump-Sum Death Benefits in a Trusteed Pension Plan," by H. L. Seal, *TSA* V, 135–42. A good survey is given in the paper "On the Mathematical Theory of Risk," by E. Lukacs in the *Journal of the Institute of Actuaries Students' Society*, VIII, 20–37. It occurred to me to apply this theory to the insurance discussed in the actuarial note.

For this purpose I shall start with the notation of the note and introduce such additional notations as may be necessary. Let B_t represent the actual sum insured in the tth year, $1 \leq t \leq n$. Then for the risk theory one considers a random variable L which takes on the values:

$$L_t = B_t v^t - \pi \ddot{a}_{\overline{t}|}, \text{ with probability } {}_{t-1|}q_x,\ 1 \leq t \leq n,$$

$$L_t = {}_n\mathrm{V} v^n - \pi \ddot{a}_{\overline{n}|}, \text{ with probability } {}_{t-1|}q_x,\ t > n.$$

It is readily shown that the mean of L is zero, but a direct calculation of its variance appears laborious. However, by the Hattendorf Theorem (see W. A. Jenkins' discussion of Menge's paper, *RAIA* XXVI, 601–602, or Lukacs' paper, page 26), one has that

$$\sigma^2(L) = \sum_{t=1}^{n} v^{2t}\,{}_t p_x q_{x+t-1}\,[B_t - {}_t\mathrm{V}]^2 . \qquad (1)$$

For the insurance discussed in the note, $B_t - {}_t\mathrm{V}$ has the fixed value F, and the variance of L reduces to

$$\sigma^2(L) = F^2 \sum_{t=1}^{n} v^{2t}\,{}_t p_x q_{x+t-1} , \qquad (2)$$

which is independent of the premium.

If continuous functions were used, formula (1) would be replaced by

$$\sigma^2(L) = \int_0^n v^{2t}{}_tp_x\mu_{x+t}\,[\bar{B}_t - {}_t\bar{V}]^2 dt\,, \tag{3}$$

where $\bar{B}_t$ is the sum payable in case of death in the instant of attaining age $x + t$. If $\bar{B}_t = F + {}_t\bar{V}$, then

$$\sigma^2(L) = F^2\bar{A}'^{\,1}_{x:\overline{n|}}\,,$$

where $\bar{A}'^{\,1}_{x:\overline{n|}}$ is calculated at the rate of interest i' such that $\frac{1}{1+i'} = v^2$.

It was a pleasure to read the note, and all the more so because it was an initial contribution of one of the authors.

(AUTHORS' REVIEW OF DISCUSSION)

PAUL W. NOWLIN AND T. N. E. GREVILLE:

We wish to thank Professor Nesbitt for his thought-provoking discussion. It is interesting to note that his random variable L_t can be interpreted as the insurer's loss if (x) dies between ages $x + t - 1$ and $x + t$, not only under the given contract, but also under a one-year term insurance for the amount F which is renewable for n years. This must be the case because the insurer is not subject to any risk under the savings fund part of the contract.

To prove this algebraically we substitute for B_t

$$F + {}_tV = F + \sum_{s=0}^{t-1} (1+i)^{t-s}(\pi - F\,v\,q_{x+s})\,,$$

and simplification gives

$$L_t = F\,(\,v^t - \sum_{s=0}^{t-1} v^s\,v\,q_{x+s}) \qquad 1 \le t \le n$$

$$L_t = -F\sum_{s=0}^{n-1} v^s\,v\,q_{x+s} \qquad t > n\,.$$

It should be noted that it is assumed there are no benefits or premiums after n years, with ${}_nV$ being payable as a pure endowment at age $x + n$. The mean-square risk, $\sigma^2(L)$, can be found by summing $L_t^2 \cdot {}_{t-1|}q_x$ with the aid of summation by parts, although the Hattendorf Theorem gives the result more easily. Similar remarks apply to the continuous case.