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PAYMENT OF CASH VALUE IN ADDITION TO FACE AMOUNT

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THE topic of this paper is of more than an academic interest, since this type of policy fulfils the specifications of a "split-dollar" or "minimum-deposit" sales approach quite adequately.

The problem of determining the level premium for a policy providing for payment of the reserve in addition to the face amount has been treated a number of times in the actuarial literature (*RAIA*, XXII, 216; *Life Contingencies*—Jordan 120; *TSA*, VIII, 10; *The Proceedings of the Conference of Actuaries in Public Practice*, X, 99, and XI, 115). However, in actual practice it is the cash value (not the reserve) which is paid in addition to the face amount as a death benefit. This is understandable, since only rarely does the insured know what the policy reserves are. Where the cash values are equal to the net level reserves the mathematics is relatively straightforward. This paper concerns the situation where cash values are less than the net level reserves.

Setting down the classic retrospective formula,

$$\sum_{1}^{n} P_x^t(1+i)^{n-t+1} = {}_nV_x + (M'_x - M'_{x+n})(1+i)^{x+n} \qquad (1)$$

where x = issue age

t = policy year

n = number of years during which additional benefit is returned

P_x^t = net premium payable in tth policy year

$$M'_x = \sum_{x}^{\infty} v^t \cdot \frac{C_t}{D_t}$$

${}_nV_x$ = terminal reserve at end of n years.

Varying the parameter P_x^t, we have the following:

If $P_x^t = P_x^{NL}$ for $1 \leq t \leq n$, then

$$P_x^{NL} \cdot \ddot{s}_{\overline{n}|} = {}_nV_x + (1+i)^{x+n}\,(M'_x - M'_{x+n})\,, \qquad (2)$$

giving the formula for the net level premium where the cash value is equal to the full net level reserve.

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If

$$P_x^1 = c_x$$

$$P_x^t = \beta_x^{FPT} \qquad \text{for} \qquad 2 \le t \le n\,,$$

then

$$\beta_x^{FPT} \cdot \ddot{s}_{\overline{n-1}|} = {}_nV_x + (1+i)^{x+n}\,(M_x' - M_{x+n}') - (1+i)^n\, c_x \qquad (3)$$

giving the formula for the net premium where the cash value is equal to the full preliminary term reserve.

If

$$P_x^1 = \beta_x - ({}_{19}P_{x+1} - c_x)$$

$$P_x^t = \beta_x \qquad \text{for} \qquad 2 \le t \le n,$$

then

$$\beta_x \cdot \ddot{s}_{\overline{n}|} = {}_nV_x + (1+i)^{x+n}(M_x' - M_{x+n}') + (1+i)^n({}_{19}P_{x+1} - c_x)\,. \qquad (4)$$

Formulas (3) and (4) will suffice for the net premiums where the cash value is equal to the CRVM reserve and where there is no uniform equivalent amount concept.

If

$$P_x^1 = P_x^{NL} - (P_x^{NL} - c_x)$$

$$P_x^t = P_x^{NL} + (P_x^{NL} - c_x)\,\frac{D_x}{N_{x+1} - N_{x+s}} \qquad \text{for} \qquad 2 \le t \le s\,,$$

and

$$P_x^t = P_x^{NL} \qquad \text{for} \qquad s < t \le n\,, \qquad (5)$$

then

$$P_x^{NL} \cdot \ddot{s}_{\overline{n}|} = {}_nV_x + (1+i)^{x+n}(M_x' - M_{x+n}')$$
$$+ (P_x^{NL} - c_x)\left[(1+i)^n - \ddot{s}_{\overline{s-1}|}(1+i)^{n-s}\,\frac{D_x}{N_{x+1} - N_{x+s}}\right]$$

gives the formula for net premiums for cash values which grade uniformly from the first full preliminary term reserve into the net level reserve at the end of the sth year. If s is greater than n (the cash values reach the net level reserve after attained age $x + n$), then the term $\ddot{s}_{\overline{n-1}|} + \ddot{a}_{x+n:\overline{s-n}|}$ is substituted for $\ddot{s}_{\overline{s-1}|}(1+i)^{n-s}$ in formula (5).

If

$$P_x^1 = {}^{AD}P_x - K\,,$$

$$P_x^t = {}^{AD}P_x \qquad \text{for} \qquad 2 \le t \le n,$$

and

$${}_nCV_x = \text{Cash value at end of } n \text{ years},$$

then

$${}^{AD}P_x \cdot \ddot{s}_{\overline{n}|} = {}_nCV_x + (1+i)^{x+n}\,(M_x' - M_{x+n}') + K(1+i)^n\,. \qquad (6)$$

Multiplying equation (6) by v^n, we get

$$ {}^{AD}P_x \cdot \ddot{a}_{\overline{n}|} = {}_nCV_x \cdot v^n + (1+i)^x (M'_x - M'_{x+n}) + K . \qquad (6.1)$$

However, the values so arrived at could possibly be negative during the early years, which presents a practical if not legal problem in defining the death benefits. Now let a be the largest integer for which ${}_aCV_x \leq 0$, then (6.1) becomes

$$ \begin{aligned} {}^{AD}P_x\left(\ddot{a}_{x:\overline{a}|} + \frac{D_{x+a}}{D_x}\ddot{a}_{\overline{n-a}|}\right) &= A^{\,1}_{x:\overline{a}|} \\ &+ \frac{D_{x+a}}{D_x}[\,v^{n-a}{}_nCV_x + (1+i)^{x+a}(M'_{x+a} - M'_{x+n})\,] + K . \end{aligned} \qquad (6.2)$$

If K is a constant, then this is the net premium formula producing a normal set of cash values which grade uniformly from an initial expense allowance.

However, if

$$ K = [(.02 + .25\,{}^{AD}P^{OL}_x) \not> .03]ELA + [.4\,{}^{AD}P_x \not> .04\ ELA] \qquad (7)$$

where ELA is the equivalent level amount, then we have net premium formulas for a face plus cash value plan generating "minimum values." Our problem now is to derive the ELA.

Now, by definition, the net single premium for the policy is

$$ A'_x = {}^{AD}P_x \cdot \ddot{a}_{x:\overline{n}|} + {}^aP_x(\ddot{a}_x - \ddot{a}_{x:\overline{n}|}) - K . \qquad (8)$$

Substituting the value of K from equation (6.2) in equation (8), we get

$$ \begin{aligned} A'_x = {}^{AD}P_x \cdot \ddot{a}_{x:\overline{n}|} &+ {}^aP_x[\ddot{a}_x - \ddot{a}_{x:\overline{n}|}] + A^{\,1}_{x:\overline{a}|} \\ &+ \frac{D_{x+a}}{D_x}[\,v^{n-a}\cdot{}_nCV_x + (1+i)^{x+a}(M'_{x+a} - M'_{x+n})\,] \\ &- {}^{AD}P_x\left[\ddot{a}_{x:\overline{a}|} + \frac{D_{x+a}}{D_x}\ddot{a}_{\overline{n-a}|}\right] \\ = A^{\,1}_{x:\overline{a}|} &+ \frac{D_{x+a}}{D_x}[\,v^{n-a}\cdot{}_nCV_x + (1+i)^{x+a}(M'_{x+a} - M'_{x+n})\,] \\ &+ {}^{AD}P_x\left[\ddot{a}_{x:\overline{n}|} - \ddot{a}_{x:\overline{a}|} - \frac{D_{x+a}}{D_x}\ddot{a}_{\overline{n-a}|}\right] + {}^aP_x\left[\ddot{a}_x - \ddot{a}_{x:\overline{n}|}\right] \\ = A^{\,1}_{x:\overline{a}|} &+ \frac{D_{x+a}}{D_x}[\,v^{n-a}\cdot{}_nCV_x + (1+i)^{x+a}(M'_{x+a} - M'_{x+n})\,] \\ + \frac{D_{x+a}}{D_x}[\,{}^{AD}P_x(\ddot{a}_{x+a:\overline{n-a}|} &- \ddot{a}_{\overline{n-a}|}) + {}^aP_x(\ddot{a}_{x+a} - \ddot{a}_{x+a:\overline{n-a}|})\,] , \end{aligned} \qquad (9)$$

or

$$ELA = A^{\;1}_{x:\overline{a}|} + \frac{D_{x+a}}{D_x}\left[v^{n-a} \cdot {}_nCV_x + (1+i)^{x+a}(M'_{x+a} - M'_{x+n}) \right]$$
$$+ \frac{\frac{D_{x+a}}{D_x}\left[{}^{AD}P_x(\ddot{a}_{x+a:\overline{n-a}|} - \ddot{a}_{\overline{n-a}|}) + {}^{a}P_x(\ddot{a}_{x+a} - \ddot{a}_{x+a:\overline{n-a}|}) \right]}{A_x}. \tag{10}$$

Equations (6.2), (7), and (10) enable us to calculate adjusted premiums for a face plus cash value policy with "minimum values." Values of 1,000 M'_x based on the 1958 CSO Table at various interest rates are presented in Table 1.

TABLE 1

VALUES OF 1,000 M'_x BASED ON 1958 CSO TABLE

Male Issue Age	2.5 Per Cent	2.75 Per Cent	3 Per Cent	3.25 Per Cent	3.5 Per Cent
0......	849.117128	695.816979	572.070059	472.011771	390.944423
1......	842.209811	688.926468	565.196273	465.154628	384.103843
2......	840.534655	687.259454	563.537341	463.503720	382.460901
3......	839.123167	685.858244	562.146309	462.122768	381.089932
4......	837.800490	684.548393	560.849129	460.838106	379.817637
5......	836.563102	683.325986	559.641485	459.645012	378.638883
6......	835.399028	682.178803	558.510907	458.530760	377.540682
7......	834.305346	681.103613	557.453853	457.491492	376.518859
8......	833.271224	680.089449	556.459215	456.515958	375.562018
9......	832.286304	679.125888	555.516499	455.593588	374.659507
10......	831.341092	678.203423	554.616181	454.714834	373.801750
11......	830.418905	677.305619	553.742059	453.863714	372.972973
12......	829.504303	676.417366	552.879335	453.025726	372.158954
13......	828.590293	675.531848	552.021355	452.194364	371.353322
14......	827.656087	674.628966	551.148674	451.350804	370.537845
15......	826.696330	673.703647	550.256477	450.490468	369.708159
16......	825.712840	672.757753	549.346656	449.615261	368.866170
17......	824.700731	671.786703	548.414905	448.721129	368.008052
18......	823.662018	670.792552	547.463303	447.810159	367.135886
19......	822.604877	669.783225	546.499520	446.889762	366.256823
20......	821.543031	668.771873	545.536147	445.971984	365.382379
21......	820.477310	667.759301	544.573953	445.057548	364.513224
22......	819.414344	666.751804	543.618905	444.152101	363.654692
23......	818.360280	665.755174	542.676452	443.260758	362.811575
24......	817.315341	664.769576	541.746693	442.383551	361.983832
25......	816.285131	663.800235	540.834489	441.524991	361.175642
26......	815.269497	662.846934	539.939557	440.684726	360.386584
27......	814.263235	661.904727	539.057188	439.858262	359.612361
28......	813.266512	660.973724	538.187426	439.045580	358.852887
29......	812.274554	660.049425	537.326024	438.242657	358.104346
30......	811.282941	659.127696	536.469102	437.445844	357.363296
31......	810.292254	658.209069	535.617136	436.655558	356.630091
32......	809.298485	657.289826	534.766668	435.868572	355.901711
33......	808.302392	656.370674	533.918349	435.085475	355.178682
34......	807.300383	655.448313	533.069134	434.303449	354.458386
35......	806.289104	654.519684	532.216224	433.519922	353.738450
36......	805.257270	653.574485	531.350202	432.726276	353.010978
37......	804.198442	652.606918	530.465838	431.917783	352.271687
38......	803.102830	651.608174	529.555193	431.087280	351.514104
39......	801.953807	650.563290	528.604790	430.222616	350.727265
40......	800.743421	649.465283	527.608492	429.318392	349.906414
41......	799.460806	648.304584	526.557865	428.367169	349.044983
42......	798.099567	647.075731	525.448247	427.364969	348.139580
43......	796.657418	645.777005	524.278382	426.310913	347.189629
44......	795.128983	644.403923	523.044541	425.201905	346.192568
45......	793.509452	642.952544	521.743509	424.035336	345.146289
46......	791.791327	641.416554	520.369973	422.806738	344.047039
47......	789.964727	639.787562	518.916806	421.510060	342.889679
48......	788.020659	638.058028	517.377694	420.140017	341.669790
49......	785.948055	636.218629	515.744786	418.690001	340.381812

TABLE 1—*Continued*

Male Issue Age	2.5 Per Cent	2.75 Per Cent	3 Per Cent	3.25 Per Cent	3.5 Per Cent
50......	783.736878	634.261023	514.011160	417.154275	339.020997
51......	781.375291	632.175343	512.168596	415.525999	337.581658
52......	778.852538	629.952747	510.209841	413.799236	336.058946
53......	776.161658	627.587796	508.130689	411.970774	334.450447
54......	773.291269	625.071216	505.923602	410.034502	332.751223
55......	770.231168	622.394837	503.582065	407.985252	330.957195
56......	766.969751	619.549325	501.098599	405.817051	329.063617
57......	763.491717	616.522204	498.463042	403.521639	327.063782
58......	759.780916	613.300351	495.664751	401.090396	324.950723
59......	755.820507	609.870145	492.692730	398.514463	322.717318
60......	751.595299	606.219495	489.537387	395.786262	320.357606
61......	747.085092	602.332083	486.185560	392.895192	317.863063
62......	742.273886	598.195324	482.627398	389.833582	315.227752
63......	737.143127	593.794543	478.851329	386.592341	312.444561
64......	731.672153	589.113368	474.844418	383.161282	309.505493
65......	725.838436	584.133960	470.592577	379.529308	306.401834
66......	719.615878	578.835579	466.079350	375.683390	303.123291
67......	712.973403	573.193408	461.284944	371.607759	299.657316
68......	705.877339	567.180621	456.188010	367.285446	295.990437
69......	698.291901	560.768805	450.766018	362.698610	292.108547
70......	690.193668	553.940189	445.005585	357.837263	288.004273
71......	681.568904	546.685284	438.900402	352.697449	283.675380
72......	672.417649	539.006240	432.453980	347.283494	279.126613
73......	662.747676	530.911670	425.675224	341.604217	274.366449
74......	652.571992	522.414502	418.576586	335.671343	269.405741
75......	641.881826	513.509438	411.155244	329.483778	264.244572
76......	630.648601	504.174765	403.394754	323.029114	258.873615
77......	618.821494	494.370497	395.263643	316.282568	253.273342
78......	606.332718	484.042916	386.719317	309.210334	247.416896
79......	593.102152	473.128535	377.711429	301.772457	241.272544
80......	579.066626	461.578280	368.201877	293.939365	234.817339
81......	564.183985	449.360709	358.167325	285.693843	228.038666
82......	548.427304	436.457090	347.595033	277.027485	220.931229
83......	531.790068	422.865504	336.486099	267.943279	213.499102
84......	514.275652	408.592128	324.848229	258.449585	205.750722
85......	495.885199	393.641290	312.687571	248.553442	197.693394
86......	476.612089	378.011009	300.005125	238.257670	189.330939
87......	456.446227	361.696502	286.799636	227.563233	180.665656
88......	435.370810	344.687632	273.065527	216.467628	171.697036
89......	413.352279	326.960869	258.786475	204.959712	162.417596
90......	390.330933	308.471857	243.929576	193.015072	152.809297
91......	366.213641	289.149765	228.440956	180.592700	142.840832
92......	340.866180	268.891507	212.241269	167.631492	132.465100
93......	314.108406	247.558225	195.223336	154.048532	121.617894
94......	285.709234	224.971218	177.249134	139.737134	110.216499
95......	255.382654	200.910101	158.148149	124.565469	98.159073
96......	222.564949	174.935885	137.578395	108.266799	85.237183
97......	186.051953	146.107274	114.803797	90.264661	70.999001
98......	142.615166	111.895497	87.842349	69.004747	54.225316
99......	84.645590	66.348813	52.033551	40.838290	32.054741

CECIL J. NESBITT:

The subject of this paper is difficult because there are several stages in the mathematics where more than one reasonable choice exists, and the various choices made lead to different outcomes. It is important, then, to have full specifications of each problem to be solved and a complete definition of notations used. To illustrate, let us consider a problem corresponding to the authors' formula (5). We might specify full preliminary term insurance for the first year with cash value of zero at the end of the year. For the next $s-1$ years, we might specify modified net premiums (cash value premiums) β such that

$$\beta = vq_{x+t}(1 + {}_{t+1}CV) + vp_{x+t}\cdot {}_{t+1}CV - {}_tCV,\ (1 \le t < s)\,, \qquad (1)$$

where ${}_tCV$ denotes the cash value at the end of year t. We have already specified ${}_1CV = 0$, and we further specify ${}_sCV = {}_sV$, where ${}_sV$ is the reserve on some net-level premium basis with annual premium P. For years s to n, we require

$$P = vq_{x+t}(1 + {}_{t+1}V) + vp_{x+t}\cdot {}_{t+1}V - {}_tV\,, \qquad (2)$$

with ${}_nV$ to be the ordinary life net level-premium reserve ${}_nV_x$, or some other appropriate value, depending on the net level-premium assumption for years $n+h$, $h>0$. From equations (1) and (2), we find

$$(\beta - vq_{x+t})(1+i)^{n-t} = \Delta[(1+i)^{n-t}{}_tCV] \qquad (1 \le t < s)\,,$$

$$(P - vq_{x+t})(1+i)^{n-t} = \Delta[(1+i)^{n-t}{}_tV] \qquad (s \le t < n)\,.$$

On summing, and noting that ${}_sCV = {}_sV$, we get

$$\beta\ddot{s}_{\overline{s-1}|}(1+i)^{n-s} + P\ddot{s}_{\overline{n-s}|} - (1+i)^{x+n}(M'_{x+1} - M'_{x+n}) = {}_nV. \quad (3)$$

As yet, we have not specified P fully. One possibility is to require that $P = P^{NL}$, as given by the authors' formula (2), then ${}_sV = {}_sV^{NL}$, the net level-premium reserve at the end of s years for an insurance of 1 plus the net level-premium reserve, with ${}_nV = {}_nV_x$. If such P^{NL} is substituted in our formula (3), then β may be determined as

$$\beta = \{{}_nV_x + (1+i)^{x+n}(M'_{x+1} - M'_{x+n}) - P^{NL}\ddot{s}_{\overline{n-s}|}\} / \ddot{s}_{\overline{s-1}|}(1+i)^{n-s}$$

and is sufficient to provide the death benefits of $1 + {}_tCV$, $2 \le t \le s$ for years 2 to s, and to provide ${}_sV^{NL}$ at the end of s years. Here, the ${}_tCV$ grade from 0 at $t=1$ to ${}_sV^{NL}$ at $t=s$, but would not necessarily agree with the authors' values based on their formula (5). Their formula resolves the indeterminacy of our formula (3), which has two unknowns, β and P, by requiring that

$$c_x + \beta \frac{N_{x+1} - N_{x+s}}{D_x} = P \frac{N_x - N_{x+s}}{D_x}, \quad (4)$$

or, that net level-premiums of P shall be equivalent to net premiums of c_x in the first year and of β in years 2 to s.

By utilizing our formula (4), to substitute for β in our equation (3), one obtains the authors' formula (5), with their P^{NL} replaced by P. The P so obtained is not the same as the P^{NL} of the authors' formula (2) but is more subtle, being the net level premium for an insurance of 1 plus the cash values which depend on β, which depends on P through our relation (4).

In the foregoing, we have assumed $s < n$. If $s > n$, further specification would be necessary, particularly in regard to $_nV$. However, if the specifications are clearly set out, then it is generally easy to write equations such as (1) and (2) for the various insurance years, and then by summation obtain the required premiums and cash values.

The authors' formulas (6) and following leave one uncertain, as they have not defined what they mean by $_nCV_x$ and ^aP_x. I expect that their formulas present only one of a number of choices that would yield adjusted premiums for a face plus cash value policy with "minimum values," and some clarification of their choice would be desirable.

The primary purpose of the paper seems to be to provide formulas for cash value premiums, and these would lead to formulas for the cash values. There remains the question as to what formulas would be used for reserves, which question is analogous for this type of insurance to that considered by D. C. Baillie in his "Actuarial Note—Cash Value as Death Benefit," which appears in this number of the *Transactions*.

FRANKLIN C. SMITH:

Since I have been doing a considerable amount of work in this same area, I was interested in the authors' treatment of this subject.

In the section of the paper which deals with the case in which the additional death benefit is the Commissioners' Reserve, the authors state that their formulas can be used "where there is no uniform equivalent amount concept." Since the plan under consideration involves varying death benefits, I fail to see how there can be no uniform equivalent amount concept. Furthermore, in my opinion the authors have not used the Commissioners' Method to produce formula (4). Since their approach would produce reserves which are larger than Commissioners' Reserves, there can be no legal objection. I merely object to the use of the label "Commissioners' Reserve."

I believe that the derivation of working formulas for the case in which the added death benefit is the Commissioner's Reserve is more complicated than the paper shows. For example, let us consider an n-pay life plan with the added death benefit in effect for n years where $n < 20$. Then, in my opinion the equation corresponding to the authors' equation (4) should be

$$\beta\ddot{a}_{\overline{n}|} = A_{x+n}V^n + (1+i)^x(M'_x - M'_{x+n}) + ELRA \cdot {}_{19}P_{x+1} - (1 + {}_1V)c_x \,,$$

which contains the three unknowns β, $ELRA$, and ${}_1V$. A second equation can be obtained by writing a prospective formula for the first reserve; thus

$${}_1V = ELRA \cdot A_{x+1} - \beta\ddot{a}_{x+1:\overline{n-1}|} \,;$$

and a third equation can be obtained by writing a retrospective formula for the same reserve; thus

$${}_1V = \{\beta - c_x - [ELRA \cdot {}_{19}P_{x+1} - (1 + {}_1V)c_x]\}(1+i) \,,$$

from which

$${}_1V = (\beta - ELRA \cdot {}_{19}P_{x+1})\mu_x \,.$$

From these three equations we can get

$$\beta = \frac{A_{x+n}V^n + (1+i)^x(M'_{x+1} - M'_{x+n})}{\ddot{a}_{\overline{n}|} - \dfrac{\ddot{a}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{20}|}} + \left(1 - \dfrac{\ddot{a}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{20}|}}\right) k_x} \,.$$

At the end of this discussion I have appended the results for a fifteen-pay life issued at age 50 with both the authors' treatment and mine. In this material P denotes the factor used to obtain the added death benefit.

An attack similar to the above can be used on any basic plan in those cases which are not eligible for full preliminary term valuation.

The authors derive formulas for grading from the first full preliminary term reserve to the sth net level reserve. The paper does not treat the problem of grading from a non-zero first year Commissioners' Reserve to a subsequent net level reserve. It may be remarked that such cases can be handled, but, since they involve five equations in five unknowns, I shall not take time to do so here.

In discussing the case in which the added death benefit is the minimum cash value, the authors introduce the symbol ${}^sP_x$ in equation (8) without defining it. Furthermore, since the basic plan has not been described, it is not possible to deduce with certainty just what this symbol represents. I suspect that the basic plan is an ordinary life plan with the added benefit payable for n years and with the premium reducing at the end of n years. If this surmise is correct, then ${}^aP_x$ must denote the adjusted premium after the nth year, and it should be pointed out that ${}^{AD}P_x$ and ${}^aP_x$ must be in the same ratio as the gross premiums for the respective periods.

APPENDIX

15-PAY LIFE, AGE AT ISSUE 50

1958 CSO 3 PER CENT

GOLD AND WILSON		SMITH	
P	= 54.11149	P	= 54.53731
β	= 54.57177	β	= 54.53731
$ELRA$	= 1132.20	$ELRA$	= 1131.48

Year	Amount of Insurance	Commissioners' Reserve	Year	Amount of Insurance	Commissioners' Reserve
1.	$1,014.13	$ 9.19	1.	$1,009.18	$ 9.18
2.	1,061.19	56.52	2.	1,056.52	56.52
3.	1,108.80	104.42	3.	1,104.43	104.43
4.	1,156.91	152.83	4.	1,152.85	152.85
5.	1,205.45	201.68	5.	1,201.71	201.71
6.	1,254.35	250.89	6.	1,250.93	250.93
7.	1,303.50	300.37	7.	1,300.42	300.42
8.	1,352.80	350.01	8.	1,350.07	350.07
9.	1,402.12	399.68	9.	1,399.74	399.74
10.	1,451.33	449.25	10.	1,449.32	449.32
11.	1,500.26	498.56	11.	1,498.63	498.63
12.	1,548.77	547.46	12.	1,547.53	547.53
13.	1,596.65	595.76	13.	1,595.82	595.82
14.	1,643.72	643.26	14.	1,643.29	643.29
15.	1,689.73	689.73	15.	1,689.73	689.73

ERSTON L. MARSHALL:

I am not entirely sure whether it is quite safe for me to make my appearance at this particular time. Several years ago one of our younger actuaries told me that I did not know how unpopular I was among the younger generation of actuaries. He himself had completed all his associateship examinations, except Part 4.

My rather insignificant actuarial note appeared in the program of the October meeting of the American Institute of Actuaries exactly thirty years ago. It did not make much impression and lay dormant for several years until the Education Committee discovered it and recommended it for reading for Part 4 of the examinations. Some of the students took a look at it and decided that, since they had not heard of any such policy as I described, they would spend their study time more profitably on some more important subject. And that was all right until the Examination Committee discovered it and commenced putting it into Part 4 of the examinations. Then they blamed me because they did not pass it.

It is quite obvious that Mr. Gold was one of those who took my actuarial note more seriously. I want to congratulate him and Mr. Wilson on their excellent paper. It is appropriate at this time of course, because, as stated in their paper, the cash value is the amount that is usually paid on that type of policy. When I presented my original paper, however, practically no one was issuing that kind of a policy, and my only excuse for presenting it was the fact that legislators of some of the states periodically submitted bills to their legislatures that would require all life insurance companies to pay the entire reserve in addition to the face amount of the policy on all forms of policies, the theory, of course, being that otherwise the company was confiscating the reserve.

So, I did not invent the policy. In fact, I did not prepare the policy for any company. But a number of years previously when I was a young consulting actuary, I was asked by a very small western company, not now in existence, to furnish reserves for a policy of this type that had been prepared for it by another actuary of whom I had never heard and who had died before furnishing the company with any table of reserves for the policy. When some of my more experienced actuarial friends were unable to tell me where to find such reserves or how to compute them, I had to try it myself. It was then that I discovered that, by preparing a new kind of commutation column, I could compute the reserve about as easily as for more usual policy forms. And I want to congratulate Professor Jordan, Professor Nesbitt, Professor Greville, and Mr. Nowlin on their improvements over my method of approach and development of the formula.

I also wish to thank Mr. Gold and Mr. Wilson for the pleasant experience of having my paper referred to thirty years after it was presented. I hope that, when they are old men, they will have a similar pleasure. It has been said that that is one form of immortality.

(AUTHORS' REVIEW OF DISCUSSION)

MELVIN L. GOLD AND DAVID T. WILSON:

The purpose of the paper was to provide some formulas for net premiums producing cash values less than net level reserves. As was so aptly remarked, "there are several stages in the mathematics where more than one reasonable choice exists." I would like to thank Professor Nesbitt and Mr. Smith for showing us what happens if we follow different tangents.

As Professor Nesbitt brought out, there is a subtle difference between the net level premiums of equations (2) and (5). Thus, if a policy is brought out where cash values grade to the net level reserves at duration s and where the reserves are net level all the way, at time s the cash values and reserves are not necessarily identical. Mr. Smith presumed correctly; the basic plan is an ordinary life plan with the added benefit payable for n years and with the premium reducing at the end of n years and ${}^{a}P_{x}$ is the adjusted premium after the nth year.

I would like to thank Dr. Marshall for giving us some background material on his classical paper and for his kind remarks. It is not every day that a younger actuary has his paper commented on by a man of Dr. Marshall's renown.