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EXPERIENCE-RATING GROUP LIFE INSURANCE

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ABSTRACT

This paper explores the principles upon which experience-rated group insurance currently is based and discusses the question of whether or not these principles are sound. With the use of two computer programs, analysis of an insurer's portfolio of one hundred identical cases is made for one policy year and for a series of fifty policy years. These analyses reveal violations of basic insurance principles. To explain how such a problem arose, a short historical study of experience rating is conducted. A mathematical analysis of the sources of group profit explains how the problem has gone unnoticed through statutory financial reports. The paper concludes with a survey of methods that will allow experience-rated group insurance to be based on sound principles.

Experience rating of large group life and health insurance risks is based on the precedent of rating workmen's compensation insurance on an individual case's own experience. Experience-rating plans for workmen's compensation insurance were developed in the early 1900's by casualty actuaries. At that time, large corporations had expressed displeasure when the manual or schedule rating plans being used by insurers developed consistent excesses of premiums over claims. Seeking a better rate for their own risks, the corporations made use of competitive pressures between insurers and the threat of self-insurance to encourage insurers to develop experience-rating plans.

Casualty actuaries who developed the original experience-rating plans were fearful of potential financial selection against the insurers. These fears were expressed in papers and discussions in the *Proceedings of the Casualty Actuarial Society*. Notable among the early papers were Winfield W. Greene's paper "Should the Compensation Premium Reflect the Experience of the Individual Risk?" (*PCAS*, II, 347) and Joseph H. Woodward's paper "The Experience Rating of Compensation Risks" (*PCAS*, II, 368). Some twenty years later, the concerns expressed by casualty actuaries were mirrored by life actuaries who borrowed experience-rating techniques from workmen's compensation insurance and applied them to group life and health insurance. William R. Williamson, in his written discussion of Ralph Keffer's paper "An Experience Rating Formula" (*TASA*, XXX, 593), expressed the same fears as did early casualty actuaries.

Despite a sound beginning, the original concern of life and casualty actuaries has been lost in the archives of actuarial literature. Group life and health insurance experience-rating plans commonly used today do not discourage the financial abuses against which the early literature cautioned. The purpose of this paper is to rediscover the original problem, to suggest how the original concern became ignored, and to offer solutions to today's problem. Actuaries who are interested in a complete bibliography of the literature on experience rating can refer to the Appendix of Paul Dorweiler's presidential address to the Casualty Actuarial Society (*PCAS*, XXI, 90).

I. HISTORICAL DEVELOPMENTS

Early experience-rating plans for workmen's compensation insurance were of two distinct types: prospective plans and retrospective plans. Prospective experience-rating plans were developed by stock companies in response to the competitive pressures of the early 1900's. A prospective plan used the individual case's actual experience for one or more experience periods to project expected claims for the succeeding experience period. Claim projections were made using a credibility formula which gave relative weights to the case's own prior experience and the insurer's portfolio experience, depending on factors influencing the claim variance of the individual case. The billed premium for the succeeding experience period was guaranteed and set equal to the projected claim experience plus a margin for expenses, profit, and adverse claims. At the end of the period, no experience refund was paid if actual experience was more favorable than the projected, and no deficit was charged to the case if actual experience was adverse.

The insurer profited from a portfolio of prospective plans as long as total billed premiums received from all policyholders exceeded total incurred claims plus expenses. Individual cases could exhibit either excesses or deficits of billed premium over actual experience while the entire portfolio remained profitable. If care was taken by the actuary to develop as unbiased and accurate a credibility formula as possible, prospective experience-rating plans would be profitable to the insurer. The importance of the credibility formula to a sound experience-rating plan is apparent from the vast amount of literature on the subject, primarily in the *Proceedings of the Casualty Actuarial Society.*

From the policyholder's viewpoint, prospective experience-rating plans had advantages and disadvantages. The major advantage was a low billed premium. The billed premium would vary with the length and number of experience periods used to project claims, the credibility formula used, and the size of any premium margins for expenses, profit, and claims. The policyholder was assured that the billed premium was close to the minimum necessary premium, and, further, that once established the premium was guaranteed. The major disadvantage of the prospective plan was that at the end of an experience period the billed premium could still be in excess of incurred claims plus expenses. In this case the policyholder would often feel that such an excess should be refunded. Naturally, however, if the policyholder incurred an excess of actual claims over the guaranteed billed premium, he was quite satisfied, since the insurer made no claim for the loss.

Surpluses on some cases were necessary to offset deficits on other cases under prospective plans. The tension between the policyholder, who felt that a surplus should be refunded, and the insurer, who required the surplus to offset other deficits, made prospective experience-rating plans an imperfect marketing vehicle. Although the principle of the plan was sound, the policyholder was often dissatisfied with the result.

Retrospective experience-rating plans were developed by mutual companies as their response to the early competitive environment. A retrospective plan used the individual case's experience for one or more experience periods to modify the rate for a completed experience period. The billed premium charged in each experience period included a large premium margin for claims; that is, billed premium included a large surplus of premium over projected claims and expenses. At the end of the experience period the excess of billed premium over actual incurred claims, expenses, and profit was returned to the policyholder as an experience refund or dividend. Retrospective experience-rating plans often used a credibility formula to pool claim experience for those cases not large enough to have developed stable dividends based on their own experience.

The life and casualty actuaries' fear of potential financial antiselection from experience-rated plans developed from retrospective plans. Negative dividends, or deficits, arose whenever actual claims plus expenses exceeded the billed premium. Since positive dividends were returned to the policyholder, they were not generally available to offset deficits as in prospective plans. A limited amount of negative dividends was recoverable from positive dividends as long as positive dividends were not reduced substantially thereby. The insurer, having failed to charge the policyholder a premium with a large enough margin for claims, was forced to hold a negative dividend. If such negative dividends were not made a legally collectible debt of the policyholder, these amounts often became uncollectible.

Policyholders found retrospective experience-rating plans more acceptable than prospective experience-rating plans. Although retrospective plans had the disadvantage of a high billed premium, the return of the case's own surplus as a dividend at the end of the experience period outweighed this disadvantage. Furthermore, large policyholders were able to bring competitive pressures to bear on insurers to reduce the billed premiums without making negative dividends a legally collectible debt of the policyholder. Over a period of years insurers also neglected to charge negative dividends from a case against positive dividends of other cases through the dividend formula.

To meet the successful competition of mutual companies, stock companies also began writing retrospective experience-rating plans. With stock and mutual insurers both writing retrospective plans, competition for large group life and health cases became intense and centered on the lowest billed premium, the range and quality of services offered the policyholder, and the provisions for disposition of losses or negative dividends. The resolution of this highly competitive situation was discussed by George E. McLean in his paper "An Actuarial Analysis of a Prospective Experience Rating Approach for Group Hospital-Surgical-Medical Coverage" (*PCAS*, XIX, 155). Mr. McLean describes a Blue Cross–Blue Shield plan developed in 1957 for large accounts that combined a billed premium based on a prospective experience-rating formula with experience refunds based on a retrospective experience-rating formula. The plan was developed because Blue Cross–Blue Shield found that an experience refund agreement alone was not sufficient inducement to retain the best risks.

George McLean's description of the Blue Cross–Blue Shield plan for large group life and health policyholders is the first discussion of the method of experience-rating large risks that is in common use today throughout the insurance industry. Retrospective experience-rating plans have had the billed premium reduced to that of a prospective experience-rating plan. This practice is not in itself necessarily pernicious, but insurers have neglected the earlier advice of actuaries and have failed to make reasonable arrangements for the recovery of negative dividends. Current practice is to carry forward the deficit on a case until it is recovered from a future surplus on the same case. This principle may seem sound, but, as will be demonstrated, it violates basic principles of insurance and finance.

II. SINGLE-YEAR ANALYSIS

A thorough analysis of the problem caused by the use of a combination prospective and retrospective experience-rating plan for a portfolio of group life and health insurance cases has four aspects: (1) an analysis of the portfolio in each policy year, assuming that each year is a statistically independent event; (2) an analysis of the portfolio over a series of policy years; (3) an analysis of statutory accounting for experience-rated plans; and (4) an analysis of the risk and profit charge. Each of these aspects is analyzed in a separate section of this paper.

The single-year analysis is based on two generally accepted experience-rating concepts described by Paul Jackson in his paper "Experience Rating" (*TSA*, V, 239): the pure accounting method and the modified pure accounting method. To analyze the experience-rating plans, a computer program was developed to calculate the probability distribution function of aggregate claims and the stop-loss premiums for pooling claims in excess of a given amount for group life and health insurance. The program is described in the Appendix.

The analysis uses a group life portfolio rather than a group health or combined life and health portfolio. Group life insurance simplifies the mathematical development, since the only random variable needing definition is the frequency of claim occurrence, the size of a given claim being fixed. Group health insurance requires a definition of both the frequency of claim occurrence and the size of a claim given that it occurs. Furthermore, a health plan is composed of various coverages such as basic hospital, surgical, and major medical, each requiring a separate analysis. The analysis of experience-rating plans using a life portfolio is equally applicable to an analysis of a health portfolio or a combined life and health portfolio, the only differences being in the mathematical complexity and the financial magnitude of the problem. A case, or a portfolio of cases, with a small claim variance will exhibit less of a financial problem than one with a large claim variance. Group life insurance, with a large individual claim amount and a low claim frequency, will have a larger claim variance in general than will group health insurance. Because of a lower claim variance, a health portfolio will have less of a financial problem than a life portfolio with the same premium.

Illustration of a single year of experience rating is based on a sample group life case described in Table 1 and an insurer's group life portfolio consisting of one hundred cases identical with the sample case. Table 2 presents the frequency function of total claims in one year, the cumulative probability distribution function, and the stop-loss premium for pooling claims in excess of a given claim amount. Figure 1 charts the frequency function of the claim data of Table 2. Figure 2 presents the cumulative probability distribution function of the claim data.

Pure Accounting Method

The pure accounting method is defined by Jackson as an experience-rating plan returning to the policyholder, in total, the excess of billed premium over incurred claims and assessed expenses during an experience period. Any excess of claims plus expenses over billed premium must be carried forward as a deficit to be recovered from future surpluses. Deficits are not recovered from surpluses on other cases in the portfolio.

TABLE 1

SAMPLE GROUP LIFE CASE
DISTRIBUTION OF LIVES
1,050 Lives

Age Group	Amount of Insurance ($000 Omitted)			
	$5	$10	$20	$40
15–19	50	......	......	......
20–29	200	50	25	......
30–39	200	100	75	......
40–49	100	50	50	......
50–70	25	50	50	25
Total	575	250	200	25

The analysis of this paper is initially concerned with the underwriting experience, not including expenses. To isolate the underwriting considerations, all premiums used in numerical examples throughout the paper are premiums for claims alone. For a new case such as the sample case, a reasonable annual premium would be the expected claim cost based on the 1960 Basic Group Mortality Table plus a small premium margin for claims. From Table 2, the expected claim cost for the case is $63,617.50 and the standard deviation is $37,310.31. A $65,000 annual premium for claims would be reasonable if a case of 1,050 lives were fully credible and without prior experience. The probability that the case will incur claims for a year less than or equal to $65,000 is 59.08 per cent. The

TABLE 2

DISTRIBUTION OF CLAIMS FOR SAMPLE CASE*

Based on 1960 Basic Group Mortality Table

Claim Amount	Claim Frequency	Cumulative Probability	Stop-Loss Premium
$ 0	.01137599	.01137599	$63,617.48
5,000	.01645537	.02783136	58,674.36
10,000	.02694041	.05477178	53,813.52
15,000	.02749243	.08226421	49,087.38
20,000	.04205203	.12431625	44,498.70
25,000	.04325447	.16757072	40,120.28
30,000	.05153941	.21911014	35,958.13
35,000	.04673943	.26584958	32,053.69
40,000	.05704469	.32289427	28,882.93
45,000	.05365845	.37655273	24,997.41
50,000	.05847281	.43502554	21,880.17
55,000	.05096266	.48598820	19,055.30
60,000	.05556274	.54155094	16,485.24
65,000	.04929648	.59084743	14,192.99
70,000	.04985879	.64070622	12,147.23
75,000	.04181338	.68251960	10,350.76
80,000	.04233273	.72485234	8,763.36
85,000	.03606388	.76091623	7,387.62
90,000	.03475389	.79567012	6,192.20
95,000	.02839859	.82406871	5,170.55
100,000	.02730536	.85137408	4,290.90
105,000	.02255299	.87392708	3,547.77
110,000	.02087666	.89480374	2,917.40
115,000	.01666696	.91147071	2,391.42
120,000	.01538302	.92685373	1,948.77
125,000	.01238849	.93924222	1,583.04
130,000	.01110901	.95035124	1,279.25
135,000	.00870337	.95905461	1,031.01
140,000	.00777612	.96683073	826.28
145,000	.00613250	.97296324	660.44
150,000	.00535124	.97831449	525.25
155,000	.00412232	.98243681	416.83
160,000	.00358306	.98601988	329.01
165,000	.00277487	.98879475	259.11
170,000	.00236575	.99116051	203.09
175,000	.00179591	.99295642	159.89
180,000	.00152492	.99448134	123.67
185,000	.00116244	.99564379	96.08
190,000	.00097096	.99661476	74.30
195,000	.00072734	.99734210	57.37
200,000	.00060503	.99794714	44.08
205,000	.00045475	.99840189	33.82
210,000	.00037301	.99877491	25.83
215,000	.00027609	.99905101	19.70
220,000	.00022554	.99927656	14.96
225,000	.00016739	.99944395	11.34
230,000	.00013508	.99957903	8.56
235,000	.00009888	.99967792	6.46
240,000	.00007947	.99975740	4.85
245,000	.00005830	.99981570	3.63
250,000	.00004636	.99986206	2.71
255,000	.00003359	.99989566	2.02

* Mean claims, $63,617.50; standard deviation, $37,310.305.

TABLE 2—*Continued*

Claim Amount	Claim Frequency	Cumulative Probability	Stop-Loss Premium
$260,000	.00002660	.99992227	$1.50
265,000	.00001931	.99994159	1.11
270,000	.00001515	.99995674	0.82
275,000	.00001087	.99996763	0.61
280,000	.00000849	.99997612	0.44
285,000	.00000610	.99998222	0.33
290,000	.00000473	.99998696	0.24
295,000	.00000336	.99999032	0.17
300,000	.00000259	.99999292	0.12
305,000	.00000185	.99999477	0.09
310,000	.00000141	.99999619	0.06
315,000	.00000099	.99999719	0.04
320,000	.00000076	.99999795	0.03
325,000	.00000053	.99999849	0.02
330,000	.00000040	.99999890	0.01
335,000	.00000028	.99999918	0.01
340,000	.00000021	.99999940	0.00
345,000	.00000015	.99999955	0.00
350,000	.00000011	.99999966	0.00

probability that the case will incur claims for a year exceeding $65,000 is 40.92 per cent.

Given that the case incurs claims exceeding the $65,000 premium, it is possible to calculate the expected underwriting deficit using Table 2 and an application of Bayes's rule. Bayes's rule states that the probability of event Y occurring, given that event X has occurred, is equal to the probability of events X and Y occurring divided by the probability of event X occurring. Bayes's rule can be applied easily to expectations rather than probabilities. The expected deficit, given that a deficit occurs, would then be calculated by dividing the expected claims in excess of the premium, $E(X, Y)$, by the probability of incurring a claim cost in excess of the premium, $E(X)$ (see accompanying tabulation). Summing the expected

Claims $= X$	Deficit $= Y$	Pr(Claims $= X$)	E(Deficit $= Y$)
$70,000	($70,000−$65,000)	0.04985879	$ 5,000×0.04985879
75,000	(75,000−65,000)	0.04181338	10,000×0.04181338
.	.	.	.
.	.	.	.
.	.	.	.
		0.40915257	$14,192.99

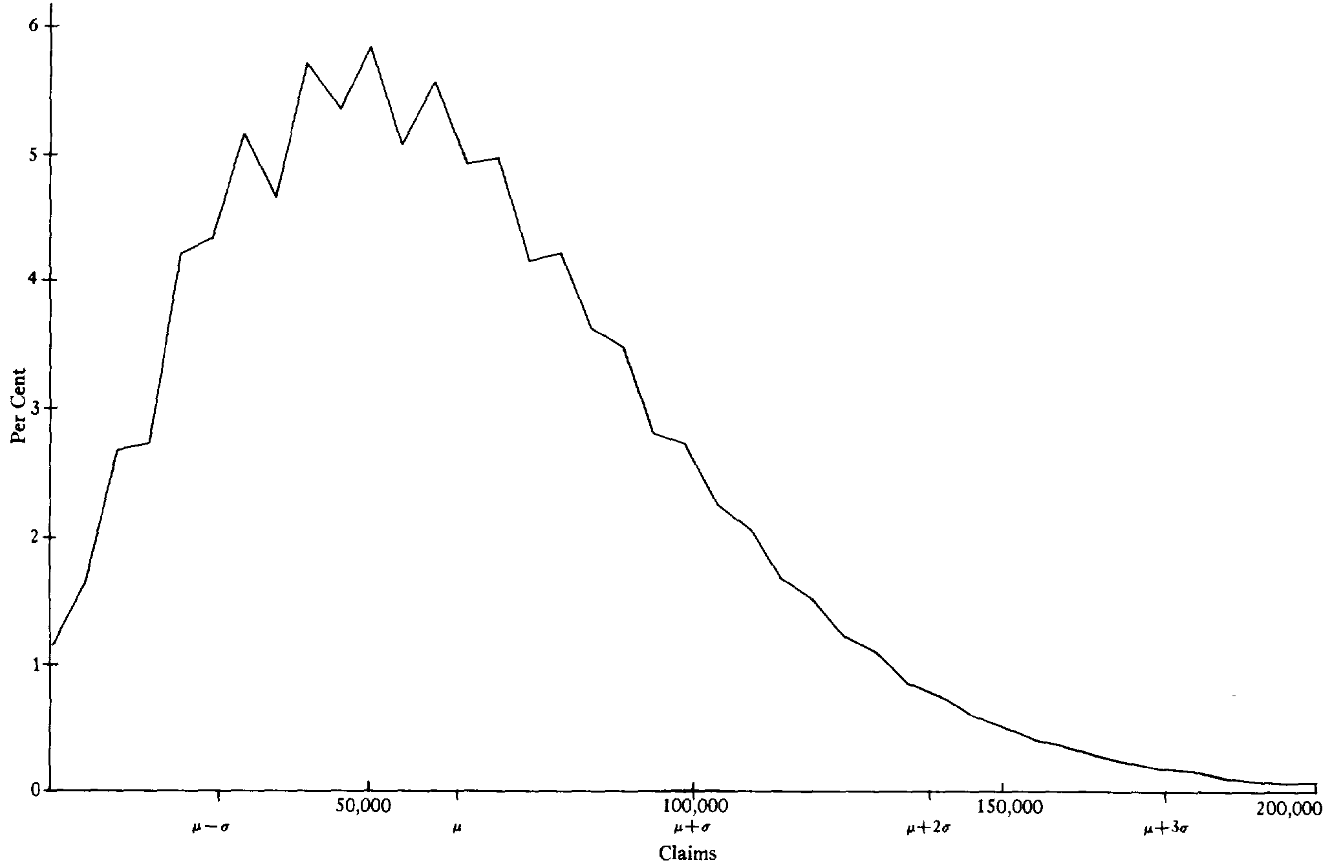

FIG. 1.—Frequency function of sample case claims

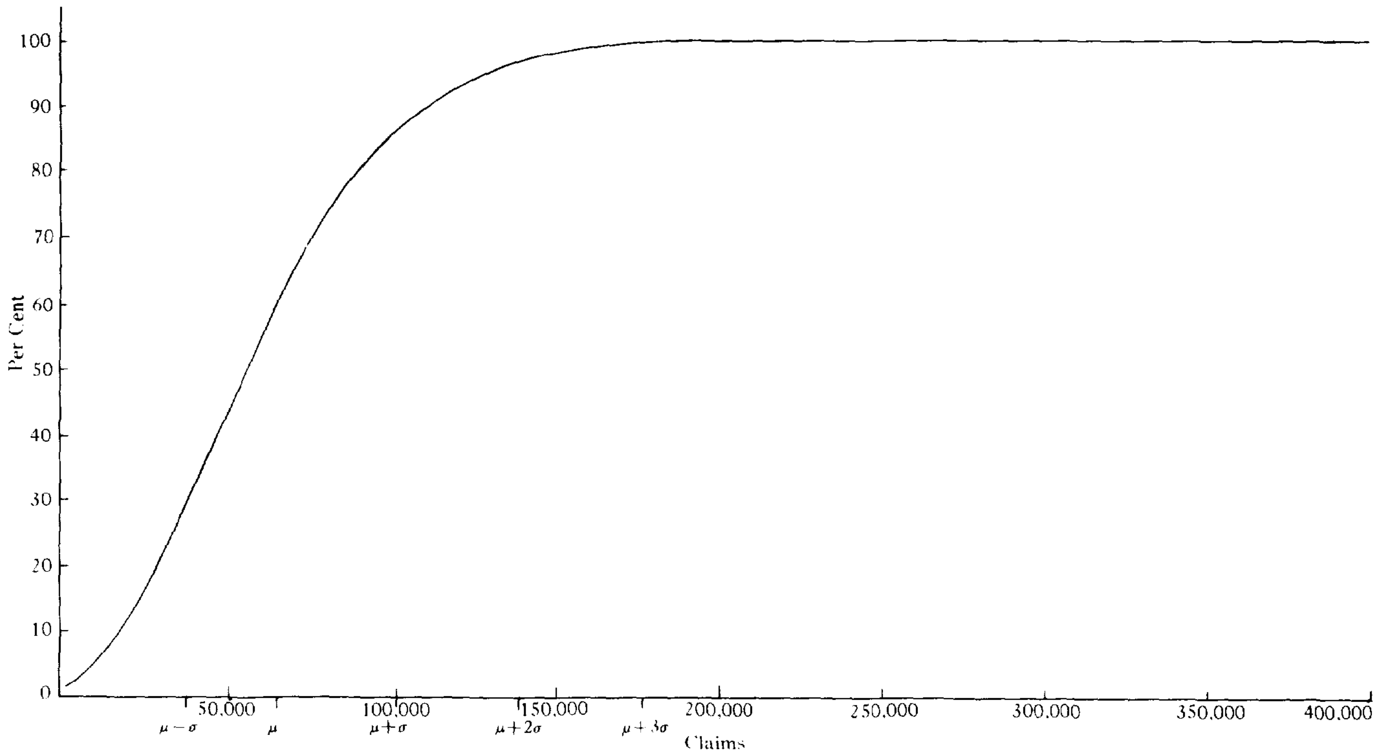

FIG. 2.—Cumulative probability distribution function of sample case claims

deficits for each claim level in excess of \$65,000, we obtain the value of the expected deficit, \$14,192.99. Then the expected deficit on a case, given that a deficit occurs, will be the expected claims in excess of the premium, \$14,192.99, divided by the probability of a case incurring claims in excess of the premium, 0.40915257, or \$34,688.75.

Similarly, for each case that has an excess of premiums over claims, the expected underwriting surplus, given that a surplus occurs, can be found by using Table 2 and the same calculation (see accompanying tabulation). The expected surplus, given that a surplus occurs, will be \$15,575.51 divided by the probability of a surplus occurring, 0.59084743, or \$26,361.31.

Claims = X	Surplus = Y	Pr(Claims = X)	E(Surplus = Y)
\$ 0	(\$65,000−\$ 0)	0.01137599	\$65,000×0.01137599
5,000	(65,000− 5,000)	0.01645537	60,000×0.01645537
.	.	.	.
.	.	.	.
.	.	.	.
60,000	(65,000− 60,000)	0.05556274	5,000×0.05556274
65,000	(65,000− 65,000)	0.04929648	0×0.04929648
	....	0.59084743	\$15,575.51

The probability that a surplus will occur on a given case in a given year is 0.59084743. The probability that a deficit will occur is 0.40915257, the complement of the probability of a surplus. A portfolio of one hundred cases, each case exactly like the sample case, will have an average of 40.915257 cases with deficits and 59.084743 cases with surpluses each policy year. Further, the total expected deficit on the cases with a deficit will be 40.915257 × \$34,688.75 = \$1,419,299.12, and the total expected surplus on the cases with surplus will be 59.084743 × \$26,361.31 = \$1,557,551.23. The total premium for the one hundred cases will be \$6,500,000.

In the first policy year after writing a hundred new cases, there are no prior deficits to be recovered from surpluses. In this year the pure accounting method provides that each one of the 59.084743 cases with a surplus be paid its total surplus as an experience refund. Each one of the 40.915257 cases with a deficit, however, carries its own deficit forward to succeeding policy years to be recovered from a future surplus. This means that in the first policy year the insurer must pay \$1,419,229.12 in claims for which reimbursement will be forthcoming only from future surpluses. If a deficit case should somehow not develop a surplus in future years or should cancel while in a deficit position, the pure accounting method does not provide for a direct recovery of the deficit.

In policy years following the first, some portion of the $1,557,551.23 surplus on 59.084743 cases will be used to recover prior years' deficits on a case rather than being paid as an experience refund. When this occurs, paid experience refunds in that year will be less than the total surplus of $1,557,551.23. The deficit on 40.915257 cases will be unchanged at $1,419,299.12. The total deficit from all years will be the sum of the current year's deficit and the unrecovered portion of prior years' deficits.

Calculation of the expected surpluses, expected underwriting deficits, and expected experience refunds in a policy year reveals two important points concerning combination experience-rating plans. First, a portfolio of group cases will require that the insurer pay some claims without being able to recover them in the year they are paid. Second, a case's deficit can be recovered only from future surpluses which can occur only if the case remains active. A case canceling with a deficit leaves the insurer in the position of having paid claims that cannot be directly recovered. Theoretically, these otherwise unrecoverable deficits are recovered indirectly over a period of time by a risk charge which is added to the annual premium. A thorough discussion of the efficacy of the risk charge is presented in Section V.

Modified Pure Accounting Method

The modified pure accounting method involves a stop-loss insurance pool against which claims in excess of a predetermined amount are charged rather than being carried forward as a deficit. The insurance pool has a charge which may be collected by either a reduction of paid experience refunds to surplus cases (Jackson's J-method) or an additional premium from each policyholder (Jackson's K-method).

Assume that the insurer now adds stop-loss coverage for claims in excess of $100,000 to each case in the sample portfolio. The amount $100,000 is approximately one standard deviation ($37,310.31) above the mean ($63,617.50). Referring again to Table 2, the stop-loss premium for pooling claims above $100,000 per case in a year is $4,290.90. The $4,290.90 stop-loss premium for each case is a non–experience-rated charge made in addition to the $65,000 premium for claims (Jackson's K-method) or a $429,090 reduction in experience refunds paid at year end for the entire portfolio of one hundred cases (Jackson's J-method). The stop-loss risks of the one hundred cases in the portfolio are combined to form an insurance pool with an annual premium of $429,090. If, during the policy year, a case incurs claims in excess of $100,000, the excess is charged to the stop-loss pool, not to the case. An extra stop-loss premium covers the following pooling arrangement on each case and is equal to $E(X, Y)$ from Bayes's rule (see the accompanying tabulation). Thus, the

Claims = X	Pooled Loss = Y	Pr(Claims = X)	E(Pooled Loss = Y)
$105,000.	($105,000–$100,000)	0.02255299	$ 5,000×0.02255299
110,000.	(110,000– 100,000)	0.02087666	10,000×0.02087666
115,000.	(115,000– 100,000)	0.01666696	15,000×0.01666696
.	.	.	.
.	.	.	.
.	.	.	.
	. . . .	0.14862592	$4,290.90

maximum annual underwriting deficit still chargeable to a case is the $100,000 stop-loss level less the $65,000 premium for claims, or $35,000.

A stop-loss pool will reduce, but not eliminate, the necessity for the company to provide funds to support deficits on experience-rated business. For each case with claims between $65,000 and $100,000 the entire underwriting deficit will have to be paid currently from the insurer's general funds and recovered eventually from future surpluses. For a case with $100,000 or more of claims, the general funds will have to provide $35,000 to pay for the case's excess claims. The expected deficit on each case after eliminating pooled claims is shown in the accompanying tabulation. Then the expected deficit on each case, given that a deficit

Claims = X	Deficit = Y	Pr(Claims = X)	E(Deficit = Y)
$ 70,000.	($ 70,000–$65,000)	0.04985879	$ 5,000×0.04985879
75,000.	(75,000– 65,000)	0.04181338	10,000×0.04181338
.	.	.	.
.	.	.	.
100,000.	(100,000– 65,000)	(1.0–0.8240637)	35,000×0.1759363
	. . . .	0.40915257	$9,902.60

occurs, is $9,902.60 divided by the probability of a deficit occurring, 0.40915257, or $24,202.71.

The introduction of a stop-loss pool reduces the expected case deficit, given that a deficit occurs, from $34,688.75 to $24,202.71. The stop-loss pool, however, does not reduce the number of expected deficits. Either experience-rating method will incur an average of 40.915257 deficits. Since the stop-loss pool is charged with $429,090 in claims, the insurer will not be required to invest general funds to pay these claims. The insurer will reduce his investment from $1,419,299.12 to $990,260.00 by arranging a stop-loss pool for the policyholders. The cost of this pool to the policyholder is either a 6.60 per cent premium increase or a reduction in paid experience refunds of 27.55 per cent.

Reviewing the funding of the stop-loss pool, it should be noted that this is a true insurance arrangement between the policyholder and the insurer. Each of the one hundred cases is charged an amount to recover the expected deficits on those cases with claims over $100,000. If more cases than expected incur a deficit in excess of $100,000, or if the deficits over $100,000 are larger than expected, then the insurer will suffer a real loss on the stop-loss pool. But, if the deficits over $100,000 are less than expected, or the number of cases which incur such a deficit is less than expected, the insurer will show a real profit from the stop-loss pool. Charges for the stop-loss pool are not experience-rated; excess charges from one case can be used to offset deficits on other cases. Since stop-loss spreads the risk across all participants in the pool, it is a true insurance product.

A Sound Insurance Scheme

One means of removing entirely the general funds investment in the portfolio is to reduce the stop-loss level to the premium for claims. If in the sample case all claims in excess of the $65,000 premium are insured through a stop-loss arrangement, there is no claim level which requires the insurer to pay claims from general funds. All claims are paid from either the $65,000 premium or the stop-loss pool. As with the modified pure accounting method, there are two basic ways to collect the premiums required for this scheme: reduce the paid experience refunds (J-method), or charge a higher premium to all cases (K-method).

If each case is to be charged an additional premium for a stop-loss pool at the premium for claims of $65,000, Table 2 shows that the charge is $14,192.99 per case. If the premium for the stop-loss pool is to be collected from experience refunds developed from surplus cases, expected experience refunds in the first policy year of $1,557,551.23 will have to be reduced by $1,419,299.12, to $138,252.11. Neither a 21.8 per cent premium increase on each case nor a 91.1 per cent experience refund reduction seems competitively feasible. Introducing a stop-loss pool at the premium level for claims, however, completely eliminates any general funds investment in the group portfolio and, by eliminating underwriting deficits, the problem of withdrawal antiselection on deficit cases. This strategy, although theoretically sound, is an expensive solution to the problem.

Since the only total insurance solution to withdrawal antiselection and general funds investment is to have the stop-loss pooling level equal to the premium level for claims, another, more feasible strategy is possible. By an increase in the premium from $65,000 to a higher level such as $85,000 (that is, an increase in the premium margin for claims), the stop-loss premium necessary to cover expected claims in excess of the premium for claims is lowered from $14,192.99 to $7,387.62. At the same time the higher premium raises the expected surplus on a case from $26,361.31 to $37,812.30 and the expected number of surplus cases from 59.084743 to 76.091623. Since the premiums for claims and the stop-loss level are still equal, each being $85,000, the insurer is not exposed to underwriting deficits which necessitate a general funds investment and cause withdrawal antiselection. This sound plan is the same retrospective plan that was developed by casualty actuaries of the early 1900's for experience-rating workmen's compensation insurance.

III. MULTIYEAR ANALYSIS

Discussion of the pure accounting and modified pure accounting methods of experience rating analyzed a single year of portfolio experience. The expected underwriting surplus and the expected underwriting deficit for the portfolio are the same in each policy year, since it is assumed that each year is a statistically independent event. In years in which the premium for each of one hundred cases remains at $65,000, the portfolio will expect 40.915257 deficit cases with a total of $1,419,299.17 in deficits and 59.084743 surplus cases with a total of $1,557,551.23 in surplus. In years after the first policy year, not all the surplus will be paid as experience refunds. Some cases that incur a surplus will be cases that entered the policy year in a deficit position. In this situation a case's surplus in the current year is used to offset the previous years' deficit, thereby reducing both the total portfolio deficit and paid experience refunds. Since the total deficit for the portfolio represents an investment of the insurer's general funds in its group life and health portfolio, insurers are justifiably concerned about its size. For this reason it is necessary to describe the total portfolio underwriting deficit over a period of years.

To study the progression of the total underwriting deficit on a group life portfolio, a second computer program was developed. This program allows testing of various experience-rating plans and pooling arrangements over a period of up to fifty policy years by drawing a sample series of claims from the distribution function of Table 2 and simulating an experience-rating plan using these claim data. The Appendix contains a description of the program.

An analysis of the sample portfolio of one hundred group life cases over a fifty-year period was completed under two sets of conditions. The first set of conditions assumes that no cases cancel during the fifty-year period; the second set of conditions assumes cancellation during the fifty years according to a specific rule based on the size of a case's year-end deficit.

No-Cancellation Situations

Seven sets of data were developed assuming that no case cancellations occur. A set of data is based on simulated claim experience for each of the hundred cases in the portfolio. A case's experience remains unchanged in each of the seven sets of data. The sole cause of differences in the total deficit, total claims, or other performance factors among the seven situations is the difference in the experience-rating plan or the pooling arrangements.

Specifications of each situation are as follows:

I. Each case has an annual premium for claims of $65,000.

II. Each case has an annual premium for claims of $65,000 and an over-all stop-loss pooling level of $100,000. An additional premium is collected from each case for the stop-loss pool.

III. Each case has an annual premium for claims of $65,000, an over-all stop-loss pooling level of $100,000, and an individual claim pooling level of $30,000. The individual claim pool removes any portion of a single claim in excess of $30,000 from the case's experience. An additional premium is collected for both the over-all and individual stop-loss pools.

IV. Each case has an annual premium for claims of $65,000, an over-all stop-loss pooling level of $100,000, an individual claim pooling level of $30,000, and a contingency reserve of a maximum of $20,000 built up from a case's surplus before payment of experience refunds at a maximum of $5,000 per year. Contingency reserves are used to offset subsequent adverse claim experience and are a refundable liability to the policyholder. An additional premium is collected for the stop-loss pools.

V. Each case has an annual premium for claims of $85,000.

VI. Each case has an annual premium for claims of $85,000 and an over-all stop-loss pooling level of $100,000. An additional premium is collected for the stop-loss pool.

VII. Each case has an annual premium for claims of $85,000 and a stop-loss pooling level of $85,000. An additional premium is collected for the stop-loss pool.

Stop-loss arrangements included above have different annual premiums. The net charges for each pool are as follows:

Over-all stop-loss pool of $100,000 only	$4,290.90
Over-all stop-loss pool of $100,000 (when used in conjunction with an individual stop-loss pool of $30,000)	2,437.58
Individual stop-loss pool of $30,000	4,505.00
Over-all stop-loss pool of $85,000 only	7,387.62

Chart I summarizes conditions of the seven situations. The total underwriting deficit at the end of each policy year for the seven situations described above is presented in Table 3. The same data are represented

CHART I

SUMMARY OF CONDITIONS
SITUATIONS I–VII

Condition	I	II	III	IV	V	VI	VII
$65,000 premium for claims	X	X	X	X			
$100,000 over-all stop-loss		X	X	X		X	
$30,000 individual stop-loss			X	X			
$20,000 contingency reserve				X			
$85,000 premium for claims					X	X	X
$85,000 over-all stop-loss							X

graphically on a semilog scale in Figure 3. A semilog scale allows an analysis of the percentage change as well as the absolute change in the deficit.

With the exception of Situation VII, where the stop-loss level is equal to the premium for claims, each situation has a total portfolio underwriting deficit in each of the fifty policy years studied. The total deficit grows rapidly in the early years, reaching an ultimate level characteristic of the experience-rating plan and the pooling arrangements in from one to twenty-five years. The total deficit then fluctuates for the remainder of the fifty years. Situation I takes the longest period of time to reach a stable level, while Situation VI takes the least time.

Table 4 summarizes the financial data for each of the seven situations. The financial importance of the total deficit becomes clear by reference to item 15 in the table. Each situation develops a fifty-year underwriting loss under normal statutory accounting practices.

TABLE 3

NO-CANCELLATION SITUATIONS

Total Underwriting Deficit of 100 Cases at Year End

Year	Situation I	Situation II	Situation III	Situation IV	Situation V	Situation VI	Situation VII
1....	$ 1,345,000	$1,000,000	$ 800,000	$ 800,000	$ 615,000	$270,000	$0
2....	2,405,000	1,655,000	1,325,000	1,175,000	1,100,000	450,000	0
3....	3,255,000	1,945,000	1,585,000	1,380,000	1,480,000	525,000	0
4....	4,370,000	2,545,000	2,005,000	1,665,000	1,730,000	510,000	0
5....	4,540,000	2,450,000	1,880,000	1,510,000	1,735,000	430,000	0
6....	5,200,000	2,820,000	2,195,000	1,670,000	1,835,000	525,000	0
7....	5,575,000	3,025,000	2,345,000	1,795,000	1,795,000	470,000	0
8....	6,285,000	3,480,000	2,600,000	2,000,000	1,670,000	375,000	0
9....	6,930,000	3,750,000	2,745,000	2,100,000	1,765,000	475,000	0
10...	7,225,000	3,455,000	2,480,000	1,840,000	1,825,000	390,000	0
11...	8,160,000	3,980,000	2,900,000	2,260,000	2,180,000	535,000	0
12...	8,695,000	4,115,000	3,125,000	2,425,000	2,205,000	525,000	0
13...	8,295,000	3,765,000	2,765,000	2,165,000	1,530,000	280,000	0
14...	8,055,000	3,635,000	2,620,000	2,125,000	1,365,000	410,000	0
15...	8,400,000	3,895,000	2,905,000	2,380,000	1,740,000	560,000	0
16...	8,445,000	3,730,000	2,725,000	2,140,000	1,455,000	450,000	0
17...	9,445,000	4,550,000	3,335,000	2,575,000	1,910,000	700,000	0
18...	10,080,000	4,630,000	3,310,000	2,550,000	2,040,000	510,000	0
19...	9,895,000	4,405,000	2,955,000	2,175,000	1,350,000	270,000	0
20...	10,210,000	4,435,000	2,930,000	2,240,000	1,545,000	400,000	0
25...	12,705,000	5,385,000	3,685,000	2,910,000	2,730,000	690,000	0
30...	13,250,000	5,205,000	3,390,000	2,660,000	1,885,000	380,000	0
35...	14,825,000	5,665,000	3,800,000	3,000,000	2,025,000	610,000	0
40...	14,430,000	4,840,000	2,850,000	2,125,000	1,490,000	305,000	0
45...	14,525,000	4,810,000	2,705,000	1,990,000	1,465,000	405,000	0
50...	16,030,000	5,435,000	3,135,000	2,445,000	1,800,000	470,000	0

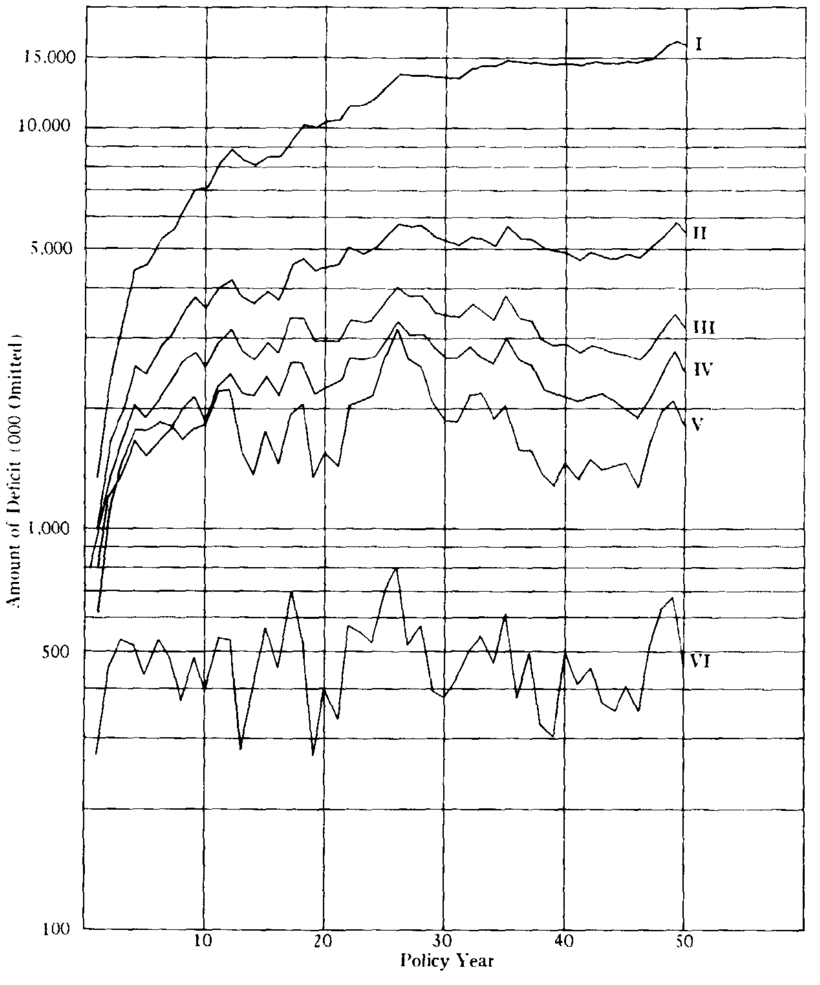

FIG. 3.—Active case underwriting deficits, Situations I–VI

TABLE 4

FIFTY-YEAR SUMMARY OF OPERATIONS

No-Cancellation Situations

	Situation I	Situation II	Situation III	Situation IV	Situation V	Situation VI	Situation VII
1. Experience-rated premiums	325,000,000	325,000,000	325,000,000	325,000,000	425,000,000	425,000,000	425,000,000
2. Claims incurred	317,530,000	295,090,000	282,465,000	282,465,000	317,530,000	295,090,000	279,450,000
3. Experience refunds paid	23,500,000	35,345,000	45,670,000	44,480,000	109,270,000	130,380,000	145,550,000
4. No. of experience refunds paid	909	1,373	1,761	1,806	2,867	3,406	3,614
5. Contingency reserves held*	0	0	0	500,000	0	0	0
6. No. of contingency reserves held*	0	0	0	49	0	0	0
7. Active case deficits*	16,030,000	5,435,000	3,135,000	2,445,000	1,800,000	470,000	0
8. No. of active case deficits*	85	69	61	51	38	23	0
9. Canceled case deficits	0	0	0	0	0	0	0
10. No. of canceled case deficits	0	0	0	0	0	0	0
11. Fifty-year underwriting gain (loss) (stop-loss not included)	(16,030,000)	(5,435,000)	(3,135,000)	(2,445,000)	(1,800,000)	(470,000)	0
12. Individual stop-loss premiums†	0	0	22,525,000	22,525,000	0	0	0
13. Individual stop-loss claims†	0	0	22,570,000	22,570,000	0	0	0
14. Over-all stop-loss premiums†	0	21,454,500	12,187,900	12,187,900	0	21,454,500	36,938,100
15. Over-all stop-loss claims†	0	22,440,000	12,495,000	12,495,000	0	22,440,000	38,080,000
16. Fifty-year stop-loss gain (loss)†	§	(985,500)	(352,100)	(352,100)	§	(985,500)	(1,141,900)
17. No. of years of statutory gains	14	22	27	24	21	23	50‡

* At the end of the fifty-year period.

† Stop-loss experience is not included in lines other than these marked.

‡ Each year has a $0 statutory profit.

§ Not applicable.

Cancellation Situations

It is unrealistic to assume that no cases cancel within a fifty-year period. In order to present more realistic results, a second set of data was developed, consisting of six situations which assumed that cases cancel. These situations also allow more sophisticated experience-rating plans than the first set of data. The same claim experience as in the first set of situations was used to allow a comparison between the two sets of data.

Specifications for each situation are as follows:

A. Each case has an annual premium for claims of $65,000, an over-all stop-loss pooling level of $100,000, an individual claim pooling level of $30,000, a maximum contingency reserve of $20,000 with a $5,000 maximum annual increase, and cancellation of the case if the year-end deficit exceeds $75,000. A fixed cancellation point was set arbitrarily to produce reasonable termination results.

B. Each case has an initial premium for claims of $65,000, with subsequent premiums based solely on the prior year's experience, an over-all stop-loss pooling level of $100,000, an individual claim pooling level of $30,000, a maximum contingency reserve of $20,000 with a $5,000 maximum annual increase, and cancellation of the case if the year-end deficit exceeds $75,000. This is the first situation to introduce a prospective experience-rating formula as well as a retrospective experience-rating formula. The premium developed by the prospective formula is based on one year's experience without any premium margin for claims.

C. Each case has an initial premium for claims of $65,000, with subsequent premiums based on the prior year's experience plus a 5 per cent premium margin for claims. Each case also has an over-all stop-loss pooling level of $100,000, an individual claim pooling level of $30,000, a maximum contingency reserve of $20,000 with a $5,000 maximum annual increase, and cancellation of the case if the year-end deficit exceeds $75,000. This situation differs from Situation B by the addition of a 5 per cent premium margin for claims in the prospective experience-rating formula.

D. Each case has an initial premium for claims of $65,000, with subsequent premiums based on the prior year's experience, plus 20 per cent of any cumulative deficit carried over from prior years. Each case also has an over-all stop-loss pooling level of $100,000, an individual claim pooling level of $30,000, a maximum contingency reserve of $20,000 with a $5,000 maximum annual increase, and cancellation of the case if the year-end deficit exceeds $75,000. This situation differs from Situation C by the addition of an extra premium to recover 20 per cent of any prior deficit and the elimination of the 5 per cent premium margin for claims.

E. Each case has an initial premium for claims of $65,000, with subsequent premiums based on the prior year's experience, plus a 5 per cent premium margin for claims, plus 20 per cent of any prior deficit. Each case also has an over-all stop-loss pooling level of $100,000, an individual claim pooling level of \$30,000, a maximum contingency reserve of \$20,000 with a \$5,000 maximum annual increase, and cancellation of the case if the year-end deficit exceeds \$75,000. This situation differs from Situation D by the restoration of the 5 per cent premium margin for claims.

F. Each case has an initial premium for claims of \$65,000, with subsequent premiums based on the average claims since inception, plus a 5 per cent premium margin for claims, plus 20 per cent of any prior deficit. Each case also has an over-all stop-loss pooling level of \$100,000, an individual claim pooling level of \$30,000, a maximum contingency reserve of \$20,000 with a \$5,000 maximum annual increase, and cancellation of the case if the year-end deficit exceeds \$75,000. This situation differs from Situation E by the use of an n-year prospective experience-rating formula rather than a one-year formula.

The premiums for the stop-loss pools are the same as for no-cancellation situations. Expected claims for each situation, after adjustment for stop-loss pools, are \$56,674.96.

Chart II summarizes the conditions of the six situations. Table 5 presents the year-end total underwriting deficit for active cases. Table 6

CHART II

SUMMARY OF CONDITIONS

SITUATIONS A–F

Condition	A	B	C	D	E	F
\$65,000 premium for claims	X					
\$100,000 over-all stop-loss	X	X	X	X	X	X
\$30,000 individual stop-loss	X	X	X	X	X	X
\$20,000 contingency reserve	X	X	X	X	X	X
\$75,000 cancellation point	X	X	X	X	X	X
One-year prospective experience rating		X	X	X	X	
n-year prospective experience rating						X
5% premium margin for claims			X		X	X
20% deficit recovery				X	X	X
Premium formula (P)	\$65,000	C_{T-1}	$1.05C_{T-1}$	$C_{T-1}+0.2D_{T-1}$	$1.05C_{T-1}+0.2D_{T-1}$	$1.05\Sigma_1^{T-1}C_i/(T-1)+0.2D_{T-1}$

NOTE.—C = Claims; D = Prior deficit.

presents the cumulative deficit on those cases that cancel during the fifty-year period. Figure 4 presents the total of the active case and cumulative canceled case deficits.

Cancellation of cases in a deficit position becomes an important factor in these six situations. In each situation, although the active case deficit is smaller than when no cancellations are assumed, large amounts of the

TABLE 5

CANCELLATION SITUATIONS

Active Case Underwriting Deficits

Year	Situation A	Situation B	Situation C	Situation D	Situation E	Situation F
1	$ 800,000	$ 800,000	$ 800,000	$ 800,000	$ 800,000	$ 800,000
2	1,175,000	1,685,000	1,563,001	1,635,000	1,524,500	1,524,500
3	1,295,000	2,135,000	1,954,000	2,032,600	1,866,450	1,453,976
4	1,495,000	2,595,000	2,287,251	2,202,800	2,038,481	1,327,862
5	1,300,000	2,100,000	1,698,503	1,590,480	1,458,918	1,038,293
6	1,165,000	2,355,000	1,963,253	1,888,040	1,731,632	1,188,580
7	1,330,000	2,205,000	1,834,503	1,854,009	1,590,125	1,218,212
8	1,255,000	2,515,000	2,042,253	2,025,901	1,711,257	1,343,255
9	1,240,000	2,430,000	1,866,254	1,690,194	1,377,765	1,264,311
10	945,000	2,165,000	1,539,254	1,632,234	1,358,357	1,070,972
11	1,285,000	2,160,000	1,639,004	1,973,521	1,726,549	1,322,247
12	1,230,000	1,855,000	1,566,003	1,530,926	1,361,689	1,080,960
13	1,015,000	1,620,000	1,276,754	1,303,859	1,152,855	887,796
14	705,000	1,540,000	1,154,006	1,385,127	1,199,117	652,895
15	495,000	1,830,000	1,370,756	1,605,669	1,490,642	806,934
16	685,000	1,550,000	1,337,255	1,632,696	1,429,822	944,238
17	980,000	1,450,000	1,584,755	1,905,823	1,718,990	1,069,242
18	830,000	1,245,000	1,229,755	1,549,919	1,366,403	884,618
19	620,000	1,195,000	1,118,256	1,224,733	1,106,882	821,933
20	675,000	1,165,000	1,216,506	1,449,791	1,228,256	795,098
25	830,000	765,000	1,095,256	1,426,264	1,245,358	935,067
30	520,000	330,000	814,754	1,019,794	988,001	507,259
35	745,000	480,000	819,254	1,305,394	1,277,727	601,403
40	780,000	385,000	631,002	1,091,168	1,021,917	625,862
45	585,000	305,000	528,002	913,336	902,413	455,861
50	725,000	285,000	467,502	712,621	576,170	445,051

insurer's general funds are invested in deficits on canceled cases. Unlike the no-cancellation situations shown in Figure 3, the total deficit of the cancellation situations continues to rise over the fifty-year period as a result of accumulated canceled case deficits. The total active and canceled deficit in each of Situations A–F is larger than all except the Situation I deficit in Situations I–VII (Fig. 3).

Table 7 summarizes the financial data for each of the six situations. As with the no-cancellation situations, each of these situations has a fifty-year underwriting loss.

Deficit Management Techniques

In each of the thirteen situations presented, with the sole exception of Situation VII, where the stop-loss level equals the premium for claims, the portfolio of group life cases has a total underwriting deficit in each of the fifty years simulated by the computer program. These simulations point up the fact that combination experience-rating plans will always result in a total portfolio deficit which will require an investment of the insurer's general fund to pay the unrecovered claims. Various strategies can be used to reduce the size of the investment, but only one, which corresponds to the actuaries' original understanding of retrospective experience-rating plans, eliminates the investment.

TABLE 6

CANCELLATION SITUATIONS

Canceled Case Cumulative Deficits

Year	Situation A	Situation B	Situation C	Situation D	Situation E	Situation F
1	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0
2	0	180,000	179,500	180,000	179,500	179,500
3	85,000	345,000	338,750	259,000	256,750	694,875
4	165,000	435,000	424,500	337,800	256,750	1,214,545
5	165,000	605,000	587,000	415,800	256,750	1,476,000
6	420,000	1,195,000	1,075,250	746,800	505,250	1,780,816
7	420,000	1,440,000	1,230,750	746,800	505,250	2,016,856
8	690,000	1,705,000	1,410,500	841,800	600,250	2,016,856
9	855,000	1,865,000	1,569,250	1,001,800	759,000	2,183,982
10	940,000	2,215,000	1,889,000	1,161,600	915,080	2,259,837
11	1,040,000	2,905,000	2,466,250	1,496,600	1,248,080	2,336,765
12	1,140,000	3,510,000	2,815,500	1,763,200	1,511,910	2,793,169
13	1,140,000	3,590,000	2,815,500	1,763,200	1,511,910	2,877,507
14	1,500,000	3,845,000	2,891,250	1,763,200	1,592,010	3,126,544
15	1,955,000	4,110,000	3,137,000	1,840,800	1,592,010	3,126,544
16	1,955,000	4,450,000	3,468,500	2,015,800	1,766,260	3,126,544
17	2,035,000	5,110,000	3,705,000	2,172,400	1,845,510	3,453,432
18	2,295,000	5,190,000	3,859,000	2,172,400	1,845,510	3,578,482
19	2,380,000	5,355,000	4,030,250	2,340,400	2,009,910	3,578,482
20	2,380,000	5,515,000	4,030,250	2,505,400	2,173,910	3,578,482
25	3,205,000	6,360,000	4,435,250	3,022,680	2,608,510	4,246,717
30	3,635,000	7,210,000	4,983,500	3,435,080	3,089,100	4,582,999
35	3,880,000	7,380,000	5,407,750	3,768,080	3,263,600	4,918,906
40	4,060,000	7,470,000	5,826,250	4,195,280	3,764,780	5,245,888
45	4,635,000	7,730,000	6,316,750	4,857,280	4,264,420	5,594,326
50	4,910,000	7,820,000	6,496,000	5,112,280	4,612,920	5,674,682
Number of cases canceling over the fifty-year period	55	92	79	61	55	66

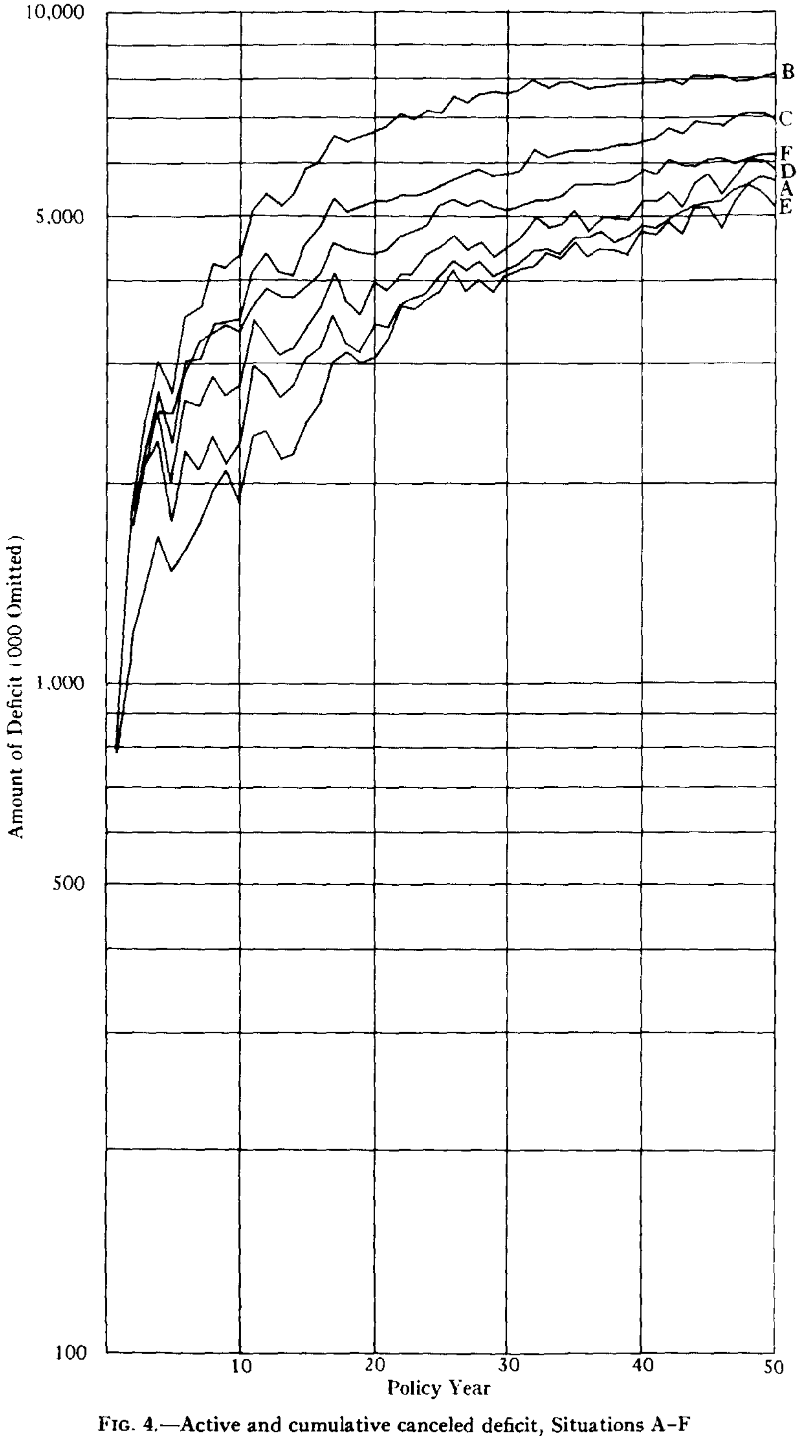

FIG. 4.—Active and cumulative canceled deficit, Situations A–F

TABLE 7

FIFTY-YEAR SUMMARY OF OPERATIONS

Cancellation Situations

	Situation A	Situation B	Situation C	Situation D	Situation E	Situation F
1. Premiums collected	226,720,000	107,105,000	151,132,112	202,056,800	222,178,300	178,200,700
2. Claims incurred	196,060,000	110,060,000	146,625,000	190,705,000	201,860,000	160,490,000
3. Experience refunds paid	36,035,000	5,145,000	11,434,114	17,098,949	25,357,860	23,645,094
4. No. of experience refunds paid	1,449	264	627	894	1,214	1,098
5. Contingency reserves held*	260,000	5,000	36,500	77,752	149,530	185,339
6. No. of contingency reserves held*	23	2	7	13	23	19
7. Active case deficits*	725,000	285,000	467,502	712,621	576,170	445,051
8. No. of active case deficits*	22	6	14	26	22	15
9. Canceled case deficits	4,910,000	7,820,000	6,496,000	5,112,280	4,612,920	5,674,682
10. No. of canceled case deficits	55	92	79	61	55	66
11. Fifty-year underwriting gain (loss)	(5,635,000)	(8,105,000)	(6,963,502)	(5,824,901)	(5,189,090)	(6,119,733)
12. Individual stop-loss premiums†	15,713,440	8,721,680	11,672,455	15,213,385	16,118,890	12,771,675
13. Individual stop-loss claims†	15,630,000	8,940,000	11,760,000	15,350,000	16,210,000	12,860,000
14. Over-all stop-loss premiums†	8,502,279	4,719,155	6,315,770	8,231,708	8,721,661	6,910,539
15. Over-all stop-loss claims†	8,840,000	5,165,000	6,560,000	8,220,000	8,715,000	7,490,000
16. Fifty-year stop-loss gain (loss)†	(254,281)	(664,165)	(331,775)	(124,907)	(84,449)	(667,786)
17. No. of years of statutory gains	10	14	15	18	19	15

* At the end of the fifty-year period.

† Stop-loss experience is not included in lines other than those marked.

Techniques for reducing the insurer's investment may be called "deficit management techniques." The effect of various deficit management techniques can be observed from analysis of the data of Situations I–VII and A–F. The following strategies reduce the total portfolio deficit:

The use of an over-all stop-loss pool: The lower the stop-loss pool limit, the lower the total deficit will be (Situation II).

The use of an individual claim stop-loss pool: Again, the lower the stop-loss pool limit per claim, the lower the total deficit will be (Situation III).

Requiring contingency reserves: The higher the contingency reserves held, and the faster they are accumulated, the lower total future deficits will be (Situation IV).

Providing for a premium margin over expected claims: The larger the premium margin for claims, the smaller the total deficit will be (Situation V).

Receiving an additional premium to recover a prior deficit: The larger the retroactive premium adjustment, the smaller the total deficit will be (compare Situation B with Situation D).

Each of the deficit management techniques described above is currently used by insurers to limit underwriting losses.

Deficit management techniques can be classified in two categories. First are those techniques which increase the total dollars available to pay claims. Included in this category are a premium margin for claims, contingency reserves, and a recovery of prior deficits through a retroactive premium adjustment. The second category includes those techniques which eliminate claims from the chargeable experience of the case. In this category are the various pooling arrangements offered at an additional premium to the policyholder. Pooling arrangements involve an implicit premium increase, since the risk to the insurer is reduced but the premium for the remaining experience-rated portion of the case is usually not reduced. An implicit premium increase results in an increase in the premium margin for claims which ultimately is returned to the policyholder in the form of higher experience refunds or lower experience premiums.

IV. SOURCES OF STATUTORY PROFIT

A group portfolio requiring an increasing investment of the insurer's funds would not be expected to produce a gain to the insurer. The incidence of profit, however, is defined by the accounting method employed. As demonstrated in this section, the statutory accounting method used to report profit for experience-rated group life and health cases is less than satisfactory in conveying accurately the nature of the business. Quite often a one-year statutory gain will emerge when no real financial gain has been realized by the insurer. The lack of accurate accounting statements is an impediment to the proper understanding of experience-rating plans.

Mathematical Analysis of the Sources of Statutory Profit on Experience-rated Groups

Assume that the policy year coincides with the calendar year. Then let

P = Gross premiums;
C^E = Expected paid claims;
C^A = Actual paid claims;
E^E = Expected expenses from retention formula (includes profit, risk charge, expenses, commissions, and premium taxes);
E^A = Actual expenses from retention formula (includes profit, risk charge, expenses, commissions, and premium taxes);
E^S = Statement expenses charged to experience-rated group business (includes expenses, commissions, and premium taxes, and *excludes* risk and profit charges);
ΔV^C = Increase in claim reserves on each case;
ΔV^S = Increase in statutory claim reserves;
ΔCR^I = Increase in contingency reserves from a decrease in experience refunds (always a positive number);
ΔCR^D = Decrease in contingency reserves to reduce a current-year deficit (always a negative number);
PR = Retroactive premium adjustments to recover a prior deficit (for accounting purposes, this charge can be assumed to be paid either at the end of the year in which the deficit occurs or at the beginning of the next policy year);
DR = Recovery of deficits by reducing paid experience refunds;
PM = Premium margin for claim fluctuation;
R = Paid experience refunds;
I = Investment income plus capital gains and losses;
FIT = Federal income tax charged to the group experience-rating line of business regardless of the insurer's tax phase;
G = Operating gain or loss from experience-rated business.

There are four basic accounting situations for group cases: surplus-surplus, surplus-deficit, deficit-surplus, and deficit-deficit. A description of each of the cases and the type of action taken by insurers to produce a statutory gain or limit a statutory loss for each case follows:

Surplus-surplus: The cases had no deficit at year end last year and incur a surplus in this year. Experience refunds are paid, and contingency reserves are accumulated by reducing the paid experience refunds.

Surplus-deficit: The cases had no deficit at year end last year and incur a deficit in the current year. No experience refunds are paid, and any existing contingency reserves will be decreased to cover the deficits. Retroactive premiums are collected at the end of the policy year in which the deficit occurs.

Deficit-surplus: The cases were in a deficit position last year and incur a surplus in the current year. The surpluses will be used first to recover the prior years' deficit, then to build contingency reserves, and, if any surplus remains, to pay experience refunds.

Deficit-deficit: The cases were in a deficit position last year and incur a deficit in the current year. No experience refunds are paid, no contingency reserves remain, and retroactive premiums are collected at the end of the policy year in which the deficit occurs.

These four cases cover all accounting possibilities, but the manner in which contingency reserves and retroactive premiums are used may differ from that described. Changes in the use of contingency reserves and retroactive premiums will not alter the sources of profit.

Statutory profit for the group line of business is calculated using the annual statement format for insurance lines of business, with one notable change. Earned premiums become gross premiums less earned experience refunds. Earned experience refunds include paid experience refunds, a reserve for earned and unpaid experience refunds, and the change in contingency reserves held. For the mathematical demonstration below it is assumed that there are no earned and unpaid experience refunds, since the calendar year and the policy year coincide. The statutory statement format is as follows:

Gross premiums		$xxx
Less: Earned experience refunds	$xxx	
Earned premiums		$xxx
Less:		
Incurred claims	$xxx	
Commissions	xxx	
Premium taxes	xxx	
Expenses	xxx	
Underwriting gain or loss		$xxx
Interest income		xxx
Less: Federal income tax	$xxx	
Operating gain or loss		$xxx

Now, for each of the four basic accounting cases, using the above statutory profit format and the above notations, we have the formulas shown in the tabulation on page 152. Substituting for gross premiums (P)

	Surplus-Surplus	Surplus-Deficit	Deficit-Surplus	Deficit-Deficit
Gross premiums	$C^E+E^E+PM+\Delta V^C$	$C^E+E^E+PM+\Delta V^C+PR$	$C^E+E^E+PM+\Delta V^C$	$C^E+E^E+PM+\Delta V^C+PR$
Paid experience refunds	$P-C^A-E^A-\Delta V^C-\Delta CR^I=(C^E-C^A)+(E^E-E^A)+PM-\Delta CR$	0	$P-C^A-E^A-\Delta V^C-\Delta CR^I-DR=(C^E-C^A)+(E^E-E^A)+PM-\Delta CR^I-DR$	0
Increase in contingency reserve	ΔCR^I	ΔCR^D	ΔCR^I	....
Earned premiums	$P-R-\Delta CR^I$	$P-\Delta CR^D$	$P-R-\Delta CR^I$	P
Incurred claims	$C^A+\Delta V^S$	$C^A+\Delta V^S$	$C^A+\Delta V^S$	$C^A+\Delta V^S$
Commissions, premium taxes, and expenses	E^S	E^S	E^S	E^S
Underwriting gain or loss	$P-R-\Delta CR^I-C^A-\Delta V^S-E^S$	$P-\Delta CR^D-C^A-\Delta V^S-E^S$	$P-R-\Delta CR^I-C^A-\Delta V^S-E^S$	$P-C^A-\Delta V^S-E^S$
Investment income	I	I	I	I
Federal income tax	FIT	FIT	FIT	FIT
Operating gain or loss	$P-R-\Delta CR^I-C^A-\Delta V^S-E^S+I-FIT$	$P-\Delta CR^D-C^A-\Delta V^S-E^S+I-FIT$	$P-R-\Delta CR^I-C^A-\Delta V^S-E^S+I-FIT$	$P-C^A-\Delta V^S-E^S+I-FIT$

and experience refunds (R) in the operating gain or loss for each case, the following sources of profit are derived:

Surplus-Surplus:

$$G_{SS}=(\Delta V^C-\Delta V^S)+(E^A-E^S)+(I-FIT)\,.$$

The sources of statutory gain from surplus-surplus cases are limited. The excess of premium charges for claim reserves on a case over the increase in statutory claim reserves results in a gain. Overreserving is common, since premium charges for reserve usually are calculated on the experience of each case which, because of smaller exposure, has a higher claim variance than the total portfolio experience used to calculate statement reserves. The excess of actual expenses charged the cases over statutory expenses includes two sources of profit. First, each case's expense charge includes a loading for risk and profit which is not a statement expense item. Second, expenses actually charged the cases usually will not coincide exactly with the statement expenses, resulting in either a gain or a loss. Investment income is allocated to the group line to the extent that income is earned on claim reserves and contingency reserves. Federal income taxes may be allocated to the group line by various techniques. Of these sources of profit from surplus-surplus cases, the largest are gains from the risk and profit charge and investment income.

Deficit-Surplus:

$$G_{DS} = (\Delta V^C - \Delta V^S) + (E^A - E^S) + DR + (I - FIT) .$$

The sources of statutory gain from deficit-surplus cases differ from those from surplus-surplus cases by the additional gain from deficit recovery. Deficit recovery is the major source of gain from deficit-surplus cases.

Surplus-Deficit:

$$G_{SD} = (C^E - C^A) + (E^E - E^S) + (\Delta V^C - \Delta V^S) + (I - FIT) + PM - \Delta CR^D + PR .$$

The source of statutory loss on surplus-deficit cases is the adverse claim fluctuation reduced by the profit from other sources. The claim deficit may be recovered in part or in total through the risk and profit charge included in initial premiums, expenses charged the case that exceed statement expenses, overreserving, and investment income less federal income taxes. In addition to these sources of loss recovery, which are also sources of gain on surplus cases, deficit management techniques provide other sources. The premium margin for claims reduces the claim loss. A reduction in an existing contingency reserve to cover remaining losses results in a further loss reduction. Also, any retroactive premiums that the insurer may collect reduce the claim loss. The premium margin, decrease in the contingency reserves, and retroactive premium payment will, in total, not exceed the adverse claim fluctuation plus the recovery of any expenses $(E^E - E^A)$ not charged in the initial premium. If all claim deficits and expenses are recovered, the profit from surplus-deficit cases will be the same as the profit from surplus-surplus cases.

Deficit-Deficit:

$$G_{DD} = (C^E - C^A) + (E^E - E^S) + (\Delta V^C - \Delta V^S) + (I - FIT) + PM + PR .$$

The sources of statutory loss from deficit-deficit cases differ from those from surplus-deficit cases by not having a loss recovery through a reduction of existing contingency reserves. This case assumed that no contingency reserves remained at the beginning of the year.

Total profit is the sum of the profit on the four specific accounting cases. Sources of profit common to each of the four cases include the following:

$(\Delta V^C - \Delta V^S)$, change in the amount of overreserving;

$(\Delta E^A - \Delta E^S)$, inaccurate expense charges plus the risk and profit charge (note that $(E^E - E^S) = (E^A - E^S) - (E^A - E^E)$ for surplus-deficit and deficit-deficit cases);

$(I - FIT)$, investment income less federal income taxes.

Sources affecting profit from a specific accounting case or cases include (subscripts refer to the specific case or cases from which the profit arises):

$(E^A - E^E)_{SD,DD}$, loss of premium expense dollars caused by the inability to recover higher expenses at year end on deficit cases;

DR_{DS}, recovery of a prior-year deficit through a reduction of paid experience refunds;

$(C^E - C^A)_{SD,DD}$, claim deficits from cases with a current-year deficit;

$PM_{SD,DD}$, premium margins for claims which reduce the current-year claim deficit;

ΔCR^D_{SD}, reduction in contingency reserves which reduce the current-year claim deficit;

$PR_{SD,DD}$, retroactive premium adjustments at year end which reduce the current-year claim deficit.

With the sole exception of $(E^A - E^E)$, which refers to recoverable expenses, the sources of profit not common to each of the four accounting cases refer to the size of the total portfolio underwriting deficit. Let ΔD be the change in the total portfolio underwriting deficit over the calendar year. The value of ΔD will be positive when the deficit decreases and negative when the deficit increases (becomes a larger negative). Then

$$\Delta D = (C^E - C^A)_{SD,DD} + DR_{DS} + PM_{SD,DD} - \Delta CR^D_{SD} + PR_{SD,DD}.$$

Since DR_{DS}, $PM_{SD,DD}$, $-\Delta CR_{SD}$, and $PR_{SD,DD}$ are always positive and $(C^E - C^A)_{SD,DD}$ is always negative, ΔD can be either positive or negative in a given policy year, depending on whether claim experience is generally favorable (that is, less deficits, more surpluses) or unfavorable (that is, less surpluses, more deficits). The gain or loss from ΔD, the change in total portfolio underwriting deficit, is a major source of statutory profit in any given year.

The value of ΔD is the net change in the portfolio underwriting deficit from the beginning to the end of the accounting period. The deficit at the end of the period is composed of deficits on active cases and deficits on cases that canceled during the accounting period. Symbolically, the net change in the deficit, ΔD, over the accounting period from time T to time $T + 1$ is

$$\Delta D = (D^A_{T+1} + D^C_{T+1}) - D^A_T ,$$

where D_T^A is the active case deficit at the beginning of the accounting period, D_{T+1}^A is the active case deficit at the end of the period, and D_{T+1}^C is the deficit on cases canceling during the accounting period. Note that a portion, or even all, of D_{T+1}^C may be included in D_T^A, since the case must have been active at the beginning of the period to contribute toward ΔD. Rearranging terms,

$$\Delta D = (D_{T+1}^A - D_T^A) + D_{T+1}^C .$$

The net change in the total portfolio underwriting deficit during an accounting period is equal to the net change in active case deficits over the period plus the total deficit on cases canceling during the period. In succeeding accounting periods D_{T+1}^C will be constant and therefore will not contribute to ΔD in those years. For this reason, deficits on canceled cases have no impact on future profit to the insurer other than to prevent the possibility of a deficit-surplus gain. An important financial indicator is not adequately reported through the statutory accounting statement.

The operation of ΔD causes statutory underwriting gains and losses. Total operating profit for the experience-rated line of business includes not only statutory underwriting profit but also profit from overreserving, expense inaccuracy, risk and profit charges, and net investment income. Total operating profit for the group line·of business on the statutory annual statement includes operating profit from the experience-rated line of business, insurance profit from stop-loss pools and other non-experienced-rated group lines of business. Depending on the mix between experience-rated business and non–experience-rated or pooled business, an insurer may have total operating profit for the group line of business in a given year dominated by ΔD. Such a profit will not represent the true financial condition of the insurer's portfolio.

Profitability of the Group Portfolio

To explore further the consequences of the statutory accounting method, an analysis of the annual statutory underwriting profit of Situation E was made. This analysis does not include any profit from expenses, risk and profit charges, overreserving, or investment income, but solely the underwriting profit emerging from the change in active and canceled case deficits (ΔD). As with the other experience-rating plans presented above, Situation E develops a statutory underwriting loss over the fifty-year experience period. This section will show that, despite the long-term underwriting loss, annual statutory underwriting profit is often generated.

Table 8 presents the annual statement as it would appear to an insurer using the Situation E experience-rating plan. As a check on the mathe-

TABLE 8

PROFIT ANALYSIS OF SITUATION E

YEAR	STATUTORY CALCULATION OF UNDERWRITING PROFIT					ALTERNATIVE CALCULATION OF UNDERWRITING PROFIT			PROFITABILITY OF STOP-LOSS INSURANCE		
	Premiums (1)	Claims (2)	Experience Refunds (3)	Δ Contingency Reserves (4)	Underwriting Gain (Loss) (5)	Canceled Case Deficit (6)	Δ Active Case Deficit (7)	Underwriting Gain (Loss) (8)	$7,289.71/ Case, 5% Margin Stop-Loss Premium (9)	Stop-Loss Claims (10)	Stop-Loss Gain (Loss) (11)
1	$6,500,000	$5,500,000	$1,510,000	$290,000	$ (800,000)	$ 0	$800,000	$ (800,000)	$728,971	$620,000	$ 108,971
2	5,934,918	5,680,000	1,169,669	(10,751)	(904,000)	179,500	724,500	(904,000)	728,971	630,000	98,971
3	6,058,820	5,495,000	968,220	14,800	(419,200)	77,250	341,950	(419,200)	714,392	795,000	(80,608)
4	6,048,458	5,775,000	516,789	(71,300)	(172,031)	0	172,031	(172,031)	707,102	795,000	(87,898)
5	6,471,365	5,080,000	741,690	70,112	579,563	0	(579,563)	579,563	707,102	640,000	67,102
6	5,625,708	5,705,000	508,387	(66,465)	(521,214)	248,500	272,714	(521,214)	707,102	640,000	67,102
7	6,021,503	5,260,000	618,642	1,354	141,507	0	(141,507)	141,507	685,232	565,000	120,232
8	5,840,953	5,655,000	416,022	(14,137)	(216,132)	95,000	121,132	(216,132)	685,232	655,000	30,232
9	6,174,929	5,400,000	578,357	21,830	174,742	158,750	(333,492)	174,742	677,943	760,000	(82,057)
10	5,735,484	4,890,000	935,193	46,963	(136,672)	156,080	(19,408)	(136,672)	663,364	850,000	(186,636)
11	5,196,106	5,485,000	498,935	(86,637)	(701,192)	333,000	368,192	(701,192)	648,784	785,000	(136,216)
12	5,689,748	5,015,000	534,420	39,298	101,030	263,830	(364,860)	101,030	619,625	605,000	14,625
13	5,223,024	4,160,000	834,666	19,524	208,834	0	(208,834)	208,834	597,756	380,000	217,756
14	4,598,512	4,195,000	549,288	(19,414)	(126,362)	80,100	46,262	(126,362)	597,756	295,000	302,756
15	4,539,509	4,600,000	279,932	(48,898)	(291,525)	0	291,525	(291,525)	590,467	590,000	467
16	5,128,067	4,645,000	585,902	10,595	(113,430)	174,250	(60,820)	(113,430)	590,467	595,000	(4,533)
17	4,953,154	5,050,000	305,192	(33,620)	(368,418)	79,250	289,168	(368,418)	575,887	710,000	(134,113)
18	5,541,241	4,465,000	680,408	43,246	352,587	0	(352,587)	352,587	568,597	755,000	(186,403)
19	4,961,472	4,300,000	548,637	17,714	95,121	164,400	(259,521)	95,121	568,597	440,000	128,597
20	4,526,321	4,345,000	478,361	(11,666)	(285,374)	164,000	121,374	(285,374)	554,018	605,000	(50,982)
21	4,603,096	3,995,000	568,430	(832)	40,498	0	(40,498)	40,498	539,439	405,000	134,439
22	4,432,247	4,230,000	474,182	(24,157)	(247,778)	0	247,778	(247,778)	539,439	575,000	(35,561)
23	4,728,551	4,135,000	598,267	(9,185)	4,469	177,800	(182,269)	4,469	539,439	645,000	(105,561)
24	4,382,352	4,145,000	370,301	(30,365)	(102,584)	90,000	12,584	(102,584)	524,859	455,000	69,859
25	4,500,367	4,210,000	430,804	5,870	(146,307)	166,800	(20,493)	(146,307)	517,569	660,000	(142,431)

TABLE 8—*Continued*

Year	Statutory Calculation of Underwriting Profit					Alternative Calculation of Underwriting Profit			Profitability of Stop-Loss Insurance		
	Premiums (1)	Claims (2)	Experience Refunds (3)	Δ Contingency Reserves (4)	Underwriting Gain (Loss) (5)	Canceled Case Deficit (6)	Δ Active Case Deficit (7)	Underwriting Gain (Loss) (8)	$7,289.71/ Case, 5% Margin Stop-Loss Premium (9)	Stop-Loss Claims (10)	Stop-Loss Gain (Loss) (11)
26	$4,470,023	$4,175,000	$555,048	$ 16,740	$ (276,765)	$169,850	$ 106,915	$ (276,765)	$502,990	$700,000	$ (197,010)
27	4,449,405	3,725,000	444,112	(17,692)	297,985	156,190	(454,175)	297,985	488,411	480,000	8,411
28	3,880,825	3,540,000	483,285	22,816	(165,276)	0	165,276	(165,276)	473,831	435,000	38,831
29	3,929,629	3,535,000	306,534	(56,933)	145,028	154,550	(299,578)	145,028	473,831	250,000	223,831
30	3,654,465	3,280,000	522,790	75,880	(224,205)	0	224,205	(224,205)	459,252	295,000	164,252
31	3,641,552	3,495,000	315,970	(43,540)	(125,878)	0	125,878	(125,878)	459,252	355,000	104,252
32	3,892,483	3,830,000	277,001	(350)	(214,168)	84,500	129,668	(214,168)	459,252	630,000	(170,748)
33	4,165,164	3,645,000	450,980	37,750	31,434	0	(31,434)	31,434	451,962	545,000	(93,038)
34	4,069,632	3,440,000	576,977	(29,576)	82,231	0	(82,231)	82,231	451,962	450,000	1,962
35	3,837,932	3,800,000	309,851	(34,074)	(237,845)	90,000	147,845	(237,845)	451,962	415,000	36,962
36	4,140,499	3,380,000	497,785	47,418	215,296	80,490	(295,786)	215,296	444,672	380,000	64,672
37	3,640,344	3,450,000	312,685	(25,419)	(96,922)	88,500	8,422	(96,922)	437,383	410,000	27,383
38	3,715,528	3,200,000	583,700	(21,636)	(46,536)	0	46,536	(46,536)	430,093	320,000	110,093
39	3,567,339	3,095,000	359,717	25,989	86,633	169,500	(256,133)	86,633	430,093	335,000	95,093
40	3,206,368	3,175,000	416,403	18,806	(403,841)	162,690	241,151	(403,841)	415,513	465,000	(49,487)
41	3,333,345	2,920,000	344,591	(18,489)	87,243	0	(87,243)	87,243	400,934	400,000	934
42	3,252,896	3,075,000	416,910	(24,666)	(214,348)	80,940	133,408	(214,348)	400,934	250,000	150,934
43	3,342,584	2,795,000	340,327	(2,179)	209,436	170,100	(379,536)	209,436	393,644	245,000	148,644
44	2,862,424	3,055,000	250,931	(17,873)	(425,634)	88,500	337,134	(425,634)	379,065	410,000	(30,935)
45	3,307,854	3,020,000	313,737	10,950	(36,833)	160,100	(123,267)	(36,833)	371,775	435,000	(63,225)
46	3,146,703	2,215,000	456,567	53,140	421,996	0	(421,996)	421,996	357,196	80,000	277,196
47	2,421,808	2,625,000	318,946	(12,000)	(510,138)	190,000	320,138	(510,138)	357,196	350,000	7,196
48	2,706,333	2,845,000	206,968	(50,941)	(294,694)	79,250	215,444	(294,694)	342,616	265,000	77,616
49	3,085,418	2,725,000	264,280	(15,500)	111,638	0	(111,638)	111,638	335,327	265,000	70,327
50	3,042,090	2,405,000	331,119	57,030	248,941	79,250	(328,191)	248,941	335,327	210,000	125,327
Total					$(5,189,090)			$(5,189,090)			$1,157,579

matical proof of the sources of profit, the annual profit is calculated two ways. First, the statutory calculation is made (premium less claims less earned experience refunds). Second, the deficit management method is used (net change in the active and the canceled deficit). A comparison of columns 5 and 8 of Table 8 reveals, as expected, that the two methods of calculating profit are identical. Each calculation results in twenty years of statutory underwriting gain over the fifty-year experience period.

Underwriting profits account for only a portion of the total statutory operating gain or loss. Stop-loss pools are also a source of statutory profit. Column 11 of Table 8 shows the stop-loss profit, assuming that a 5 per cent margin for claims is added to the net stop-loss premium of $6,942.58. Combining the stop-loss profit with the underwriting profit, the number of years which result in a statutory gain remains at twenty. Other sources of profit discussed above, such as interest income and the risk and profit charge, will result in additional years of statutory operating profit.

To this point, each set of data studied has had identical claim data. To demonstrate that the results have not been spurious, Situation E was rerun with four new sets of simulated claim data (E1, E2, E3, E4) drawn from the original probability distribution function. Tables 9–11 present the results of the new data.

Situation E resulted in twenty years of statutory underwriting gain, compared with nineteen, twenty-one, eighteen, and twenty-three years of underwriting gain under the new claim data (Table 9). Also, the number of years of combined underwriting and stop-loss profit (adding a 5 per cent stop-loss premium margin for claims) are comparable, that is, twenty, twenty-four, twenty-one, twenty-three, and twenty-three years of statutory gain (Table 10). Also note that in each new situation there is an over-all underwriting deficit over the fifty-year period, even though there are years of substantial underwriting profit.

Statutory underwriting profit is a result of favorable fluctuations in the total portfolio deficit. As Figures 3 and 4 have illustrated, the active case deficit reaches an ultimate level characteristic of the experience-rating plan and then fluctuates randomly. Random fluctuations can result in statutory gains. Since favorable fluctuations occur more often in later policy years when there is no upward bias in the total deficit, statutory gains would be expected to occur more often in later years. Tables 11 and 12 verify this expectation for Situations E–E4. Profit occurs more frequently after the first five policy years. The same tables also fail to reveal any significant pattern in the size of a gain by duration. The random nature of gains explains the lack of a pattern.

Sample Situations E–E4 show an underwriting gain in 101 out of 250

TABLE 9

SUMMARY OF UNDERWRITING GAINS (LOSSES)
SITUATIONS E, E1, E2, E3, E4
Number of Gains (Losses) of Various Sizes

Size of Gain (Loss)	E	E1	E2	E3	E4
$(Over 1,000,000)		1			
$(1,000,000)-$ (900,000)	1				
(900,000)- (800,000)	1		1		1
(800,000)- (700,000)	1		2	1	
(700,000)- (600,000)		3		2	3
(600,000)- (500,000)	1	2	1	1	1
(500,000)- (400,000)	4	3	1	2	1
(400,000)- (300,000)	1	2	8	3	4
(300,000)- (200,000)	10	3	4	8	6
(200,000)- (100,000)	8	4	7	8	10
(100,000)- (0)	3	13	5	7	1
0- 100,000	7	7	4	7	9
100,000 - 200,000	6	6	6	5	8
200,000 - 300,000	4	1	8	2	3
300,000 - 400,000	2	3	2	1	1
400,000 - 500,000		2	1	1	
500,000 - 600,000	1			1	1
600,000 - 700,000				1	
700,000 - 800,000					1
800,000 - 900,000					
900,000 - 1,000,000					
Over $1,000,000					
Total	50	50	50	50	50
Number of gains	20	19	21	18	23
Number of losses	30	31	29	32	27
Average size gain	$ 181,811	$ 181,133	$ 196,860	$ 191,794	$ 177,125
Average size loss	$ (294,177)	$ (254,696)	$ (285,921)	$ (255,796)	$ (298,859)
Number of cases canceling	55	42	38	51	38
Cumulative canceled deficit	$ 4,612,920	$ 3,389,525	$ 3,097,190	$ 4,152,216	$ 3,097,418
Total 50-year gain (loss)	$(5,189,090)	$(4,456,596)	$(4,160,433)	$(4,765,794)	$(4,077,239)

TABLE 10

SUMMARY OF COMBINED UNDERWRITING AND STOP-LOSS TOTAL GAINS (LOSSES)
SITUATIONS E, E1, E2, E3, E4

Number of Gains (Losses) of Various Sizes

Size of Gain (Loss)	E	E1	E2	E3	E4
$(Over 1,000,000)		1			
$(1,000,000)-$ (900,000)		1	1		
(900,000)- (800,000)	2				1
(800,000)- (700,000)		1	1	1	1
(700,000)- (600,000)	1	3	1	2	1
(600,000)- (500,000)	1	2	2	1	3
(500,000)- (400,000)	6	2	3	4	2
(400,000)- (300,000)	3	2	4	5	2
(300,000)- (200,000)	6	2	6	3	5
(200,000)- (100,000)	5	6	6	5	6
(100,000)- (0)	6	6	5	6	6
0 - 100,000	4	12	3	7	6
100,000 - 200,000	6	3	6	5	5
200,000 - 300,000	3	3	2	5	6
300,000 - 400,000	4	2	3	2	2
400,000 - 500,000	1	1	5	3	1
500,000 - 600,000		1	1		1
600,000 - 700,000	2	2	1		
700,000 - 800,000				1	
800,000 - 900,000					2
900,000 - 1,000,000					
Over $1,000,000					
Total	50	50	50	50	50
Number of gains	20	24	21	23	23
Number of losses	30	26	29	27	27
Total 50-year gain (loss)	$(4,031,511)	$(4,150,490)	$(3,057,605)	$(2,545,340)	$(1,608,666)
Stop-loss gain (loss) with 5% margin for claims	$ 1,157,579	$ 306,106	$ 1,102,828	$ 2,220,454	$ 2,468,573
Underwriting gain (loss)	$(5,189,090)	$(4,456,596)	$(4,160,433)	$(4,765,794)	$(4,077,239)

TABLE 11

DISTRIBUTION OF UNDERWRITING GAINS
BY POLICY YEAR OF OCCURRENCE
SITUATIONS E, E1, E2, E3, E4
Number of Gains

Policy Year	E	E1	E2	E3	E4	Total	Average Size Gain
1–5	1	1	0	0	1	3	$325,045
6–10	2	1	2	2	3	10	136,154
11–15	2	2	3	1	2	10	133,075
16–20	2	1	3	3	3	12	192,875
21–25	2	2	3	2	1	10	156,329
26–30	2	2	2	2	3	11	188,888
31–35	2	3	2	2	2	11	217,731
36–40	2	2	2	1	2	9	224,629
41–45	2	3	3	2	3	13	116,219
46–50	3	2	1	3	3	12	265,621
Total number of gains	20	19	21	18	23	101	....
Average size gain	$181,811	$181,133	$196,860	$191,794	$177,125	$185,525	$185,525

TABLE 12

DISTRIBUTION OF COMBINED UNDERWRITING AND STOP-LOSS GAINS
BY POLICY YEAR OF OCCURRENCE
SITUATIONS E, E1, E2, E3, E4
Number of Gains

Policy Year	E	E1	E2	E3	E4	Total
1–5	1	2	1	2	1	7
6–10	2	4	2	4	3	15
11–15	3	1	3	1	2	10
16–20	2	1	3	3	3	12
21–25	1	3	3	3	1	11
26–30	2	2	1	2	3	10
31–35	1	4	2	2	2	11
36–40	3	1	2	1	2	9
41–45	2	4	2	2	3	13
46–50	3	2	2	3	3	13
Total number of gains	20	24	21	23	23	111

accounting periods and a combined gain in 111 out of 250 accounting periods. Gains emerge even though, as an example, Situation E has a fifty-year underwriting deficit of $5,189,090, including $4,612,920 in canceled case deficits. When the insurer writing the sample portfolio includes a risk and profit charge, investment income, and non-experience-rated business in the total operating profit from the group line of business, a statutory gain will be reported to management most of the time. The statutory statement will report neither the investment of general funds in the portfolio nor the canceled case deficits. What will be reported is the utilization of deficit management techniques and random fluctuations in the active case deficits.

Statutory statements do not report adequately the financial health of the group line of business to management of the insurance company. A significant number of years of statutory gains can be expected from any combination experience-rating plan, since favorable fluctuations in the active case deficit occur even with plans resulting in very large deficits. Line 17 of Tables 4 and 7 reports the number of years of statutory underwriting gain for the original thirteen situations. As with the gains developed by Situations E1–E4, a statutory gain in any one year is not particularly meaningful.

V. RISK AND PROFIT CHARGE

Insurers theoretically recover losses from underwriting a combination experience-rating plan through the use of a risk and profit charge. An analysis of a suitable risk charge for Situation E reveals problems concerning the traditional approach to loss recovery. The risk portion of the charge is designed to compensate the insurer for the risk assumed in managing a group portfolio. There are two such risks: uncollectible canceled case deficits, and persistent active case deficits. The profit portion of the charge is designed to provide a fair return to the insurer for managing the portfolio. Since the insurer must invest general funds in active and canceled deficits, the profit charge should provide a fair rate of interest on the investment plus a fair return on administrative services provided to the policyholder. Table 13 presents, for Situation E, the charge by source of risk and profit as a percentage of the cumulative premium.

The risk charge to recover the cumulative canceled case deficit for Situation E results in a fairly stable charge as a percentage of premium. A charge equal to 2 per cent of premium would be sufficient. This portion of the risk charge must be made, since canceled case deficits are otherwise unrecoverable. In any given year a level risk charge cannot always be adequate to recover deficits on the cases actually canceling in that year.

TABLE 13

RISK CHARGE FOR SITUATION E

By Source of Risk

Year	Cumulative Premium	Cumulative Canceled Case Deficit	Canceled Deficit as % of Premium	Active Case Deficit	Active Deficit as % of Premium	Simple Interest on Active Deficit at 6%	Interest as % of Premium
5	$ 31,013,561	$ (256,750)	0.83%	$(1,458,918)	4.70%	$ 461,300	1.49%
10	60,412,138	(915,080)	1.52	(1,358,357)	2.25	927,445	1.54
15	85,659,037	(1,592,010)	1.86	(1,490,642)	1.74	1,343,294	1.57
20	110,769,292	(2,173,910)	1.96	(1,228,256)	1.11	1,754,314	1.58
25	133,415,905	(2,608,510)	1.96	(1,245,358)	0.93	2,137,579	1.60
30	153,800,252	(3,089,100)	2.01	(988,001)	0.64	2,441,509	1.59
35	173,407,015	(3,263,600)	1.88	(1,277,727)	0.74	2,800,138	1.61
40	191,677,093	(3,764,780)	1.96	(1,021,917)	0.53	3,088,851	1.61
45	207,776,196	(4,264,420)	2.05	(902,413)	0.43	3,366,015	1.62
50	222,178,300	(4,612,920)	2.08	(576,170)	0.26	3,592,664	1.62

Over an extended period, however, a level charge may be adequate to recover the insurer's losses as long as canceled case deficits are predictable.

Active case deficits do not lend themselves to a simple form of recovery of the insurer's losses. Situation E has a fairly stable active case deficit, fluctuating around $1,200,000. Because the active deficit is relatively level, the percentage of premium required to recover the deficit decreases with increasing duration. The insurer must either decide on the number of years over which recovery of the deficit is expected, or wait to recover the deficit when active cases cancel. The progression of the active deficit is predictable in situations where no cases cancel, but cancellation adds a large element of chance to any prediction.

If the insurer chooses to recover active case deficits by waiting for a case to cancel, a risk charge of 2 per cent of premium will be sufficient in Situation E. Addition of a charge for recovery of active case deficits will raise the charge to over 3 per cent of premium.

A proper profit charge depends on which option the insurer chooses for the risk charge. If only canceled case deficits are to be recovered, the profit charge should provide interest income on the insurer's investment in active case deficits. The actual rate of interest should be based on alternative investments of general funds over an extended time period. Long-term interest rates provide a sound base for the interest charge. The base must be inflated to recognize the risk inherent in predicting active case deficits. The 6 per cent interest rate used to calculate the profit charge for Situation E is less than most insurers would require. A profit charge of 1.60 per cent of premium is developed using the 6 per cent interest rate. The total charge for risk and profit requiring no active case deficit recovery is 3.60 per cent for Situation E over a twenty-year period.

A risk charge of over 3 per cent of premium to recover both active and canceled deficits would require a small profit charge for foregone interest on the outstanding active case deficit. A flat percentage of administration expenses should be added to the basic 3 per cent risk charge to provide some profit to the insurer over a period of time for the services provided to the policyholder (such as claim payment, underwriting, and the like). Thus, either method of ascertaining risk and profit results in a theoretical charge of at least $3\frac{1}{2}$ per cent of premium.

Other situations will require a higher charge than Situation E because of generally higher active and canceled case deficits and lower cumulative premiums. For comparison purposes, the components of a theoretical risk charge at the end of twenty years for Situations A–F would be as shown in the tabulation on page 165. The theoretical risk charge for each situa-

Situation	Simple Interest on Active Deficit	Canceled Case Deficit	Active Case Deficit
A	1.06%	2.06%	0.58%
B	2.73	6.86	1.45
C	1.99	4.45	1.34
D	1.90	2.41	1.39
E	1.58	1.96	1.11
F	1.36	3.77	0.84

tion will be well in excess of the charge insurers would normally make for a case as large as the sample case.

In practice it is not possible to predict accurately a risk charge, because of the unpredictable nature of case cancellation. A theoretical risk charge is based on the assumed predictability of canceled case deficits. Group insurance contracts do not limit the right of a policyholder to cancel the contract. Cancellation of a contract with an insurer often means a savings in future insurance costs to the policyholder. A reduction occurs whenever the policyholder has a cumulative deficit larger than the acquisition costs that would be incurred in rewriting the case with a new insurer. On a case which has incurred a deficit, the current insurer needs to charge future premiums adequate to avoid future deficits and to recover the prior deficit. If the policyholder writes a new insurance contract, the new insurer is concerned only with the adequacy of future premiums to pay future claims. By canceling the contract, the policyholder reduces future costs by the amount of the current deficit less new acquisition costs. The financial rationale for a cancellation grows with the size of the deficit. Since the right of a policyholder to cancel is not impeded by the group contract, selection against the company must be expected.

The predictability of the occurrence of an event which is encouraged by a contract is questionable. Insuring an event which can be controlled by the insured violates the basic principles of insurance. A risk charge does not provide a sound method of recovering active case and canceled case deficits unless the termination antiselection problem can be solved. In Situation E, the stable risk charge for case cancellation is the result of an arbitrary termination rule rather than any inherent stability in a theoretical charge.

VI. REVISION OF CURRENT EXPERIENCE-RATING PLANS

An experience-rating plan using both a prospective formula and a retrospective formula results in underwriting losses to the insurer over an extended period of time. Underwriting losses can be attributed to two primary sources: an unrecoverable loss occurring when cases cancel in a deficit position and a persistent deficit on active cases resulting from the experience-rating plan. Elimination of underwriting losses must be approached by eliminating the two sources of loss.

Cases cancel in a deficit position because the experience-rating plan produces deficits and there are no legal impediments to policy termination. Lack of legal impediments has evolved historically from competitive pressure which forced the insurers into financially unsound concessions to obtain and to maintain business. Canceled case deficits are the largest source of financial losses and can be completely eliminated by the addition of a termination clause to the group contract. The termination clause should make the deficit a legally collectible debt of the canceling policyholder. It should be noted that insured pension funds do exactly this through their universally accepted market-value termination clauses. The effect of a termination clause on statutory profits for Situations A–F can be determined by using Table 7 as shown below:

Situation	Fifty-Year Statutory Underwriting Loss (1)	Canceled Case Deficit (2)	Fifty-Year Statutory Underwriting Loss with Termination Clause (3) = (1) − (2)
A	($5,635,000)	($4,910,000)	($725,000)
B	(8,105,000)	(7,820,000)	(285,000)
C	(6,963,502)	(6,496,000)	(467,502)
D	(5,824,901)	(5,112,280)	(712,621)
E	(5,189,090)	(4,612,920)	(576,170)
F	(6,119,733)	(5,674,682)	(445,051)

A termination clause will reduce, but not eliminate, the long-term statutory underwriting loss.

Active case deficits can be eliminated through the use of a cost-plus contract requiring a full retroactive premium adjustment. At the end of any policy year in which a case incurs a deficit, the insurance contract could stipulate that the policyholder remit an additional premium for the prior policy year equal to the deficit. Retroactive premium adjustments are fairly common, although few contracts stipulate a full retroactive premium. Many such agreements are the result of a policyholder's understanding that an experience-rating plan requires that the policyholder ultimately pay all claims under the contract.

The cost-plus contract eliminates the small persistent active deficits, and the termination clause eliminates the large canceled case deficits. Both clauses combined will eliminate the financial problems of experience rating. However, the fifty-year underwriting profit will be equal to zero, this being the largest statutory underwriting profit possible under a retrospective experience-rating plan.

A presumption exists that as long as a case remains active under an experience-rating plan, the case ultimately will pay all claims under the contract from either future premiums or a reduction of future experience refunds. An employer who realizes this may find it advantageous to self-insure his life and health risk and purchase administrative services from an insurance company or an independent administrator. Insurers can pursue this business profitably by offering administrative services to self-insured cases coupled with stop-loss coverage to insure the plan against adverse claim fluctuations. An administrative services contract avoids both the active and the canceled deficit problems of experience rating and can produce a profit from both the administrative service charges and the stop-loss insurance.

Those cases that the insurer chooses to insure under an experience-rating plan should be handled in one of two ways in lieu of requiring a termination clause and a cost-plus contract. The acceptable methods correspond to the original plans used in the early 1900's: either a prospective experience-rating plan or a retrospective experience-rating plan. With the elimination of retrospective experience refunds, which would not be paid under a prospective plan, the insurer's profit depends on total premiums from all cases exceeding total claims and expenses. A deficit on one case can be offset by a surplus on another case, eliminating the necessity of the insurer's holding deficits.

An acceptable retrospective experience-rating plan would involve a basic premium for claims and expenses which includes a large premium margin for claims and a premium for stop-loss coverage in excess of the basic premium. Such a plan was presented in Situation VII. Under the plan, both the small persistent active deficits and the large canceled case deficits are eliminated.

The financial problems caused by a combined experience-rating plan are eliminated by each of the following: a cost-plus contract with a termination clause, an administrative services contract with or without stop-loss insurance, a prospective experience-rating plan, and a retrospective experience-rating plan with stop-loss coverage. Within these categories a multitude of plans can be developed applying the sound principles discussed above.

VII. CONCLUSION

Combination prospective and retrospective experience-rating plans commonly used throughout the industry result in a long-term underwriting loss to the insurer. Underwriting deficits are essentially a loan from the insurer's general funds used to pay claims for each deficit policyholder. The loan is presumably repaid from future surpluses of the policyholder but is forgiven upon cancellation of the policy. Since cancellation is an unrestricted option of the policyholder, financial selection against the insurer cannot be avoided. A combination experience-rating plan does not constitute an insurance contract but rather a finance contract between the insurance company and the policyholder. Treating experience-rated business as insurance rather than as a finance contract results in large unrecoverable deficits to the insurer. A sound experience-rating plan would remove the necessity of financing the payment of policyholder claims or would treat such a loan as any other loan made by the insurer.

ACKNOWLEDGMENT

The author would like to acknowledge with thanks the invaluable mathematical and programming assistance of William J. Falk, A.S.A.

APPENDIX

PROGRAM DESCRIPTIONS

I. SINGLE-YEAR ANALYSIS PROGRAM

This program uses a collective risk theory approach to analyze a group's mortality experience over a one-year period. It calculates the claim amount frequencies and the stop-loss premium.

The probabilities are calculated under the assumptions that (1) upon death a new life exactly like the old one is added to the group, (2) the probability of a claim in a time interval is dependent only on the length of the interval, and (3) simultaneous deaths do not occur. These assumptions lead to the compound Poisson model. The formula for the distribution function of the aggregate claim amount is

$$F(x) = \sum_{y=0}^{\infty} [e^{-m} m^y P^{y*}(x)/y!] ,$$

where m is the expected number of claims and $P^{y*}(x)$ is the yth convolution of the distribution of claim amounts, given that a claim occurs. These are calculated until the probability of another claim is less than 1×10^{-7}.

The over-all stop-loss premiums are then calculated as

$$\sum_{x>SL} f(x)(x - SL) ,$$

where SL is the stop-loss level and

$$\begin{aligned} f(x) &= F(x) - F(x - D) & x > 0 \\ &= F(0) , & x = 0 \\ &= 0 , & x < 0 ; \end{aligned}$$

D is the amount difference.

II. MULTIYEAR SIMULATION PROGRAM

This program simulates the experience of a portfolio of identical cases over a period of up to fifty years. It uses a random number generator to determine the number and amount of claims. These are based on the following distributions:

$$F(x) = \Pr(\text{Number of claims} \le x) = \sum_{y=0}^{x} e^{-m} m^y / y! ,$$

$$P^{y*}(z) = \Pr(\text{Claim} \le z \mid \text{a claim occurs}) = \sum_{i \le z} \left(\frac{\sum_{j=0}^{w} C_{i,j} q_j}{m} \right)$$

where m is the expected number of claims and

$$m = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} C_{i,j} q_j ,$$

$C_{i,j}$ is the number of insureds aged j with coverage i, and q_j is the mortality rate for age j. Note that these are two components of the compound Poisson distribution used in the single-year analysis. The program calculates the distributions and then reads the parameters. It then simulates the requested number of cases year by year. First the number of claims is determined, followed by the claim amounts and reserves. The experience refund or deficit is then determined, with cancellation occurring if necessary. Finally, the next year's premium is calculated. While doing this, the program retains the starting random numbers, so that the user may run the data with new parameters and the same claims.

The following assumptions are used in the development of this program:

1. Each case is statistically independent.
2. Each year is statistically independent.
3. The cases have stationary populations with replacement of individuals terminating.
4. The probability of a claim is dependent only upon the length of the time interval.
5. Simultaneous deaths do not occur.

A fifty-year experience period using a 1,050-life sample case develops significant exposure for each simulation run. The following tabulation shows the number of case-years and life-years of exposure contributing to the data in the paper.

Situation	Case-Years	Life-Years
I	5,000	5,250,000
II	5,000	5,250,000
III	5,000	5,250,000
IV	5,000	5,250,000
V	5,000	5,250,000
VI	5,000	5,250,000
VII	5,000	5,250,000
A	3,488	3,662,400
B	1,936	2,032,800
C	2,591	2,720,550
D	3,377	3,545,850
E	3,578	3,756,900
F	2,835	2,976,750
E1	3,740	3,927,000
E2	4,031	4,232,550
E3	3,778	3,966,900
E4	3,908	4,103,400

DISCUSSION OF PRECEDING PAPER

MYRON H. MARGOLIN:

Mr. Bolnick's paper is a well-intended effort to bridge one of the many serious gaps between theory and practice in actuarial science. However, his principal conclusion will come as a great surprise to most practicing group actuaries. Applying a certain mathematical model to group life insurance, Mr. Bolnick has concluded that "combination prospective and retrospective experience-rating plans commonly used throughout the industry result in a long-term underwriting deficit to the insurer." Of course, the fact is that the major carriers have consistently written group life on a profitable basis. The group life branch annual statement earnings (after dividends and taxes) of ten leading carriers have been positive in each of the last five years, 1969-73.

To determine why Mr. Bolnick's model leads to this erroneous conclusion, we must consider the three major components of the model. Mr. Bolnick assumes that the frequency distribution of the annual claims of each risk can be faithfully represented by a well-behaved mathematical function—in this instance, by the compound Poisson distribution favored by collective risk theory. On the surface this is a plausible assumption. Indeed, to many it will seem entirely self-evident that claim experience must follow such a distribution function. However, there is no empirical evidence that it does. More on this point later. Second, Mr. Bolnick's calculations assume a premium margin of approximately 2 per cent for a case of 1,050 lives. This in my judgment is a much smaller margin than is generally in use for cases of this size under full experience rating. Third, Mr. Bolnick utilizes two alternative experience refund techniques—the "pure accounting" method and the "modified pure accounting" method. My own company (and perhaps others too) employs a much more sophisticated dividend formula than either of these two techniques.

Mr. Bolnick also postulates that "because of a lower claim variance, a health portfolio will have less of a financial problem than a life portfolio with the same premium." Again, the facts seem to be otherwise. Many of the major carriers have experienced sizable group health underwriting losses in recent years. In the years 1969-73, among ten leading group writers, there were nineteen instances of negative gains from operations in the group health branch.

Perhaps there is no simple explanation as to why group health earnings have tended to be worse than group life. Three possible reasons might be the setting of lower earnings objectives for medical coverages, the persistent inflation of medical care costs, and the existence of certain tax and other competitive advantages enjoyed by the Blues; but none of these applies to group long-term disability insurance, for which many carriers have sustained especially serious losses. With regard to long-term disability, it is evident that much of the problem stems from unpredictably large changes in year-to-year claim experience on large cases, changes which are far too large to be attributable to "random fluctuation." Thus a simple stochastic model of the type used by Mr. Bolnick cannot adequately depict long-term disability claim experience. This raises doubts as to whether any such model can represent properly other health coverages or even group life. (This question has been discussed at length by Mr. Ernest Arvanitis and myself.)[1]

We welcome efforts like Mr. Bolnick's to bridge the gaps between theory and practice. However, we must insist that any such bridge be firmly grounded in the facts and take adequate account of the realities of the situation.

WILLIAM SCHREINER:

It is difficult to know how to approach Mr. Bolnick's paper. Both its scope and the mathematical techniques illustrated recommend it; yet it is a seriously flawed paper. Simply put, its thesis is false.

The author states that "an experience-rating plan using both a prospective formula and a retrospective formula results in underwriting losses to the insurer over an extended period of time." He further suggests that this result is attributable to the termination antiselection of groups with deficits. He contends: "A risk charge does not provide a sound method of recovering active case and canceled case deficits unless the termination antiselection problem can be solved." A slight modification of an example given in the paper will indicate the author's error.

Under the heading "A Sound Insurance Scheme" in Section II, the author demonstrates that a premium of \$79,192.99 (\$65,000 basic premium and \$14,192.99 stop-loss charge) is sufficient for an insurer to avoid loss with respect to the hypothesized portfolio when the stop-loss level is set at \$65,000. He indicates that this is true because no case can ever create a deficit, since, by definition, no group can ever be charged with claims in excess of the premium available to meet claims. Let us modify this example so that deficits may accumulate and look at the results.

[1] Discussion of Myron H. Margolin, "On the Credibility of Group Insurance Claim Experience," *TSA*, XXIII (1971), 229.

We will continue to charge each policyholder $79,192.99. We will also retain the same refund formula ($65,000 less claims equals refund, if positive), but this time we will set the stop-loss level at $75,000, not $65,000. What are the implications of this approach? Our premium income is the same as before; our claims will be unchanged, and, by hypothesis, there will be no change in our refunds for the current period. Thus our financial results will be the same, and we will have again avoided an underwriting loss. However, one important element will have changed: the insurer is now carrying forward deficits on every group with claims of more than $65,000. In other words, deficits will exist in 41 per cent of the groups insured. Indeed, deficits of $10,000 will exist on 36 per cent of the groups.

Obviously, the carrier in this example is in a stronger financial position than the carrier that used a $65,000 write-off level, since the accumulated "deficits" will serve to decrease future years' refunds. An analysis of the $79,192.99 premium should be helpful to an understanding of what has taken place. The $65,000 represents the basic premium for the group, and $10,350.76 represents the stop-loss premium for absorbing aggregate claims in excess of $75,000. The remaining $3,842.23 represents the risk and profit charge required to build up the insurer's surplus to the point where it has sufficient funds to cover fully all the expected deficits from the year's operation. To the extent that some groups subsequently repay their deficit, the insurer will subsequently obtain a profit of like amount.

Many additional examples of sound combination prospective/retrospective rating are possible, but the foregoing example should be sufficient to demonstrate that the author's statement that "the only total solution to withdrawal antiselection . . . is to have the stop-loss pooling level equal to the premium level for claims" is clearly false and that such combination plans do not *automatically* result in underwriting losses, even when no attempt is made to solve the termination antiselection problem. The author's construction of hypothetical illustrations merely serves to show that it is possible, without too much difficulty, to construct combination prospective/retrospective rating plans which are unsound.

The basic of a sound group insurance portfolio is not that of avoiding termination of groups in "deficit." It is really much simpler (and more difficult) than that; it is a question of obtaining an income (via premiums and investment income) that is greater than one's costs (claims, expenses, and refunds). No principles of insurance or finance are violated when "deficit" cases terminate, *provided* that sufficient funds have been created to fund such "losses." The basic theory underlying this process is indicated in the Society's Part 9E Study Note "Group Insurance Premium Development" (9E 1-6-72, pp. 10–16), written by this discussant.

The author's concern for the general questions of the management of an experience-rated portfolio and, particularly, for the management of deficit cases is well founded. He has failed to recognize, however, that a total portfolio can be managed under the combination approach on a financially satisfactory basis even when individual groups may terminate in deficit. Since it is probable that nearly every group insurance writer of large cases in North America uses a combination experience-rating plan, it is somewhat comforting to be able to demonstrate that the experience-rating methods used by the actuaries of these companies are not inherently unsound.

THEODORE W. GARRISON:

Mr. Bolnick is certainly doing our industry a service by bringing up the important subject of risk theory for our renewed consideration and by sharing with us the results of his research and his mathematical models. He may also be doing us a service by drawing controversial conclusions which are certain to stimulate considerable thinking and discussion. I do not agree, however, with his unequivocal conclusion that "an experience-rating plan using both a prospective formula and a retrospective formula results in underwriting losses to the insurer over an extended period of time."

Before making specific comments on the paper, I want to describe an experience-rating framework that differs slightly from the one used by Mr. Bolnick. In the common vernacular of the industry, as I understand it, a gross premium should include (1) expected incurred claims, (2) provision for expenses, (3) a margin for fluctuation and error, and (4) a charge for risk and profit. The margin for fluctuation and error should, on average, be returned to the policyholder as dividends or rate credits. This margin reduces the number of cases that go into a deficit position, and it provides money for deficit recovery and for the accumulation of premium stabilization funds. The premium stabilization funds (or contingency funds) are policyholder money and should be returned eventually to the policyholder in the form of benefit payments, dividends, or rate credits.

If a policy is issued on a basis that does not anticipate payment of any dividends or rate credits, then, by the above definition, there is no "margin" in the rates. Any "fat" added to the rates is simply an increase in the charge for risk and profit.

As we all know, it is possible to lose money on an experience-rated case. The most substantial losses come from cases that terminate while in a deficit position. Other losses may come from having actual expenses exceed the expense allowances that are built into the retentions or from having interest earnings that are less than were planned for when setting the expense-recovery and risk-charge factors. An annual statement loss on an experience-rated case will arise whenever the case slips into a deficit position; however, an offsetting annual statement gain will occur when the deficit is recovered. As long as the case remains in force, these deficits are comparable to unrealized capital losses on investments. The loss is realized, without hope of future recovery, whenever a case cancels while in a deficit position.

The realized losses that are incurred from a block of experience-rated cases should be recovered from that block of cases. This must be done through a risk-sharing or true-insurance process. Every case in the block should contribute something to cover the carrier's risk and profit. If over an extended time period the sum of the charges for risk and profit from all cases exceeds the realized losses, then the carrier will have made an operating profit from the experience-rated risks; otherwise, the carrier will have suffered a loss.

A theoretically proper scale of risk charges will take into account the probabilities that a case will get into a deficit position and that it will cancel while in the deficit position. The calculation probably will be based on a series of a priori conditional probabilities, with the probability of cancellation increasing as the size of the deficit increases.

It is true that policy terminations are the result of deliberate decisions by policyholders and are not chance occurrences subject to the laws of probability. However, we know from past experience that many, and perhaps most, policyholders will stay with the carrier while it recovers deficits. We also know that heavy-handedness on the carrier's part will lead to more cancellations than will a conciliatory attitude toward working out the deficit problem.

The size of the risk charge probably will vary by coverage and by size of case. It also might vary according to the amount of margin that is included in the prospective rates (or in a protective retrospective premium agreement). It should be possible to offer a trade-off between risk charge and margin. Thus, for a case where the policyholder is more concerned with premium level than with retention, it should be acceptable to reduce the margin by 3 or 4 or 5 per cent, and increase the risk charge by 1 or 2 per cent. If the reverse situation exists, the process can go the other way.

In theory it might be proper to reduce the risk charge for a case that has built up a large premium stabilization fund. However, that would be a dangerous practice because it would be difficult, and perhaps self-defeating, to impose a larger risk charge against the case that is already in a deficit position. Of course, it is appropriate to increase the risk charge for those cases that resist the usual accumulation of a premium stabilization fund.

A block of experience-rated cases should create a positive cash flow. The premium stabilization reserves on the positive cases, and the claim reserves on all the cases, should be greater than the deficits on the negative cases. Interest is commonly paid on the premium stabilization funds but not on claim reserves. The actuary should consider the actual amount of investment income being derived from the assets generated by a block of experience-rated cases, and the amount of interest payments being made to those cases. The difference will probably be a modest amount of excess investment income which may be used to satisfy partially the company's risk and profit requirements.

From time to time there is pressure for the payment of interest on claim reserves. If this is done, the actuary should be careful that the total interest paid to a block of cases does not exceed the interest earned on the assets generated by that block of cases. It is not possible to charge interest against the deficit cases because this would not be a collectible debt; therefore, the interest that is available to be paid on the positive cases is less than the amount of interest earned on the assets of only those positive cases. If the actuary is relying on the excess interest to meet part of his risk requirements, then even this excess amount is not available to be paid to the positive cases.

Using the logic and opinions set forth above, I will now comment on some of the statements made by Mr. Bolnick.

1. The premium for a prospective experience-rating plan as defined by Mr. Bolnick should not contain any "margin."
2. Mr. Bolnick did not identify the amounts of the margin for fluctuation and error that are included in the premium for each of his model situations. According to my calculations, they are as follows:

Situation	Margin for Fluctuations	Situation	Margin for Fluctuations		
			1st Year	Average Renewal	
				Positive Cases	Deficit Cases
I	2.2%	A	14.7%	14.7%	14.7%
II	9.6	B	14.7	Nil	Nil
III	14.7	C	14.7	5 %	5 %
IV	14.7	D	14.7	Nil	10 %
V	33.6	E	14.7	5 %	15 %
VI	43.3	F	14.7	5 %	15 %
VII	51.2				

The margins shown are expressed as a percentage of expected claims. As an example of how these numbers were calculated, here are the details for Situation III:

Situation III experience-rated net premium	$65,000
Total expected claims	$63,618
Claims covered by stop-loss	2,438
Pooled claims over $30,000	4,505
Expected experience-rated claims	$56,675
Margin ($65,000 − $56,675)	$ 8,325

$$\frac{\text{Margin}}{\text{Expected claims}} = \frac{\$\ 8,325}{\$56,675} = 14.7\%$$

The amount of margin included in Situations B, C, D, and E must go up and down like a yo-yo. These are the situations in which premiums after the first year are based solely on the prior year's experience. It appears that the average renewal margin for Situation B is nil, and the average margin for Situation C is 5 per cent greater than that for Situation B. Under Situation D, the average margin for positive cases will be nil and the margin for negative cases will depend on the size of the deficit. Situation E has 5 per cent more margin than Situation D. The renewal formula for Situation F is a little more realistic because the renewal premiums are based on the average claims since inception.

As we might reasonably expect, there is a direct relationship between the amount of margin in the rates and the size of the deficits that are generated. In the no-cancellation situations, Table 3 of the paper shows that Situation I, which has only 2.2 per cent margin, is a total disaster. The other situations have progressively lower deficits because of their progressively higher margins.

The following tabulation compares the margins with the canceled case deficits for the six cancellation situations. The risk charge shown here is defined as the percentage of net premium required to cover the canceled case deficits. The relationships between the margins and the risk charges for the various situations are interesting.

	A	B	C	D	E	F
1. Experience-rated premium ($000)	226,720	107,105	151,132	202,057	222,178	178,201
2. Canceled case deficits ($000)	4,910	7,820	6,496	5,112	4,613	5,675
3. Risk charge [(2)÷(1)]	2.17%	7.30%	4.30%	2.53%	2.08%	3.19%
4. Approximate margins (% of claims)						
a) 1st year	14.7%	14.7%	14.7%	14.7%	14.7%	14.7%
b) Renewal						
i) Positive cases	14.7%	Nil	5%	Nil	5%	5%
ii) Deficit cases	14.7%	Nil	5%	10±%	15±%	15±%

a) In comparing Situation A with Situation C, we see that the reduction in margin is from 14.7 to about 5 per cent. The indicated increase in risk charge is about 2.1 per cent. Thus, over this range, it takes a change in the margin of nearly 5 per cent to be worth as much as 1 per cent in the risk charge.

b) In going from Situation C to Situation B, the margin is reduced from 5 per cent to nil, and the increase in the indicated risk charge is about 3 per cent of premium. Thus, over this range, a change in margin of only 1.7 per cent is worth as much as 1 per cent in the risk charge.

c) The difference between the risk charges for Situations D and E goes in the right direction but is less than might be expected. However, when Situation E was rerun four times (see Table 9 of the paper), the average deficit for the five runs was only $3,670,000 instead of $4,613,000. This lower deficit is 1.65 per cent of the experience-rated premium for Situation E. Thus the 5 per cent change in margin between Situations D and E appears to be worth 0.9 per cent in the risk charge.

d) The indicated risk charge for Situation A appears consistent with Situations D and E (as modified above).

e) I do not have any good explanation, except possibly fluctuation, for the size of the indicated risk charge for Situation F. I would expect the risk charge for Situation F to be about the same as that for Situation A.

Of course the cases that are in a positive position will have more safety than is shown in the tabulation above because of their contingency (premium fluctuation) reserve. The amount of the contingency fund will usually be 9, 18, 26, or 35 per cent of the expected claims. If the contingency reserve was built up at a faster rate, then fewer cases would go ultimately into a negative position and the risk-charge requirements would be reduced.

3. Mr. Bolnick's cancellation assumption is not particularly realistic, and this could affect the validity of the above risk/margin relationships. He assumed that no case would cancel until the deficit reached $75,000 (equal to 1.32 times the expected annual experience-rated claim cost), and all cases cancel when they reach that deficit level. A more realistic assumption would have about 5 per cent of the positive cases canceling each year, with the cancellation rate for deficit cases increasing along with the size of the deficit, up to a maximum rate of about 40 or 50 per cent per year.
4. Mr. Bolnick states that "combination experience-rating plans will always result in a total portfolio deficit which will require an investment of the insurer's general fund to pay the unrecovered claims." This may be true in his simulations, but he has not included any risk charges (which are essential), and he has not considered the delay in claim payments, which results in the accumulation of claim reserves. In real life, a block of experience-rated cases should produce a positive cash flow. The premiums received should be greater than the claims and expenses actually paid. Therefore, assets are available to be invested. If the amount of interest

paid to the various policyholders (usually paid on premium stabilization funds) is less than the amount of interest earned on the invested assets generated by the block of cases, then the residual interest is available to help satisfy the company's risk and profit requirements. As long as the block of cases is producing a net investment gain, it is not necessary to worry about trying to recover any loss of interest on invested surplus. If the situation deteriorates so as to cause a drain of assets that were generated from other sources, then it is appropriate to worry about providing "interest income on the insurer's investment in active case deficits."

We might point out here that the deficits do offset much of the claim reserves, so that paying interest on claim reserves is not a practical alternative.

5. The treatment of deficits on active cases is deserving of further discussion. As experience fluctuates from year to year, there will be times when increases in the total active case deficits will cause annual statement losses. Probably three-fourths or more of these active case deficits will ultimately be recovered, and, when the recovery occurs, there will be an offsetting statement profit. Depending on the circumstances of the company and the attitudes of management, the appearance of a temporary statement loss may be merely serious or it may be considered a disaster. In any event it is undesirable, and the group actuaries and underwriters must take appropriate steps to avoid its occurrence and/or to recover from it as quickly as possible.

   Mr. Bolnick talks about a risk charge being used to recover active case deficits as well as canceled case deficits. If these active case deficits are recovered via a risk charge that is assessed against all cases, should the losses also be recovered from the cases that incurred them? If this were done, we would be recovering twice. In the short run this might help recover the statutory losses, but in the long run it would result in excessive profits and noncompetitive retentions.

6. Mr. Bolnick's list of "deficit management techniques" is incomplete. An additional essential technique is the assessment of a charge for risk and profit against every case so as to produce funds to cover the deficits on canceled cases. A portion of the risk charge may come from the net investment income generated by the block of cases.

7. I agree that deficit management techniques can be classified into two categories. I would like to observe that the first category is aimed at making the policyholder pay his own claims—for example, self-insurance. The second category involves some form of pooling or the spreading of risks.

8. Mr. Bolnick expresses concern over the financial selection exercised by policyholders who cancel in a deficit position. I disagree that "in practice it is not possible to predict accurately a risk charge, because of the unpredictable nature of case cancellation." We have past experience to guide us. We know that many deficit cases do not cancel and that panic on our

part in trying to force a rapid recovery of deficits can increase the likelihood of cancellation. In practice, it is essential for us (consciously or unconsciously) to estimate the frequencies of cancellation for various coverages and for various possible loss positions and profit positions. I believe that we have the capability of setting fairly good risk charges. The real problem is that too often we yield to competitive pressure and use risk charges which we know are inadequate.

It may not be in the best interest of a policyholder to change insurance carriers unless the size of his deficit with the old carrier is substantial, because of the extra expenses involved in the change. These include new first-year commissions, other sales expenses, revised contracts and certificates, and the cost for the new carrier to set up his administrative records. There also may be claim problems during the transition.

9. Mr. Bolnick's models and most of his discussion are related to group life insurance. Early in his article he acknowledges that "because of a lower claim variance, a health portfolio will have less of a financial problem than a life portfolio with the same premium." It happens that a health portfolio is affected by external forces and forms of antiselection that are difficult to cope with statistically. The problems on health are not necessarily less than those on life, but they are different. A combined portfolio of life and health is easier to manage than a pure portfolio of either. In most years the life policy will produce margins to help support the health, and in the other years, the health insurance will help support the life insurance.

10. In practically all his examples Mr. Bolnick used a stop-loss feature. I do not think that the use of stop-loss, or even its availability, is all that prevalent in the insurance industry. Stop-loss involves the tail of the claim distribution, and the size of the tail is extremely sensitive to the underlying probabilities. Those probabilities are never known and must be estimated. If the assumptions made concerning the probabilities are in error by even a small amount, the resulting stop-loss premium will be in error by a larger amount. The theory of stop-loss insurance may be sound, but the risks involved to the insurer are substantial. The only solution of which I am aware is to overcharge for the product. Of course, overcharging makes the product unattractive to the customer. Trying to produce long-term profits on stop-loss insurance might present more problems than trying to produce long-term profits on experience-rated insurance.

    We have observed that the probability that a case will cancel increases along with the size of the deficit. As a practical matter, it may be in the insurer's best interest at some point to forgive part or all of the deficit on some particular case in lieu of having the case cancel. This philosophy amounts to de facto stop-loss insurance. The insurer might be better off to formalize his stop-loss program and charge a specific pooled premium for it. Otherwise, any deficit that is forgiven on an active case is equivalent to a loss on a canceled case and must be recovered through the risk charge.

11. Mr. Bolnick refers to his Situation VII as being an acceptable retrospective

experience-rating plan. This plan has an experience-rated net premium of \$85,000 and expected experience-rated claims of \$56,230. Thus the margin is \$28,770, or 51.2 per cent of the expected claims. This plan may be acceptable to a conservative actuary, but I cannot imagine its being accepted in the marketplace. Also, I do not see any need for a 51 per cent margin. The risk theory discussed in item 2 above indicates that a moderate risk charge is sufficient if the margin is as high as 5–10 per cent.

12. Some of the important keys to success in writing experience-rated group insurance are the following:
    a) An adequate scale of charges for risk and profit.
    b) Adequate margins in the first-year and renewal premiums.
    c) Moderately conservative rules for interpreting prior experience when setting renewal rates.
    d) An adequate set of rules for accumulating and maintaining premium fluctuation funds (or contingency reserves) on each case that has favorable experience.
    e) A reasonable rule for recovering the deficits on the cases that have had unfavorable experience.
    f) Possible use of stop-loss limits.
    g) Possible use of pooling for individual benefits.
    h) A thorough consideration of the sources and disposition of interest income for each block of cases.
    i) An adequate expense recovery formula.

    The total experience-rating plan includes a mixture of these elements. Mr. Bolnick chose some mixtures which, under his assumptions, produced long-term losses. I am convinced that other mixtures are available which in real life will produce long-term gains.
13. In his parting shots Mr. Bolnick implies or states that combined experience-rated insurance does not work and that our only chance for salvation is to use either some form of pure pooling or some form of pure self-insurance. I believe that the form of experience-rated insurance that has evolved into common usage can be made to work with the inclusion of margins and risk charges that are low enough to be acceptable to the policyholders and, at the same time, high enough to protect the insurance company from long-term losses. Most employers appreciate this form of insurance and are willing to pay the insurer a modest charge for risk and profit. Success in this business requires us to walk a narrow line, but I believe it can be done.

JOHN C. ANGLE:

Mr. Bolnick deserves our thanks for opening the door on an arcane subject, the experience-rating of group life insurance. Mr. Bolnick is particularly helpful in tracing the right-tailed frequency distribution of claims, in tabulating stop-loss premiums for the sample group life case, and in illuminating the risk charges needed to offset canceled case deficits and the loss of interest on accumulated deficits. There are, however, instances in which Mr. Bolnick appears to have reached for conclusions that are not manifest in the evidence he offers for our inspection.

These conclusions all turn on the purported inability of group insurers and their actuaries to charge adequate premiums for group life insurance. Without presenting any evidence, Mr. Bolnick implies that all North American insurers are locked in a no-win competitive struggle, commercially akin to the Israeli-Arab confrontation, but without any hope of a Secretary Kissinger who might mediate a settlement among insurers.

This simply seems an unwarranted generalization to me. Certainly any number of insurers have taken the plunge into group insurance by relaxing underwriting standards and shaving group health premium rates. But such spectacles are usually short-lived. Few company managements enjoy explaining continuing group insurance underwriting losses to their boards of directors, stockholders, or policyholders. There are, on the other hand, any number of examples of group insurers who seem able to turn a modest profit while charging adequate premium rates. Apparently some actuaries are able to articulate a sound group pricing strategy regardless of the siren calls of the marketplace.

On the management side, some actuaries recall Bertram M. Pike's excellent treatise "Gain and Loss Analysis and Related Concepts for Group Insurance."[1] Here Mr. Pike recommended carefully separating gains and losses from deficit increases or recoveries from other sources of gain or loss. He also recognized gains from interest. In analyzing group gains from interest as recommended by Mr. Pike, one may find that interest losses on deficits carried forward are fully offset by interest gains on claim and unearned premium reserves. Unless one wishes to grapple with the prickly pear of the interest element in group health premium rates, the matter of an interest charge on experience-rating deficits may best be left as a moot question.

Certainly, as Mr. Bolnick urges, some insurers may have lost sight of "the fundamental principles of finance and insurance" in insuring groups of 1,000 lives or more. As Mr. Bolnick amply demonstrates, there is no defense of the position that such groups deserve a credibility of 1. Somehow, laymen quite readily assume a deterministic outcome in which most groups will hit the target loss ratio but a few, proved somehow inherently better by low loss ratios, will deserve full refunds. It would be a real service to all of us if the elements of Mr. Bolnick's demonstration could be grasped, say, by the attendees at the next HIAA Group Forum.

[1] *TSA*, XIII (1961), 412–24.

Surely there is still hope that the unrepentant can be converted by logic and demonstration. I cling to the notion that instruction remains an alternative superior to the proposition that group insurers must abandon their risk-bearing and loss-sharing roles to become solely claim-paying, administrative agents of employers.

Let me add one final point. The matter of group experience-rating may bedevil some insurers because of the fictional nature of the projections of experience contained in their proposals to prospective contractholders. I have, for instance, seen proposals that assume a 60 per cent group health loss ratio for each of the next ten years. Certainly such projections are so unlikely to occur as to breed policyholder and broker disenchantment when the inevitable rate increases take place. Perhaps, then, part of the problem whose symptoms Mr. Bolnick describes is best cured by more adequate description and projection of experience-rating results.

EDWARD J. PORTO:

Mr. Bolnick is to be congratulated on the contribution his paper makes to actuarial science, particularly in its scope and the techniques utilized, in an area in which actuarial analysis has been relatively sparse.

In my interpretation, the main point and conclusion of the paper is that experience-rating techniques "commonly used throughout the industry" for group life insurance are unsound and will result in long-term underwriting losses. This is in the face of an attitude that I believe is prevalent—that group life insurance is profitable as written and experience-rated today, even though it encompasses features and methods that the author claims, and attempts to prove, unsound.

I do not think that Mr. Bolnick has quite succeeded in substantiating his main conclusion; in my opinion he has not selected a sufficiently representative set of examples or situations to prove his point. A more valid point to be made, I would think, is one that is somewhat less sweeping and categorical: that there is a tendency for the mortality fluctuation margins traditionally used in group life premium rates to be eroded as competition for group life business becomes ever more keen, and that if such practice continues to the extreme that Mr. Bolnick has carried it—that is, by using a $65,000 premium for claims when the expected claims are $63,617.50—long-term underwriting losses will indeed result under traditional experience-rating methods, particularly if risk charges are not increased to reflect the increase in risk.

Much as I am grateful to the author for the scope of the situations he has illustrated in the multiyear analysis, I think that the situations indicated in Table 1 of this discussion would also have been of interest.

In Table 1 the factors 1.34, 1.18, and 1.10 are suggested in situations a–d, e–h, and i–l, respectively, to provide the same percentage margin in each renewal premium as in the first-year premium; that is, in the ratio of \$85,000, \$75,000, and \$70,000, respectively, to \$63,617.50. In the renewal premium formulas for situations d, h and l, Z represents an appropriate credibility factor.

I would regard the use of \$85,000 as the annual premium for claims as corresponding to the type of margins inherent in the 1961 Standard Group Life Insurance premium rates (prior to the modifications made in the table in 1971, for use as a first-year minimum table, by the New York Insurance Department), and \$75,000 and \$70,000 as intermediate levels between the 1961 table and the extremely low margin that Mr. Bolnick is using in his main examples (with \$65,000 as the premium for claims).

TABLE 1

A. NO-CANCELLATION SITUATIONS

Situation	Premium for Claims	Over-all Stop-Loss	Individual Stop-Loss	Contingency Reserve
1	\$85,000	\$100,000	\$30,000	...
2	85,000	100,000	30,000	\$20,000
3	75,000	...	...	...
4	75,000	100,000	...	...
5	75,000	100,000	30,000	...
6	75,000	100,000	30,000	20,000

B. CANCELLATION SITUATIONS

(All Including Over-all Stop-Loss, Individual Stop-Loss, and Contingency Reserve)

Situation	First-Year Premium for Claims	Renewal Premium Formula
a	\$85,000	\$85,000
b	85,000	$1.34C_{T-1}$
c	85,000	$1.34C_{T-1}+0.2D_{T-1}$
d	85,000	$1.34[Z(\Sigma_1^{T-1} C_i)/(T-1)+(1-Z)\ (63{,}617.50)]$
e	75,000	\$75,000
f	75,000	$1.18C_{T-1}$
g	75,000	$1.18C_{T-1}+0.2D_{T-1}$
h	75,000	$1.18[Z(\Sigma_1^{T-1} C_i)/(T-1)+(1-Z)\ (63{,}617.50)]$
i	70,000	\$70,000
j	70,000	$1.10C_{T-1}$
k	70,000	$1.10C_{T-1}+0.2D_{T-1}$
l	70,000	$1.10[Z(\Sigma_1^{T-1} C_i)/(T-1)+(1-Z)\ (63{,}617.50)]$

Even with the $65,000 premium for claims, in Situation E, Mr. Bolnick does demonstrate that a risk charge of about 3½ per cent would cover the "underwriting loss" that would occur. In fact, this appears to be an overstated figure, since the author apparently is including as part of the "underwriting loss" the profit made by the over-all and individual stop-loss pools. Thus, in Table 10 of the author's paper, I would interpret Situation E to show an underwriting loss of $4,033,511, composed of a stop-loss pool gain of $1,157,579 and an experience-rating deficit of $5,189,090, whereas Mr. Bolnick refers to the $5,189,090 as an underwriting loss.

In Situations c, g, and k indicated above, the mortality fluctuation margins are greater than those under the author's Situation E (2 per cent first year, 5 per cent renewal years), and the corresponding appropriate risk charge therefore would be less than the appropriate risk charge for Situation E. While Situation c would not be representative of current rate levels today (since it would imply use of the original 1961 rate table in both first and renewal years), I would expect current rate levels generally to be closer to the $70,000–$75,000 levels than to the $65,000. This is certainly the case at my company.

It is to be noted that some of the rate reductions occurring on group life are not so much an erosion of mortality fluctuation margins as a searching out of varying basic mortality levels by industry. This is where Mr. Bolnick's investigation is necessarily limited: the simulated mortality experience is based on random fluctuation only, whereas the basic purpose of experience rating is to adjust the rate toward the individual group's true mortality level. Mr. Bolnick's results correspond to true situations only to the extent that the true mortality level of each group is as assumed; in actual practice, they would be scattered in mortality level about the average represented by the 1960 Commissioners Standard Group basic mortality level.

Mr. Bolnick decries the practice, in experience-rating individual groups, of carrying deficits forward to be recovered out of future years' profits. He refers to this as an unsound financial arrangement and suggests that either the insurance element in the group contract be enlarged by having a stop-loss equal to the basic premium or the insurance element be reduced by making the accumulated deficit a legally collectible debt of the policyholder in event of cancellation or, as an even more drastic measure, by making any deficit a legally collectible debt to be paid at the end of the year in which it arises. I share some of Mr. Bolnick's aversion to the concept of carrying deficits forward, but I have some doubts as to the marketability of the alternatives that he proposes. Certainly the proposal to have a stop-loss equal to the basic premium has far more appeal to me than the other two alternatives. A possible compromise approach would be to limit the accumulated deficit carried forward to a certain multiple of the excess of the first year's retention charge over the renewal retention charge. Any excess deficit would be covered by a smaller pooling charge than would be required for having a stop-loss equal to the basic premium.

WILLIAM A. BAILEY:

My discussion will focus especially on Situation E of the paper but will indicate how appropriate risk charges may be calculated without recourse to random numbers.

Under Situation E, each case has an individual claim pooling level of \$30,000 and an over-all stop-loss pooling level of \$100,000. Thus we need a frequency distribution of total claims for the group life case shown in Table 1 of the paper, using the following forces of mortality which were assumed in calculating Table 2 of the paper:

Age Bracket	μ
15–19	.00098
20–29	.00105
30–39	.00154
40–49	.00429
50–59	.01802

(The value of μ for the age bracket 50–59 may be a bit high.)

A Method for Calculating Frequency Distributions Such as That Shown in Table 2 of the Paper

We will define a few operators on discrete frequency distributions. For this purpose we assume that a matrix $[x_i, p_i]$ represents a 1-dimensional discrete frequency distribution, where i refers to the number of the row (or line); the x_i are real numbers, and each p_i is the probability or frequency of occurrence of the particular value of x_i. We shall take the liberty of referring to such a matrix as a frequency distribution even when the sum of the frequencies (i.e., $\Sigma_{i=1}^{n} p_i$, where n is the number of rows or lines) is less than unity.

The operator "Transform" is defined by $(ax \rightarrow x)\,[x_i, p_i] = [ax_i, p_i]$, where a is a given real number; transformations involving different formulas would be analogously defined.

The operator "Convolute for Sums" ($\oplus$) is defined as follows:

$$[x_i^{(1)}, p_i^{(1)}] \oplus [x_j^{(2)}, p_j^{(2)}] = [x_i^{(1)} + x_j^{(2)}, p_i^{(1)} p_j^{(2)}] ,$$

where i assumes each integer value from 1 to the number of lines in the first matrix, and, for each such value of i, j assumes each integer value from 1 to the number of lines in the second matrix. Thus the resulting matrix is obtained by calculating the pair of values

$$(x_i^{(1)} + x_j^{(2)}, p_i^{(1)} p_j^{(2)})$$

for each combination of i and j. The superscripts (1) and (2) merely identify whether the value originates from the first or second matrix, respectively. The Convolute for Sums operator $\oplus$ is associative and commutative in an algebraic sense.

For ease of discussion assume that a life replacing a death will be assigned the same certificate number, will be insured for the same amount of insurance, and will be subject to the same force of mortality as the original life. The 1-dimensional frequency distribution $A1$ of number of deaths in one year from one certificate within the age bracket 15–19, where $\mu_{18} = .00098$, is given below.

k	Frequency
0	.9990204800
1	.0009790401
2	.0000004797
3	.0000000002

These values are given by the Poisson distribution, which can be expressed as $\mu^k e^{-\mu}/k!$, where $k = 0, 1, 2, \ldots$, and μ is the expected number of deaths.

Let $B1 = A1 \oplus A1 \oplus \ldots \oplus A1$ (50 times) be the frequency distribution of the number of deaths arising from 50 such certificates.

Let $C1 = (5{,}000x \rightarrow x)B1$; this symbolism merely indicates that each line (x_i, p_i) in the $B1$ matrix is replaced by $(5{,}000x_i, p_i)$. The resulting $C1$ is the combined frequency distribution of total claims in a one-year period for the 50 certificates having a death benefit of \$5,000 and subject to a force of mortality of .00098.

For each of the fourteen age/amount categories in Table 1 of the paper, a frequency distribution similar to $C1$ is constructed; referring to these frequency distributions as $C2, C3, \ldots, C14$, let $D = C1 \oplus C2 \oplus \ldots \oplus C14$. The resulting D is the combined frequency distribution of total claims in a one-year period for the 1,050 certificates implied by Table 1. Thus D is equivalent to Table 2 of the paper. Table 2E, in this discussion, was calculated using the procedure described above for frequency distribution D, except that (1) the lives subject to a death benefit of \$40,000 were assumed to be subject to a death benefit of \$30,000 and (2) where the amount of total claims from the group in one year exceeded \$100,000, the amount was replaced by \$100,000.

TABLE 2E

Mean from Table = 56,674.91306; Theoretical Mean = 56,674.91306
Standard Deviation from Table = 27,855.82664
Theoretical Standard Deviation = 27,855.82664
Table Variance = 775,947,078.05036; Variance = 775,947,078.05071

Amount	Frequency	Cumulative	Cumulant
$ 0	.0113759932	.0113759932	0
5,000	.0164553742	.0278313674	82.27687
10,000	.0269404124	.0547717799	351.68099
15,000	.0274924386	.0822642186	764.06757
20,000	.0420520332	.1243162519	1,605.10824
25,000	.0432544702	.1675707221	2,686.46999
30,000	.0566643018	.2242350240	4,386.39905
35,000	.0541525832	.2783876072	6,281.73946
40,000	.0640564670	.3424440742	8,843.99814
45,000	.0586306557	.4010747300	11,482.37765
50,000	.0652805962	.4663553262	14,746.40746
55,000	.0580634554	.5244187817	17,939.89752
60,000	.0609911887	.5854099705	21,599.36884
65,000	.0525362716	.6379462421	25,014.22650
70,000	.0527639406	.6907101828	28,707.70234
75,000	.0443805815	.7350907643	32,036.24596
80,000	.0429301568	.7780209212	35,470.65851
85,000	.0353288917	.8133498129	38,473.61431
90,000	.0330455343	.8463953472	41,447.71239
95,000	.0266529179	.8730482652	43,979.73960
100,000	.1269517346	.9999999998	56,674.91306

Since the Convolute for Sums operator is associative, the meanings of the above formulas for D and the B's are well defined. Some alternative ways of deriving frequency distribution D would be the following:

1. Define $B1 = (5{,}000x \rightarrow x)A1$ and $C1 = B1 \oplus \ldots \oplus B1$ (50 times), and perform the calculations in that order; the results would be the same.
2. Since the A's are Poisson distributions, the calculation of $A_k \oplus \ldots \oplus A_k$, for any k from 1 to 14, can be done directly without using the Convolute for Sums operator; that is, simply calculate the frequency distribution $\lambda^k e^{-\lambda}/k!$, where λ is the expected number of deaths within a particular age/amount category in Table 1 of the paper, and where k is the number of deaths in a one-year period from such age/amount category. The resulting frequency distribution of number of deaths would be transformed by multiplying the number of deaths by the individual amount of insurance. If the A's were binomial distributions, then once again the Convolute for Sums operator would be unnecessary in deriving the B's. However, the Convolute for Sums operator would be unnecessary in deriving the B's. However, the Convolute for Sums operator would presumably be required to derive the D's in any case.

A Method for Calculating Risk Charges of the Type Shown in Table 13

Define a "Special Convolute" operator ξ_1 as follows:

$$[x_1^{(1)}, y_i^{(1)}, z_i^{(1)}, p_i^{(1)}] \; ⓔ_1 \; [x_j^{(2)}, 0, 0, p_j^{(2)}] = [x_k^{(3)}, y_k^{(3)}, z_k^{(3)}, p_k^{(3)}]$$

where

$$z_k^{(3)} = z_i^{(1)} + y_i^{(1)} - x_j^{(2)} \qquad \text{if } y_i^{(1)} - x_j^{(2)} < 0$$

$$= \min \{z_i^{(1)} + 5{,}000; 20{,}000; z_i^{(1)} + y_i^{(1)} - x_j^{(2)}\}$$

$$\text{if } z_i^{(1)} > 0 \text{ and } y_i^{(1)} - x_j^{(2)} \geq 0$$

$$= \min \{5{,}000; z_i^{(1)} + y_i^{(1)} - x_j^{(2)}\} \text{ if } z_i^{(1)} < 0 \text{ and } y_i^{(1)} - x_j^{(2)} \geq 0 \, ;$$

$$x_k^{(3)} = x_i^{(1)} + y_i^{(1)} \, ;$$

$$y_k^{(3)} = 1.05x_j^{(2)} - 0.20 \min \{0; z_k^{(3)}\} \, ;$$

i assumes each integer value from 1 to the number of lines in the first matrix, and, for each such value of i, j assumes each integer value from 1 to the number of lines in the second matrix. The Special Convolute operator $ⓔ_1$ is neither commutative nor associative in an algebraic sense. The superscripts (1), (2), and (3) merely identify whether the value is from the first, second, or third matrix, respectively. (To the extent that the volume of calculations implied by this or other definitions in this discussion would be inordinate, suitable meshes can be superimposed on each of the coordinate axes, and sufficiently small probabilities ignored; control can be maintained by calculating various moments both before and after imposition of the meshes.)

Consider the matrix (or frequency distribution) $[x_i^{(1)}, y_i^{(1)}, z_i^{(1)}, p_i^{(1)}]_t$, where

$x_i^{(1)}$ = Total premiums from issue to the end of the tth policy year;
$y_i^{(1)}$ = Prospective premium at the beginning of the $(t + 1)$st policy year;
$z_i^{(1)}$ = Accumulated contingency reserve, if +; accumulated deficit, if −; both at the end of the tth policy year;
$p_i^{(1)}$ = Probability (or frequency) at issue of the occurrence of the triplet (x_i, y_i, z_i) at the end of the tth policy year.

For $t = 0$, let $[x_1^{(1)}, y_1^{(1)}, z_1^{(1)}, p_1^{(1)}]_0 = [0; 65{,}000; 0; 1.00000]$, that is, a one-line 3-dimensional frequency distribution.

Consider the frequency distribution $[x_j^{(2)}, y_j^{(2)}, z_j^{(2)}, p_j^{(2)}]$, where

$x_j^{(2)}$ = Total claims in a one-year period from the 1,050 lives implied in Table 2 of the paper;
$y_j^{(2)} = 0$;

$z_j^{(2)} = 0$;
$p_j^{(2)}$ = Probability (or frequency) of the occurrence of $x_j^{(2)}$.

Calculate $T01 = [0;\ 65{,}000;\ 0;\ 1.00000]$ Ⓔ1 $[x_j^{(2)},\ 0,\ 0,\ p_j^{(2)}]$. Define a "Horizontal Split" operator, $\rightleftarrows$, as follows:

$$\text{3-d } [x_i, y_i, z_i, p_i] \begin{cases} \xrightarrow{z_i \geq m} [x_j, y_j, z_j, p_j]\,, & \text{where } z_j \geq m \text{ for all } j \\ \xrightarrow{z_i < m} [x_k, y_k, z_k, p_k]\,, & \text{where } z_k < m \text{ for all } k\,, \end{cases}$$

where m is a given real number.

Split $T01$ into the two 3-dimensional frequency distributions $A01$ and $W01$:

$$T01 \begin{cases} \xrightarrow{z_i \geq -75{,}000} A01 \\ \xrightarrow{z_i < -75{,}000} W01\,. \end{cases}$$

Save $W01$, and calculate $T02 = T01$ Ⓔ1 $[x_j^{(2)}, 0, 0, p_j^{(2)}]$.

Split $T02$ into two 3-dimensional frequency distributions, $A02$ and $W02$:

$$T02 \begin{cases} \xrightarrow{z_i \geq -75{,}000} A02 \\ \xrightarrow{z_i < -75{,}000} W02\,. \end{cases}$$

Save $W02$, and proceed recursively until $t = n$ $(=10$, say); the size of n will depend on how far into the future the projection is to extend.

In the resulting 3-dimensional frequency distributions $W01, W02, \ldots, W10$ and $A10$, the x-values represent total premiums, the y-values represent prospective premiums, and the z-values represent accumulated contingency reserves (if $+$) or accumulated deficits (if $-$), in each case at the end of the indicated policy year. Of course, since the W's represent group withdrawals, the y-value in the W's will not be used in the calculations.

Question 1: If our portfolio consists of a single group life case (of the composition of the group underlying Table 2 of the paper), (*a*) What is the expected value of the risk charges of the type shown in Table 13 of the paper? (*b*) How much variability can we expect in such risk charges?

Define a 3-dimensional "Merge" operator Ⓜ as follows:

$$[x_i^{(1)}, y_i^{(1)}, z_i^{(1)}, p_i^{(1)}] \text{ Ⓜ } [x_j^{(2)}, y_j^{(2)}, z_j^{(2)}, p_j^{(2)}] = \begin{bmatrix} x_i^{(1)}, y_i^{(1)}, z_i^{(1)}, p_i^{(1)} \\ x_j^{(2)}, y_j^{(2)}, z_j^{(2)}, p_j^{(2)} \end{bmatrix},$$

that is, the resulting matrix is simply the union of the first and second matrices.

Determine $Y10 = W01 \circledM W02 \circledM \ldots \circledM W10 \circledM A10$.

Determine $Z10 = (-\min \{0, z\}/x \rightarrow x)Y10$. This last operation replaces any contingency reserves (i.e., positive values of z) with zero, because zero is the required risk charge wherever there is a contingency reserve; although $Y10$ is a 3-dimensional frequency distribution, $Z10$ could be treated either as a 3-dimensional frequency distribution $[x, 0, 0, p]$ or a 1-dimensional frequency distribution $[x, p]$.

$Z10$ was calculated using the above procedure; the mean of the x-values was 6.44152 per cent, and the standard deviation was 14.78815 per cent. The accompanying tabulation is an abridged version of the results. The

FREQUENCY DISTRIBUTION OF RISK CHARGES FOR A PORTFOLIO OF ONE GROUP WITH t = 10

Cumulative Frequency	Risk Charge	Cumulative Frequency	Risk Charge
.001	127.76%	.30	5.81%
.01	98.14	.40	3.21
.10	13.56	.50	1.93
.20	8.41	.60	0.62

probability of a 0 per cent risk charge was .378185; 127.76 per cent was the largest risk charge emerging. $Z10$ is the frequency distribution of risk charges sought in Question 1(*b*). The mean value (6.44 per cent) of $Z10$ is the expected value sought in Question 1(*a*).

Some corresponding statistics for $Z1, \ldots, Z9$ are shown below.

SOME RISK-CHARGE STATISTICS FOR A PORTFOLIO OF ONE GROUP

t	Mean	Standard Deviation	t	Mean	Standard Deviation
1	12.50%		6	7.65%	
2	13.91		7	7.17	
3	11.30		8	6.85	
4	9.51		9	6.62	
5	8.40	15.42%	10	6.44	14.79%

Question 2: If our portfolio consists of an infinite number of group life cases (each of which has the composition of the group underlying Table 2 of the paper), what risk charge of the type shown in Table 13 of the paper would be appropriate?

The answer to this question is obtained by calculating the ratio

$$\frac{\Sigma_i\,(-\min\,\{0, z_i\}\,p_i)}{\Sigma_i\,(x_i p_i)}$$

obtained by operating on the 3-dimensional frequency distribution $Y10$. For illustration, the results of such calculations for $t = 1$–10 are shown below.

PORTFOLIO OF INFINITE SIZE

t	Risk Charge	t	Risk Charge
1.	12.50%	6.	5.50%
2.	10.93	7.	4.93
3.	8.93	8.	4.51
4.	7.39	9.	4.17
5.	6.30	10.	3.89

Question 3: If our portfolio consists of 100 group life cases (each of which has the composition of the group underlying Table 2 of the paper), (*a*) What is the expected value of the risk charges of the type shown in Table 13 of the paper? (*b*) How much variability can we expect in such risk charges?

Define a 3-dimensional Convolute for Sums operator $\oplus$ as follows:

$$[x_i^{(1)}, y_i^{(1)}, z_i^{(1)}, p_i^{(1)}] \oplus [x_j^{(2)}, y_j^{(2)}, z_j^{(2)}, p_j^{(2)}]$$
$$= [x_i^{(1)} + x_j^{(2)}, y_i^{(1)} + y_j^{(2)}, z_i^{(1)} + z_j^{(2)}, p_i^{(1)} p_j^{(2)}] ,$$

where i assumes each integer value from 1 to the number of lines in the first matrix, and, for each such value of i, j assumes each integer value from 1 to the number of lines in the second matrix; thus the resulting matrix is obtained by calculating the quadruplet of values

$$(x_i^{(1)} + x_j^{(2)}, y_i^{(1)} + y_j^{(2)}, z_i^{(1)} + z_j^{(2)}, p_i^{(1)} p_j^{(2)})$$

for each combination of i and j. The superscripts (1) and (2) merely identify whether the value originates from the first or second matrix, respectively. The 3-dimensional Convolute for Sums operator $\oplus$ is associative and commutative in an algebraic sense.

Determine $U10 = Y10 \oplus Y10 \oplus \ldots \oplus Y10$ (100 times), using the 3-dimensional Convolute for Sums operator.

Determine $V10 = (-\min\{0, z\}/x \rightarrow x)U10$. (Note that it would not be strictly correct to reverse the order, doing the transformation to 1 dimension first and then the Convolution for Sums $\oplus$ on the 1-dimensional results; that is, the frequency distribution of risk charges for a portfolio of 100 groups is *not* simply the Convolution for Sums $\oplus$ of the frequency distribution of risk-charge percentages for one group.)

*V*10, considered as a 1-dimensional frequency distribution $[x, p]$, is the frequency distribution of risk charges sought in Question 3(*b*). The mean value of *V*10 is the expected value sought in Question 3(*a*). *V*10 was calculated using the above procedure; the mean was 3.89747 per cent, and the standard deviation was .45905 per cent. The accompanying tabulation is an abridged version of the results.

FREQUENCY DISTRIBUTION OF RISK CHARGES FOR A PORTFOLIO OF 100 GROUPS

Cumulative	Risk Charge	Cumulative	Risk Charge
10^{-6}	6.30%	.60	3.79%
10^{-5}	6.05	.70	3.62
10^{-4}	5.66	.80	3.48
10^{-3} = .001	5.42	$1-10^{-1}$ = .90	3.42
10^{-2} = .01	5.10	$1-10^{-2}$ = .99	3.10
10^{-1} = .10	4.59	$1-10^{-3}$ = .999	2.67
.20	4.34	$1-10^{-4}$	2.39
.30	4.19	$1-10^{-5}$	2.15
.40	3.92	$1-10^{-6}$	1.98
.50	3.92*		

* The repetition of 3.92 occurs because a frequency of .194 . . . is attached to this outcome.

Both the author's calculations and mine reflect the assumption that, if there were no claims in a given year, the premium for the following year would be set equal to the premium charged in the previous year. Another assumption was that the premium charged could be as high as $120,000, in spite of the fact that the maximum total one-year claims could not exceed $100,000. These assumptions should be borne in mind in interpreting the results.

Cases were assumed to terminate when the cumulative deficit exceeded $75,000. An alternative type of assumption might have been that the probability of terminating is a function of the size of the cumulative deficit; for example:

Probability of Case Withdrawal	Cumulative Deficit as a % of One-Year's Premium
.05	$\geq$ 0, $<$ 25%
.10	$\geq$ 25%, $<$ 50%
.20	$\geq$ 50%, $<$ 75%
.30	$\geq$ 75%, $<$100%
.40	$\geq$100%

Thus even cases with contingency reserves may withdraw. Using the convolution approach, this type of assumption can be handled easily by defining a "Vertical Split" operator as follows:

$$[x_i, y_i, z_i, p_i] \xrightarrow{p_i q \to p_i} [x_i, y_i, z_i, p_i q] \qquad \text{for each } i$$

$$[x_i, y_i, z_i, p_i] \xrightarrow{p_i(1 - q) \to p_i} [x_i, y_i, z_i, p_i(1 - q)] \qquad \text{for each } i\,,$$

where q is a given real number. In our particular situation, q would successively be one of the assumed withdrawal probabilities; the Horizontal Split operator, previously defined, would be used to determine to which portion of the trivariate frequency distribution the particular withdrawal probability applies.

General Remarks

1. To focus on certain underwriting results, the author adds interest on deficits as a final step in his calculations. A complete analysis would presumably reflect interest earned on funds accumulated ($\pm$), building in federal income taxes in a manner appropriate for the insuring company, and might tend to produce lower risk charges. In any event, the risk charges would have to be increased for direct percent-of-premium expenses.

2. The author distinguishes among four categories, namely, surplus-surplus, surplus-deficit, deficit-surplus, and deficit-deficit. It is not clear whether the word "deficit" in the first position of these categories refers to a one-year deficit or a cumulative deficit.

3. Risk charges would not necessarily have to be level; that is, risk charges could vary by policy year and be set each year to cover the increases in deficits during that year among active and terminated cases. Theoretically, risk charges could be varied according to several parameters, one of which might be the accumulated deficit under the case.

4. Almost any risk charge can turn out to be inadequate, under sufficiently adverse experience.

5. The author correctly underscores the fact that almost no amount or type of "deficit management" will be adequate to produce profits, unless suitable risk charges are imposed.

6. The present value of profit to the insurer at issue of the group contract is presumably the present value of premiums minus expenses, actual claims, and refunds, taking into account cash in and cash out, together with the probability and incidence thereof. The rate of interest at which such cash flow is discounted can be selected by the insurer and does not necessarily have to be equal to the rate of interest added to the losses to be carried forward. The main thing is for the present value of profit (in dollars or related to the present value of premiums) to be at a level satisfactory to the insurer.

7. It would be interesting to know the form of the random number generator used by the author. Also, when illustrative financial results are based on generating random numbers, it would be helpful to have some idea of the range of error or probable variation which can be expected. The convolution approach includes, as a by-product, some answers to this type of question.

8. In the author's Section III on "Multiyear Analysis," he generates random numbers first to obtain the number of deaths and then to obtain the total claim amounts given the number of claims. The calculation might have been done more efficiently if one random number had been generated for each group for each year, entering the equivalent of my Table 2E to determine the total claims. The formulas in Section II of the Appendix would then be unnecessary, since the formulas in Section I of the Appendix would apply.

9. Making the cumulative deficit a legally collectible debt may defeat the purpose of the insurance in the first place; that is, if the insured company must put up the deficits as a liability, the swings in earnings may not be insulated to the extent desired.

10. The author states that "in practice it is not possible to predict accurately a risk charge, because of the unpredictable nature of case cancellation." Certainly this statement is true for any given group case. However, for portfolios of one or more group cases, as I have indicated, frequency distributions of risk charges can be constructed as a guide in the setting of risk charges. Bayesian or personal probability weights may have to be assigned to different sets of actuarial assumptions (e.g., probabilities of deaths and of case terminations), but risk charges can be determined in ways to give the insurer a reasonable expectation of profits. Of course, competition may be willing to underwrite the business at inadequate rates, in which case the best you can do is avoid losses.

11. Although the assumption of a stationary distribution of total claims from case to case and from year to year might be called into question, presumably a rate/refund structure should be at least adequate to produce a reasonable expectation of a profit (over a selected observation period), under the assumption of a stationary random variable for claims. Further modifications may be required in order to provide for nonrandom effects.

12. The question of variation in the level of risk charge by size of group case has not been addressed by the author, except insofar as he has studied a group of a certain size. It would be of interest to investigate groups of different sizes and compositions, to see the effect on the risk charges.

13. The subject of credibility and the interrelationship among risk charge, credibility, and dividend margin has not been investigated or referred to by the author.

14. A simple type of situation in which premiums are constant from year to year, no interest on loss carry-forwards is involved, and no withdrawals are assumed is rather neatly handled by analytical means in the paper entitled "Expected Value of Dividends," being presented at this meeting by Messrs. Jones and Gerber.

Conclusion

Mr. Bolnick has shown us from first principles that some "solutions" to the "deficit management" problem are not solutions at all. He uses a Monte Carlo technique to calculate some level risk charges, ignoring the effect of interest and expenses, and concludes that the level of risk charges being charged currently may be inadequate. His suggestion that deficits on termination of the group case be treated as a collectible debt may or may not be practical, depending on the needs of the particular insured group.

Mr. Bolnick has reinforced my own conclusion that there is real need for a considerable amount of research in the whole area of group insurance rate/refund structures. Although this paper focuses on group life insurance, the same techniques can be applied to group health, long-term disability income, other group insurance, and reinsurance. This paper should stimulate a vigorous exchange of ideas.

HANS U. GERBER AND DONALD A. JONES:

The author is to be congratulated for his extensive coverage of the subject. Losses carried forward in dividend formulas are reminiscent of negative actuarial reserves in life contingencies, which create caution in the mind of every actuary. One remedy for this situation would be a group life insurance contract that not only defined the premiums, benefits, and dividends but also specified a limited duration (say, 1, 3, or 5 years) for its validity. It appears that the following ideas (in chronological order) would lead to a "sound" contract.

1. Specification of the duration
2. Definition of the benefits
3. Definition of the dividends, as a function of claims (but not as a function of the premiums, which are not known at this stage!)
4. Analysis of benefits, dividends, and other costs (the computation of expected values seems to be a minimum requirement!)
5. Determination of the premiums
6. Optional: an alternative (possibly picturesque) description of the dividends (that were defined in the third step) in terms of premiums, claims, "risk charge," and so on

We believe that Bolnick's paper, with some modifications on technical points, could be a valuable supplement to the Syllabus for the Part 9E candidate who lacks experience in group insurance. We would suggest modifications on the following points:

1. Group health insurers may question the validity of the statement (preceding Table 1) that "because of a lower claim variance, a health portfolio will have less of a financial problem than a life portfolio with the same premium." The statement would be true for portfolios of independent claims, but changes in utilization and fee levels of medical care cause dependence among health contract claims. Hence the law of large numbers is not applicable, and convergence to the mean as the portfolio size increases cannot be inferred.
2. The key idea of a recent paper by John Mereu ("Algorithm for Computing Expected Stop-Loss Claims under a Group Life Contract," *TSA*, XXIV (1972), 311–20) was that the right-hand side of the following identity is a more efficient tool to compute net stop-loss premiums than the left-hand side (i.e., that shown in Bolnick's Appendix I):

$$\sum_{x>SL}(x-SL)f(x) = \sum_{x=0}^{\infty}(x-SL)f(x) - \sum_{x\leq SL}(x-SL)f(x)$$

$$= 63{,}617.48 - SL - \sum_{x\leq SL}(x-SL)f(x)\ .$$

   The use of the right-hand side saves computer time and avoids truncation errors.
3. The extensive calculations of the expected deficit and expected surplus, as displayed in the subsection on the pure accounting method, may mislead some readers. First, the expected deficit (over the \$65,000 premium) is, by definition, the net premium for stop-loss coverage at the \$65,000 level, as given in Table 2 without calculation. Then the expected surplus may be calculated from the identity

$$E[65{,}000 - x] = E[\text{Surplus}] - E[\text{Deficit}]\ .$$

4. The multiyear "simulation" should probably be labeled an illustration, since only one replication has been calculated. Usually "simulation" implies that the results of many replications have been averaged or otherwise analyzed for presentation.
5. Also, the description of the simulation given in the Appendix does not state exactly how the one-year claim amounts were determined. It could have been done by generating one random number and using the cumulative distribution function in Table 2 to determine the total claims. On the other hand, a random number to determine the number of claims and then a set of random numbers to determine the size of each of these claims could have been generated. Either of these procedures would fit the description in the Appendix.

Our comments on the technical points are made as suggestions to increase the value of a paper that appears headed for the Reading List of the Society Examinations.

JAMES E. JEFFERY:

I commend Mr. Bolnick for daring to say what has gone unsaid so often in the past, namely, that many of the experience-rating plans in common use today are probably founded on unsound actuarial principles. This documentation of that fact will confirm the suspicions of many, and hopefully will open the eyes of others who have heretofore been non-believers.

I and my associates at London Life have recently concluded a lengthy examination of group life dividend and premium strategies. Our investigations followed a remarkably similar course to Mr. Bolnick's. We too constructed a computer model designed to simulate realistic application of various experience-rating strategies to particular sample groups and portfolios of groups. We too were forced to make many of the choices faced by Mr. Bolnick in the construction of the model, and the fact that most of our choices agreed, although they were made independently, is a testimony to their propriety.

I had been contemplating writing a paper for the purpose of presenting the results of our investigations, but Mr. Bolnick has kindly relieved me of that task. With the indulgence of Mr. Bolnick and the Editor of the *Transactions*, I will use this opportunity to present a very abridged version of our considerations and conclusions.

Our approach to the investigations was necessarily pragmatic. We knew that we needed a new experience-rating approach. Our previous formula combined a t-year cumulative smoothing formula of the credibility type, with pooling of individual amounts of insurance in excess of a certain limit. It had been regularly criticized from all sides on the following grounds:

1. It permitted accumulation of very substantial deficits, arising both from single disastrous years and from multiple occurrences of marginally poor years.
2. Apparently unjustified rate increases were required when a single unfortunate year early in a group's career adversely affected the cumulative average experience and thus resulted in substandard charges for claims for years to come.
3. Pooling of individual amounts of insurance in meaningful volumes resulted in complete removal of a substantial portion of the premium from the experience-rating process. Moreover, this technique is administratively burdensome.

4. Different time sequences of otherwise identical claim experiences resulted in different total charges for claims.

Because our approach was to be pragmatic rather than theoretical, we did not feel compelled to examine the wide range of strategies examined by Mr. Bolnick. Higher premium margins for claims, withholding of claim fluctuation or contingency reserves, and recovery of prior deficits through retroactive premium adjustments were all rejected at the early stages for competitive reasons. Pooling of individual amounts of insurance and credibility smoothing were rejected, partly for reasons stated above, but more importantly because it was felt that these techniques were less efficient as deficit management techniques for a given dollar of premium removed from the experience-rated portion than some form of over-all stop-loss coverage would be for the same dollar. Intuitively this would appear to be so, because each of these techniques pools some portion of claims in years when no pooling is necessary to prevent a deficit and fails to pool enough claims in some other years.

Our desire for practical results also dictated that we examine many sizes and types of group. Accordingly, an interesting feature of our model was its ability to pick out certificate data of a number of actual live groups from our group master file and automatically simulate claims and apply selected strategies to the entire portfolio carried through a number of years of experience.

Lapses were simulated on the basis of a probability function, applied to each group separately, related to the size of the deficit, the excess of claim charges over actual claims, and a fixed amount related to the size of the group.

We felt that we could not increase our present risk charge, and we knew that we could not increase our premium rates in the marketplace. Given that we had settled on some form of over-all stop-loss pool, the problem became one of finding the over-all stop-loss level which would limit deficits to a manageable level, return a reasonable margin to the insurer, and yet give the policyholder maximum participation in his own financial results.

We did not share Mr. Bolnick's apparent distaste for deficit management techniques in general. We realized that lapse experience worse than expected would compromise the success of our suggested formula. However, we were prepared to take that risk, provided that it could be minimized to a reasonable degree, measured, and provided for in the loadings. We take this risk all the time in individual insurance when we guarantee cash values in excess of the theoretical asset share. In my opinion, this is perfectly proper. It is seldom possible to eliminate this risk completely in a competitive environment.

Consider the following results of an early run of our model. The dividend formula consisted of a variation of Jackson's J-type stop-loss arrangement applied to the current year only. The premium margin for claims and the risk-charge level may be considered as representative of the levels achievable in the Canadian market.

For this run, claims were simulated with a dispersion somewhat wider than that which random chance alone would dictate. This was accomplished by simulating claims for some groups at 85 per cent of the tabular expected level, for some others at 115 per cent and for the balance at 100 per cent, the total simulation to reproduce close to 100.0 per cent of tabular expected. I felt that this feature would introduce a desirable element of conservatism into the results, and it may well be that experienced levels of dispersion about expected are higher than the theoretical. Tests indicated that this wider dispersion produced dividends paid approximately 12.5 per cent higher (for a stop-loss level of 115 per cent) than the dividends which would be paid assuming the theoretical dispersion.

The stop-loss levels shown in the accompanying tabulation are actually maximum charge levels. That is, the sum of the charge for claims plus the risk charge is not allowed to exceed the stated maximum in any policy year. This run assumed that premium levels for each group, and with them the corresponding stop-loss levels, would be adjusted each year by a credibility formula granting full credence to an exposure with 196 expected claims. Assumptions were the following:

1. Number of different groups (i.e., sets of certificate data)—10
2. Size range of groups—smallest 231 lives, largest 4,002 lives
3. Number of ten-year histories of each group—ranging from 5 to 50
4. Total number of policy years—2,150
5. Total number of life-years—1,900,100
6. Total number of claims—8,646
7. Premium margin for claims—5 per cent of tabular expected claims
8. Risk-charge level—$1\frac{1}{2}$ per cent of tabular expected claims
9. Interest rate for deficit accumulation—6 per cent
10. Actual total claims simulated—101.2 per cent of tabular expected

Stop-Loss Level*	Cash Dividends Paid*	Average Deficits O/S*	Lapses (% by Number)	Deficits Written Off*	Deficits O/S at End of Study Period*
145%......	8.55	18.38	8.79	3.90	2.43
135%......	7.66	14.62	8.28	2.98	1.96
125%......	6.39	11.11	6.98	1.70	1.53
115%......	4.78	6.47	5.49	0.55	0.91
105%......	3.62	0.00	4.74	0.00	0.00

* Figures quoted as per cent of tabular expected claims for total study period.

The most notable feature of these results is that, for any stop-loss level higher than about 117 per cent of expected claims, the cash dividends paid would exceed the premium margin for claims. It is my contention that this total result is reasonably representative of a mature portfolio, since it contains about 15 per cent first policy years, 13.5 per cent second policy years, . . . , and about 5 per cent tenth policy years. Accordingly, an insurer who now has a mature portfolio and who expects to continue to write new business at or around 15 per cent of his in-force must expect to continue to lose money if he chooses a stop-loss level higher than 117 per cent (under the assumptions specified).

An actuary who is an eternal optimist might argue that a stop-loss level as high as 145 per cent is acceptable because the sum of deficits written off (3.90 per cent) plus anticipated nonrecoveries on deficits outstanding at the end of the study period (for argument, say 2.43 per cent $\times$ 3.90 $\div$ 18.38) does not exceed the premium margin for claims. I believe that this argument fails, except in the case in which the insurer is expected to eventually stop writing new business and yet hold on to its remaining business long enough to recover most existing deficits. Nevertheless, I will concede that the argument presented in the paragraph above is somewhat conservative in its implied assumption that no part of deficits outstanding at the end of ten policy years will ever be recovered.

As for the suggestion that competitive pressures rule out a J-type stop-loss arrangement which has a very low stop-loss level because policyholders will not accept the consequent high level of reduction in actual surplus returnable as a dividend, we concluded that the suggestion had insufficient merit for two reasons. In the first place, we were able to demonstrate that, compared with our former credibility smoothing formula, the proposed formula was as sensitive or more sensitive to a single year's favorable experience for cases of 1,000 lives or less, and only slightly less sensitive to a longer run of favorable results. For larger cases, no pooling formula could compete with the existing formula, because such cases received very high or even complete credibility on current results under that formula. In the second place, we could argue with considerable justification that any policyholder who would not accept such a reduction in actual surplus was not worth having as a policyholder because the risk of loss under a more generous formula would be too great. In effect, we concluded that the proposed levels of reduction in actual surplus would be more salable than an increase in the premium margin for claims.

I will close by indicating my agreement with many of the findings of Mr. Bolnick, with the exception of those already stated. In addition, I solicit his comments with respect to the following points:

1. Why were credibility smoothing and other averaging formulas of one kind and another essentially ignored in the investigations?
2. What about my suggestion that the dispersion of actual claim results about the expected may be wider than the theoretical? Should your results be modified accordingly before practical application?
3. I question whether comparisons of different pooling formulas assuming different levels of total premium are entirely fair. I suggest that the premium margin for claims should be treated as a factor independent of the particular pooling method employed when comparing one method with another. For example, Jackson's J-method and K-method can both be applied to the same situation with no increase in the total premium in either case. As a second example, consider pooling of individual amounts of insurance with no increase in total premium. Conversely, the comparison between the J-method and others can be made fairer by increasing the total premium to the same level under all methods.

I wish to thank Mr. Ivan R. Taylor and Mr. John A. Mereu, who were instrumental in the conception and construction of the model and the direction of our investigations.

(AUTHOR'S REVIEW OF DISCUSSION)

HOWARD J. BOLNICK:

I would like to thank the eight discussants of the paper for their time and effort in preparing written statements. The diversity of opinion presented in the discussions provides a welcome addition to the paper.

In reviewing the eight discussions, two general areas of comment arise in more than one discussion: comment on the concept of underwriting deficits, and support for the risk charge/deficit carry-forward scheme of experience rating. Since an understanding of these two areas is basic to an understanding of the paper, an answer to these general comments will precede my reply to the specific points offered by each discussant.

Underwriting profit is a specific, well-defined subset of the statutory or GAAP statement profit for group life and health insurance. The underwriting loss from inception of a group portfolio is simply the sum of all deficits being carried forward on active cases and all deficits that have become unrecoverable through cases canceling in a deficit position.

Section IV of the paper, "Sources of Statutory Profit," calculates the *operating* gain or loss from experience-rated business, G, as

$$G = (\Delta V^c - \Delta V^s) + (E^A - E^S) + (I - FIT) - (E^A - E^E)_{SD,DD+\Delta D} \,.$$

The ΔD term is the annual *underwriting* gain or loss and is merely the change in deficits being carried forward on active cases and deficits lost on canceled cases. The underwriting profit is therefore only one factor in G, the operating gain or loss from experience-rated business. A negative ΔD does not necessarily imply a negative G. Further, an annual statement gain for the group line of business will not only include G as a source of profit but will also include the gain or loss from non-experience-rated pools operated in conjunction with experience-rated business (e.g., a stop-loss pool), the gain or loss from non-experience-rated insurance, and the gain or loss from prospective experience-rated insurance coverages issued by the insurer.

My statement that combination prospective and retrospective experience-rating plans result in long-term underwriting losses to the insurer merely means that the plan will have deficits being carried forward and cases terminating in a deficit position. These active case and canceled case deficits are described and demonstrated by the model in the paper. The paper also demonstrates that no amount of deficit management short of operating a stop-loss pool at the premium margin for claims or charging a full retroactive premium will eliminate the underwriting deficit. To produce a consistent gain on the annual statement, the insurer must devise adequate and equitable deficit recovery techniques to offset the inevitable underwriting deficit.

The spectrum of deficit recovery techniques varies from self-insurance (administrative services only or a full retroactive premium each year) to pure insurance (a prospective experience-rating plan or a retrospective experience-rating plan with a stop-loss pool at the premium margin for claims). Limited only by the imagination of the actuary, various experience-rating plans can be developed at the extremes or within the spectrum from self-insurance to pure insurance.

The risk-charge approach to deficit recovery lies midway between self-insurance and pure insurance on the spectrum. An element of self-insurance is involved, since a risk charge is calculated assuming that an active case deficit will be carried forward and repaid from subsequent surpluses on the case through reduced experience refunds. The recovery of canceled case deficits through an additional charge to all cases in the portfolio involves some aspects of an insurance scheme. For this reason, I think it is fair to say that the risk-charge approach to deficit recovery is a compromise developed by insurance companies to retain elements of insurance while allowing elements of self-insurance to be incorporated into an experience-rating plan.

Chart I of this discussion compares the range of deficit recovery techniques discussed in the paper. The most important point developed in the chart is that any deficit recovery scheme must pay all claims in excess

CHART I

Analysis of Deficit Recovery Techniques

	Self-insured ←—— Spectrum of Techniques ——→ Fully Insured			
	ASO or Full Retroactive Premium	Canceled Case Termination Charge	Risk Charge	Stop-Loss Pool at Premium Margin for Claims
Extra premium received annually from each case	N.A.	N.A.	Charge for canceled case deficits	Charge for all claims in excess of premiums for claims in the entire portfolio
Extra charge from each case upon occurrence of specific event	Charge for all claims in excess of premiums in year claims incurred	Charge for all unpaid claims upon termination Reduction in experience refund when surplus is run while case is in a deficit position	Reduction in experience refunds when surplus is run while case is in a deficit position	N.A.
Total claims paid in excess of premium for claims	All claims in excess of premium	All claims in excess of premium	All claims in excess of premium	All claims in excess of premium
Insurer's risk in scheme	None	Carrying active case deficits	Carrying active case deficits Prediction of canceled case deficits	Prediction of the tail of a claim distribution
Accounting problems with scheme	None	Changes in active deficits create gains and losses	Changes in active and canceled case deficits create gains and losses	None

of the premium in order for the insurer to earn a real long-term operating gain on the experience-rated line of business. An analysis of Situation A of the paper will emphasize this point.

Situation A has an annual premium of $65,000 and uses the following deficit management techniques: (*a*) a contingency reserve is built from surpluses, and (*b*) an over-all and an individual stop-loss pool is operated by the insurer. Section II of the paper, on "Single-Year Analysis," shows that a case with a $65,000 premium for claims can expect a deficit of $14,192.99 or a surplus of $15,575.51. Using these expectations and other expected data (with the exception of canceled case deficits and the number of case-years of experience, 3,488, for which we must use actual data from the multiyear simulation) an analysis of the sources of claim payment can be made (see the tabulation on page 206).

In each scheme all of the $49,505,149.12 in deficits is recovered and all of the $54,327,378.88 in surpluses is disposed. The only question is *who* pays for deficits—the case incurring the deficit or the portfolio as a whole? —and *how* are surpluses deposed of—are they returned to the policyholder who incurs them, or are they used to recover prior deficits?

The job of the group actuary is to develop an experience rating formula which (1) is based on sound principles, (2) adequately recovers all paid claims, and (3) equitably apportions the payment of all claims. It is my contention that a risk charge/deficit carry-forward scheme does not fulfill the "sound principle" requirement.

To support my contention, let us examine two policyholders' views of a risk-charge scheme. A group insurance carrier has explained to Corporation X and Corporation Y exactly how a risk charge/deficit carry-forward scheme operates—especially that the risk charge involves the pooling of canceled case deficits. Corporation X's response to the presentation is disappointment, since he expects to maintain a lasting relationship with the insurer and accepts the fact that he must *eventually* pay *all* his own claims. Corporation X cannot condone being charged for someone else's perfidy and suggests to the group insurer that a self-insurance scheme would be most suitable to the corporation's needs. By agreeing to pay all claims through a full retroactive premium or by agreeing to any of a multitude of financially sound loan arrangements, Corporation X can save money by not participating in the pooling of canceled case deficits.

Corporation Y, on the other hand, feels that changing economic conditions may force him to cancel his group contract, or, worse yet, the insurance manager may realize that it is advantageous for Corporation Y to switch insurers when a deficit occurs. For either reason, Corporation Y

ANALYSIS OF SITUATION A

$$E[\text{deficit}] = \$14{,}192.99 \times 3{,}488 = \$49{,}505{,}149.12$$
$$E[\text{surplus}] = \$15{,}575.51 \times 3{,}488 = \$54{,}327{,}378.88$$

Risk charge deficit carry-forward scheme	
Recovery of E[deficits]	
1. Stop-loss pools, \$6,942.58 × 3,488	\$24,215,719.04
2. Canceled deficits paid through a risk charge	4,910,000.00
3. Recovered from surpluses	20,379,430.08
	\$49,505,149.12
Disposition of E[surplus]	
1. Paid experience refunds	\$33,947,948.80
2. Payment of prior deficits	20,379,430.08
	\$54,327,378.88
Pure insurance scheme	
Recovery of E[deficits]	
1. Stop-loss pool, \$14,192.99 × 3,488	\$49,505,149.12
2. Canceled deficits	0
3. Recovered from surpluses	0
	\$49,505,149.12
Disposition of E[surplus]	
1. Paid experience refunds	\$54,327,378.88
2. Payment of prior deficits	0
	\$54,327,378.88
*Self-insurance scheme**	
Recovery of E[deficits]	
1. Stop-loss pool	\$ 0
2. Canceled deficits	0
3. Retroactive premium—additional charge	49,505,149.12
	\$49,505,149.12
Disposition of E[surplus]	
1. Paid experience refunds	\$54,327,378.88

* Full retroactive premium for each policy year, including year of cancellation. A sound self-insurance scheme can allow for deficit carry-forward arrangements if proper care is taken contractually to treat deficits as loans.

chooses to accept the risk-charge contract. With this knowledge of the motivation of Corporation Y and other knowledgable companies participating in the canceled deficit pool, the insurer can make only one truly sound prediction of canceled case deficits—that whenever a company that chooses to participate in the pool runs a deficit, the contract will be canceled. This implies that each year the expected deficits will be assumed to coincide with the canceled deficits. The proper charge to each case to compensate the insurer adequately for this risk is exactly the premium for a stop-loss pool at the premium margin for claims.

From this exercise, I will postulate that there are only two types of group insurance markets: those clients who fully accept the risks and rewards of self-insurance and those clients who want or need pure insurance, that is, a mutually satisfactory pooling of the risks and rewards. The reason these two classes and only these two classes have not become apparent is because most large policyholders do not understand the real nature of the risk-charge concept and because insurers have not pursued adequately the sale of experience-rating plans that emphasize insurance or that emphasize self-insurance.

Despite the compelling logic of the above exercise, not all cases with an active deficit do cancel. This fact results in actual risk charges used by insurers being less than the charge for a stop-loss pool. The basis of an adequate risk charge of less than a stop-loss charge is the prediction of canceled case deficits. There are two fundamental barriers to the use of sound statistical procedures to develop a risk charge.

1. *Cancellation of a case in a deficit position is a zero-sum game.* As pointed out in the paper, a new insurer will not require the repayment of an existing deficit as does the current insurer. Both insurers will require an adequate payment for expected future claims. When the policyholder cancels, the policyholder's gain is equal to the insurer's loss—a zero-sum game.
2. *Normally, group contracts do not limit the right of policyholders in a deficit position to cancel.*

Neither characteristic of a risk-charge scheme is in itself dangerous. Together, however, they do not result in an event which can be predicted by normal statistical methods.

Insurance is based on the ability of a statistical methodology to predict probabilities of the occurrence of an event. It is a fundamental principle that the event to be predicted must not be controlled to any significant degree by the insured. This principle can be amply demonstrated in daily practice: insurers do not issue life insurance or health insurance to individuals who know they are ill; insurers insist on suicide clauses in life insurance; insurers do not issue health insurance policies to individuals who can obtain free medical care; insurers limit the amount of disability income payable to an individual so that it does not become profitable to stay disabled. Despite these sound insurance examples, group insurers do issue contracts to policyholders while expecting deficits to occur and further allow the policyholder to completely control the outcome of the occurrence of that event. I must therefore conclude that case cancellations cannot be adequately predicted and that risk charges based on such predictions cannot be statistically meaningful.

A sound experience-rating plan should therefore avoid basing the profitability of a group portfolio on the ability of the actuary to develop an adequate risk charge. The actuary's imagination and skills should be directed instead toward developing experience-rating plans based on the sound principles presented in the paper.

With these general comments in mind, I will respond to the specific comments of each discussant.

Mr. Margolin's discussion questions whether or not a well-behaved mathematical model can represent adequately an actual group life portfolio. The point of departure for Mr. Margolin's criticism is that the annual statements of the ten largest group carriers from 1969 through 1973 do not seem to support the existence of long-term underwriting deficits on combination prospective and retrospective experience-rated plans. The general comments made above on this point draw a clear distinction between underwriting losses and statement losses. The link between the concepts is the efficacy of deficit recovery techniques. By itself, however, Mr. Margolin's statement has no bearing on the accuracy of the model. Properly analyzed, the statistics of the ten largest group carriers would verify the existence of long-term *underwriting* losses.

Advancing from this erroneous beginning, Mr. Margolin asks for a demonstration of the faithfulness of the compound Poisson distribution function. To establish a solid empirical link, I refer the reader to Mr. Arvanitis' discussion of Mr. Margolin's paper, cited in the discussion. To quote: "In studying the mean and standard deviation by size of group . . . for group life coverage . . . the rate at which the standard deviation decreases with increase in size of group seems to be reasonably consistent with what might be expected *if chance fluctuations were the major contributing factor*" (emphasis added). Discussions following Paul Jackson's paper "Experience Rating" (*TSA*, Vol. V) and Ralph Keffer's paper "An Experience Rating Formula" (*TASA*, Vol. XXX) support the link as Mr. Arvanitis does. These prior discussions provide empirical data supporting the contention that group life insurance can be represented adequately by a well-behaved mathematical model.

That there are difficult problems using a risk-theory model for group health insurance is beyond question. The unstable character of the actual mean and standard deviation of a health case results in larger deficits than would be expected by a well-behaved mathematical model. Deficits larger than expected imply larger underwriting losses, larger losses on canceled cases, and larger active case deficits. In addition, instability of the mean and standard deviation implies that accurate prediction of the losses of a combination experience-rating plan for health insurance is almost impossible. These facts make the adequacy of any group health risk charge suspect. Thus a logical extension of Mr. Margolin's criticism only reinforces my theoretical conclusions and my practical aversion to an experience-rating plan using a risk charge. The use of a well-behaved mathematical model for group health insurance, however, does provide useful information concerning minimum reasonable levels of risk charges and stop-loss pools.

The apparent validity of a random model for group health insurance and the apparent lack of validity of a simple random model for group health insurance bring up a basic question of the usefulness of statistical models. This is a question that has not been explored adequately in actuarial literature. The validity of random models rests on the fulfillment of one of two conditions for the variables in question. A strong condition to be met is that the model dissects reality to the point where an irreducible randomness exists in the world. A weak condition will be met when a random model can be used as proxy for an extensive investigation of actual causes of the occurrence of an event. Further investigation of these two conditions would require a philosophical dissertation which is beyond the scope of this paper.

Mr. Margolin's second point, concerning the small premium margin is also inaccurate. I refer the reader to Mr. Garrison's calculation of the margins in his discussion and my own calculation in the reply to Mr. Garrison.

Third, the use of relatively few experience-rating formulas was dictated by the limitations of a written document. To enable the paper to provide a conceptual framework which can be used as a standard for judging all types of dividend formulas, the use of familiar, readily available methods such as the "pure accounting" method and the "modified pure accounting" method is necessary. There exist literally thousands of possible sound alternatives. "Right" answers should be avoided in this situation. "Wrong" answers and the framework for the actuary to develop sound alternatives should be emphasized.

Mr. Schreiner, like Mr. Margolin, confuses underwriting profit with operating profit by not making a distinction between deficit management and deficit recovery. Some insights into Mr. Schreiner's comments can be gained by analyzing the example used in his discussion. The premium for his scheme can be broken down as follows:

Experience-rated premium	\$65,000.00
Stop-loss premium at \$75,000	10,350.76
Risk charge	3,842.23
	\$79,192.99

This set of premiums will result in a sound insurance scheme. The risk charge premium of \$3,842.23 is sound, since it requires the recovery of the expected underwriting deficit each year without assuming any recovery of deficits through future surpluses. The implicit assumption embodied in such a charge is that each deficit case cancels at year end. The universal cancellation assumption reduces the risk charge to a stop-loss pool, in Mr. Schreiner's case a stop-loss pool for claims between the \$65,000 and the \$75,000 stop-loss levels. The risk charge plus the \$75,000 stop-loss premium combined constitute a stop-loss pool at the premium margin for claims. Such a risk charge/stop-loss combination is sound specifically because it eliminates the withdrawal antiselection problem by assuming universal termination of deficit cases.

Since the "risk charge" purchases insurance coverage for all deficits in excess of premium each year, it is inappropriate to carry forward deficits. To do so double-charges the policyholder.

Allowing for deficit carry-forward and ignoring the risk charge, the scheme presented will incur a long-term *underwriting* deficit. Using the same claim data as in the paper, a \$65,000 annual premium, and \$75,000 stop-loss pooling level (no contingency reserves, no cancellation), the following results are obtained over a fifty-year period:

Premiums............	\$325,000,000
Claims..............	266,375,000
Experience refunds....	59,525,000
Underwriting loss.....	(\$ 900,000)*

* At the end of the fifty-year period there are fifty-seven active case deficits with an aggregate total of \$900,000.

Just as with other situations in which the stop-loss level does not equal the premium margin for claims, long-term underwriting losses are incurred. Mr. Schreiner's deficit recovery scheme (i.e., premiums of \$3,842.23 × 5,000 case-years or \$19,211,150 for a new stop-loss pool between the \$65,000 premium and the \$75,000 stop-loss level) overly compensates the insurer for the underwriting deficit unless active deficits are forgiven and not carried forward.

Mr. Schreiner's criticism is particularly puzzling after review of his Part 9 Study Note. At the bottom of page 15 of the Study Note he advances the following equation as one embodying sound principles:

$$\int_0^P [1 - g(x)](P - x)f(x)dx > \int_P^\infty (x - P)f(x)dx ,$$

where P is the net premium margin available for claims and dividends (premium for claims), $f(x)$ is the distribution function of claim experience, and $g(x)$ is a dividend distribution function operating on the gain before dividends. The basic principle of this equation is that all claims in excess of the premium margin for claims, P, must be recovered by reducing experience refunds. This scheme is exactly the same as paying for a stop-loss pool at the premium margin for claims by reducing experience refunds.

I am forced to conclude that both Mr. Schreiner's example and his Study Note agree with my assertion that a sound *insurance* scheme must solve the withdrawal antiselection problem and does so by developing a stop-loss pool at the premium margin for claims.

Mr. Garrison makes the same error as the previous discussants by confusing underwriting losses with statement losses. It is a fact, amply demonstrated by the model, that a combination experience-rating plan will result in underwriting losses to the insurer over an extended period of time unless the insurer uses the deficit management technique of either forming a stop-loss pool at the premium margin for claims or receiving a full retroactive premium. In lieu of using these deficit management techniques, deficit recovery techniques must be used to recover the underwriting deficit for the insurer to have long-term operating gains for the group experience-rated line of business.

The majority of Mr. Garrison's introductory comments and criticisms are repeated in his comments. For reading ease I will comment on each of Mr. Garrison's comments in his numerical order.

1. A prospective experience-rating plan must include some margin for adverse claim fluctuation and for profit to the insurer. Since there are no stop-loss pools in a true prospective plan, the margin must be added to the experience-rated premium.
2. Mr. Garrison's calculation of the margin for fluctuation in each situation is appreciated. Three notes of caution must be introduced, however. First, the margin for fluctuation for those cases with contingency reserves should be increased, since holding a contingency reserve has the same effect on future deficits as increasing the premium margin for claims. The result of adding contingency reserves to the premium margin for claims will be that the calculated margin for Situations IV and A–F in Mr. Garrison's table will be the minimum effective margins. To find the maximum margin, 35 per cent must be added to the minimum (35 per cent is the maximum \$20,000 contingency reserve divided by \$56,675 in expected experience-rated claims). Second, the margin for those situations with a 20 per cent recovery of prior deficit will vary significantly depending on the size of the deficit at the beginning of a policy year. Mr. Garrison's 10 per cent figure is an accurate average margin according to my calculations. For example, the deficit recovery margin for Situation D should be: expected experience-rated premium = \$56,675; expected deficit given that one occurs = \$25,884.70; margin = 0.2 × \$25,884.70/\$56,675.00 = 9.13 per cent. This calculation is based on the claim frequency distribution from Table 2, adjusted for the \$30,000 individual stop-loss pool. Mr. Bailey presents a

	Situations					
	A	B	C	D	E	F
1. Case-years of experience	3,488	1,936	2,591	3,377	3,578	2,835
2. Average annual premium	$65,000.00	$55,322.83	$58,329.60	$59,833.28	$62,095.58	$62,857.50
3. Per cent case-years with contingency reserve	56.8 %	22.0 %	31.0 %	35.8 %	57.8 %	48.4 %
4. Average contingency reserve	$10,598.48	$ 4,894.37	$ 5,715.70	$ 5,536.36	$ 6,044.93	$ 9,407.73
5. Average dollars available to pay claims (weighted by % of case-years with contingency reserve)	$71,019.94	$56,399.59	$60,101.47	$61,815.30	$65,589.55	$67,410.84
6. Average claims	$56,209.86	$56,849.17	$56,590.12	$56,471.72	$56,416.99	$56,550.39
7. Average renewal margin (including contingency reserve)	26.3 %	(0.8 %)	6.2 %	9.5 %	16.3 %	19.2 %

table of cumulative frequencies for this situation in his discussion. Third, Mr. Garrison's figures are based on expected margins using the claim distribution of Table 2. His risk charges, however, are based on the actual claims and actual premiums from the simulation program. This approach nevertheless does not change the analysis significantly from an analysis based solely on actual claim data.

In evaluating the trade-off between the risk charge and the premium margin, the information on actual margins shown in the table on page 212 supplements Mr. Garrison's calculations. Using the average renewal margin including contingency reserves, the comparison of risk charge with premium margin becomes the following:

Situation	Risk Charge	Average Renewal Premium	Average Renewal Margin
B........	7.30%	$56,399.59	(0.8%)
C........	4.30	60,101.47	6.2
F........	3.19	67,410.84	19.2
D........	2.53	61,815.30	9.5
A........	2.17	71,019.94	26.3
E........	2.08	65,589.55	16.3

The average renewal margin behaves in relation to risk charge as expected, with the exception of Situations A and F. The explanation for this discrepancy is simply that Situations B, C, D, and E have premiums based on a single-year experience formula, while Situations A and F have premiums based on a multiyear or fixed premium formula. The source of the discrepancy can be demonstrated by comparing Situations E and F, which utilize the same deficit management techniques but different premium formulas:

	Premium Formula	Average Renewal Premium	Risk Charge	Average Renewal Margin
Situation E.......	Single-year	$65,589.55	2.08%	16.3%
Situation F.......	*n*-year	67,410.84	3.19	19.2

For the same claim data a fixed premium or a stable premium plan will result in cancellation occurring as the result of a number of small recurring deficits. A single-year premium formula, which does not have a built-in premium lag behind a slowly rising claim level, requires much larger single deficits to reach the cancellation point. Because of the different premium formulas, the results expected by Mr. Garrison can be obtained by comparing Situations B, C, D, and E separate from the comparison of Situations A and F.

3. The cancellation assumption used in the paper is admittedly arbitrary and

not realistic. It was conceived for illustrative purposes only. Any actuary who finds himself in a position of having to calculate risk charges, despite the theoretical weakness of the approach, should use a model such as the one presented by Mr. Garrison. The upper limit on termination rate, however, probably should be 1 per cent rather than Mr. Garrison's 40–50 per cent.

4. The cash flow from a portfolio of group business will differ from the results of the model in the paper. Two assumptions are inherent in the model:
   a) Premiums are received only as needed throughout the policy year.
   b) Claims are paid in full when incurred.

   Leads in the payment of premiums before they are needed and lags in the payment of claims after they are incurred result in a cash flow different from that used in the model. Premium leads and claim lags also result in paper liabilities on the annual statement (which is, by law, on an incurred basis). The model accurately depicts the incurred basis of accounting. Investment of the insurer's general funds is on an incurred accounting basis, so that an underwriting deficit from the model will require an annual statement general funds investment.

   Large lags in claim payments may create a positive cash flow despite actual losses on an incurred basis. A prime example of this is long-term disability coverage where the incurred claims far exceed paid claims. To the extent that a positive cash flow is generated on a group portfolio, real assets, which earn interest, are also generated. However, there is no assurance that a positive cash flow will develop on a group portfolio. To create it would require
   a) Products with a long claim lag.
   b) Products with a small claim variance.

   As Mr. Angle notes in his discussion, it is quite difficult for the pricing actuary to predict whether or not a positive cash flow will occur and how much interest will be earned on the assets created by a positive cash flow.

5. I agree with Mr. Garrison that active case deficits create accounting losses which are in a sense akin to unrealized capital losses. Contrary to the treatment of unrealized capital losses, which do not flow through the income statement, active deficits do flow through the income statement. The fact that active deficits become "realized" at the option of the policyholder certainly enters into consideration of this difference. But it is also apparent that fluctuation in active deficits can cause gross inaccuracies in the insurer's annual statement.

   A proper risk charge should not result in double-charging the policyholder. Mr. Garrison is correct in saying that a risk charge that recovers active deficits without forgiving the already recovered active deficits commits this error.

6. The introductory comments to my reply make a distinction between deficit management and deficit recovery techniques. The risk charge is clearly a deficit recovery technique.

7. The division of deficit management techniques into those of a self-insurance

nature and those of an insurance nature is an important observation, for which I thank Mr. Garrison.

8. It is interesting to note that, although Mr. Garrison agrees "that policy terminations are the result of deliberate decisions by the policyholders and are *not chance occurrences subject to the laws of probability*" (emphasis added), he insists on the validity of the risk-charge approach to group insurance. In fact, he insists that a proper risk charge can be developed using probabilities of withdrawal based on the relative size of the active deficit. Such a contradictory attitude seems inappropriate in light of the fact that there are sound alternatives available that make no change in the total amount of claims paid to the policyholder and which, properly presented, can be marketable.
9. The implications of the inability of a mathematical model to predict the behavior of a group health insurance portfolio is discussed in my reply to Mr. Margolin. As for the complement of life margins and health margins, I must be convinced that such a neat arrangement occurs more often than would be predicted by chance. In the absence of any evidence, my opinion is that a group portfolio of life and health coverages is no easier to manage than a pure portfolio of either.
10. I am aware of numerous insurers who offer stop-loss coverage as a part of their experience-rating package. One prime example is the experience-rating formula proposed by Mr. Jeffery in his discussion. Stop-loss coverage on life insurance is becoming fairly common. Stop-loss coverage on health insurance, however, is not readily available.

    There is little question that the determination of adequate stop-loss premiums is a difficult actuarial task. There are many well-behaved mathematical models that can be used to predict claims and new calculation techniques using these models (see John Mereu, "Algorithm for Computing Expected Stop-Loss Claims under a Group Life Contract," *TSA*, XXIV, 311). It is curious that Mr. Garrison should shy away from calculating stop-loss premiums, which are based on sound insurance principles, while advocating the calculation of risk charges which he admits are not based on sound insurance principles.
11. Whether or not the retrospective experience-rating plan of Situation VII is acceptable in the marketplace today, it is (*a*) a sound experience rating plan and (*b*) the retrospective experience-rating plan originally used by participating insurers.
12. Mr. Garrison's list of considerations needs no comment.
13. Perhaps most employers appreciate the risk-charge approach to group insurance because they do not understand the product or the alternatives. Employers buy what is commonly offered in the marketplace. Mr. Garrison's argument ought not to be used to perpetuate a scheme which is not based on sound principles.

Mr. Angle points out an apparent overstatement in my criticism of techniques used by insurance companies in managing their group insurance portfolios. To the extent that I have inadequate knowledge of the dividend formulas and experience-rating techniques utilized by other insurers, I stand corrected.

A general criticism which is appropriate, however, is reliance on a risk-charge deficit carry-forward scheme which allows substantial deficits to be carried and exposes the insurer to the risk of termination. Without implying that all insurance companies place their owners in jeopardy by using this scheme, I will caution those insurers who feel that doing so is sound. An in-depth analysis of the consequences of such a scheme is one purpose of the paper.

Mr. Angle cites Mr. Pike's article on "Gain and Loss Analysis and Related Concepts for Group Insurance." Mr. Pike discusses the same accounting problems as does the paper. It is true that interest earnings on claim reserves and premium reserves may offset interest losses on deficits carried forward, but the argument that this justifies ignoring foregone interest on the deficit does not necessarily follow. Interest earned on claim reserves and premium reserves is a source of profit common to all group coverages. The interest earnings on reserves would exist even if deficit were not carried forward. The loss of interest income because of deficit carry-forwards is therefore a loss that occurs only under specific types of contracts, and this loss should be analyzed independent of interest earnings on reserves. Without an independent analysis, the insurer implicitly would be accepting less profit on experience-rated business than on non-experience-rated business.

I would like to thank Mr. Angle for presenting these important points. Perhaps an airing of possible alternatives to a risk-charge approach to group business will result in putting the reality of insurance rather than just the appearance of insurance back into group insurance.

I would like to thank Mr. Porto for his well-thought-out discussion. Two points should be mentioned concerning the margins contained in the situations of the paper. First, the actual margins for Situations A–F are calculated in my reply to point 2 of Mr. Garrison's discussion. The margins for Situations I–VII are also substantial, with the exception of Situation I, which has a $65,000 premium and $63,617.50 in expected claims. Second, although the 1971 Standard Group Life Insurance premiums do include substantial margins, insurers often use the life margins as an excuse to reduce the health margins. The effective margin for life insurance is often substantially reduced through this technique.

With the above comments concerning margins in mind, Mr. Porto's Situations 1–6 and a–l (with the exception of Situations d, h, and l) will result in the summary of operations shown in Tables 1 and 2 of this

TABLE 1

FIFTY-YEAR SUMMARY OF OPERATIONS: NO-CANCELLATION SITUATIONS

	SITUATION					
	1	2	3	4	5	6
1. Experience-rated premiums	$425,000,000	$425,000,000	$375,000,000	$375,000,000	$375,000,000	$375,000,000
2. Claims incurred	$283,445,000	$283,445,000	$318,150,000	$296,580,000	$823,445,000	$283,445,000
3. Experience refunds	$141,905,000	$140,235,000	$ 60,910,000	$ 79,755,000	$ 92,430,000	$ 91,095,000
4. No. experience refunds paid	3,721	3,716	1,959	2,554	2,964	3,001
5. Contingency reserves held*	$ 0†	$ 1,370,000	$ 0	$ 0	$ 0	$ 930,000
6. No. contingency reserves held*	0	93	0	0	0	75
7. Active case deficit*	$ 350,000	$ 50,000	$ 4,060,000	$ 1,335,000	$ 875,000	$ 470,000
8. No. active case deficits*	26	7	62	46	36	25
9. Canceled case deficits	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
10. No. canceled case deficits	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
11. Fifty-year underwriting gain (loss)	($ 350,000)	($ 50,000)	($ 4,060,000)	($ 1,335,000)	($ 875,000)	($ 470,000)
12. Individual stop-loss premiums‡	$ 22,525,000	$ 22,525,000	N.A.	N.A.	$ 22,525,000	$ 22,525,000
13. Individual stop-loss claims‡	$ 22,660,000	$ 22,660,000	N.A.	N.A.	$ 22,660,000	$ 22,660,000
14. Overall stop-loss premiums‡	$ 12,187,900	$ 12,187,900	N.A.	$ 22,660,000	$ 12,187,900	$ 12,187,900
15. Overall stop-loss claims‡	$ 12,045,000	$ 12,045,000	N.A.	$ 21,570,000	$ 12,045,000	$ 12,045,000
16. Fifty-year stop-loss gain (loss)‡	$ 7,900	$ 7,900	N.A.	$ 1,090,000	$ 7,900	$ 7,900
17. No. years of statutory gains	22	25	19	23	26	25

* At the end of the fifty-year period.

† Each year has a $0 statutory profit.

‡ Stop-loss experience is not included in lines other than those marked.

TABLE 2

Fifty-Year Summary of Operations: Cancellation Situations

	Situation								
	a	b	c	e	f	g	i	j	k
1. Experience-rated premiums	$425,000,000	$344,884,920	$351,837,128	$359,625,000	$281,376,018	$292,716,072	$287,630,000	$237,953,454	$270,717,656
2. Claim incurred	$283,445,000	$253,895,000	$253,895,000	$272,155,000	$236,055,000	$237,700,000	$232,615,000	$215,160,000	$233,568,000
3. Experience refunds paid	$140,235,000	$ 93,029,923	$ 99,915,623	$ 87,805,000	$ 48,772,449	$ 58,174,355	$ 57,700,000	$ 27,767,457	$ 40,821,490
4. No. experience refunds paid	3,716	2,555	2,636	2,896	1,801	1,988	2,128	1,261	1,638
5. Contingency reserves held*	$ 1,370,000	$ 454,698	$ 479,606	$ 850,000	$ 224,999	$ 258,000	$ 520,000	$ 83,499	$ 173,579
6. No. contingency reserves held*	93	52	54	68	29	31	45	15	27
7. Active case deficit*	$ 50,000	$ 711,701	$ 670,001	$ 465,000	$ 1,087,230	$ 907,683	$ 445,000	$ 1,211,502	$ 1,145,313
8. No. active case deficits*	7	26	24	24	39	38	23	37	40
9. Canceled case deficits	$ 0	$ 1,783,000	$ 1,783,100	$ 720,000	$ 2,589,200	$ 2,508,600	$ 2,760,000	$ 3,846,000	$ 2,700,000
10. No. canceled case deficits	0	22	22	8	32	31	32	48	33
11. Fifty-year underwriting gain (loss)	($ 50,000)	($ 2,494,701)	($ 2,453,101)	($ 1,185,000)	($ 3,676,430)	($ 3,416,283)	($ 3,205,000)	($ 5,057,502)	($ 3,845,413)
12. Individual stop-loss premiums†	$ 22,525,000	$ 20,128,340	$ 20,128,340	$ 21,601,475	$ 18,731,790	$ 18,866,940	$ 18,511,045	$ 17,033,405	$ 18,533,570
13. Individual stop-loss claims†	$ 22,660,000	$ 20,490,000	$ 20,490,000	$ 21,680,000	$ 18,960,000	$ 19,100,000	$ 18,620,000	$ 17,250,000	$ 18,740,000
14. Over-all stop-loss premiums†	$ 12,187,900	$ 10,891,107	$ 10,891,107	$ 11,688,196	$ 10,135,458	, 10,208,585	$ 10,016,016	$ 9,216,490	$ 10,028,204
15. Over-all stop-loss claims†	$ 12,045,000	$ 10,755,000	$ 10,755,000	$ 11,510,000	$ 10,065,000	$ 10,075,000	$ 9,785,000	$ 9,305,000	$ 9,955,000
16. Fifty-year stop-loss gain (loss)	$ 7,900	($ 225,553)	($ 225,553)	$ 99,671	($ 157,752)	($ 99,475)	$ 122,061	($ 305,105)	($ 133,226)
17. No. years of statutory gains	25	23	23	23	15	19	21	15	15

* At the end of the fifty-year period.

† Stop-loss experience is not included in lines other than those marked.

discussion. Situations d, h, and l are not considered, since the credibility formula, important as it is in day-to-day operations, is not a concern of the paper. My comments on credibility formulas will be presented at the 1974 Berkeley Conference on Credibility Theory sponsored by the Committee on Research.

In his remarks on the risk charge for Situation E, Mr. Porto contends that the underwriting loss should be offset by any expected gain from stop-loss pools in calculating the risk charge. A stop-loss pool represents a non-experience-rated insurance line of business distinct from the experience-rated line of business. Any profit from the insurance pools should accrue to the insurer's benefit, not to the policyholder's benefit. It is inappropriate for the policyholder to participate in the profits of a non-experience-rated line of business, especially if no provision is made for sharing in the losses.

To the extent that no credibility formula is used in the situations tested and to the extent that there is only one level of expected claims (i.e., 100 per cent of the 1960 CSG Basic Table), the portfolio used does not represent a real portfolio. The question of credibility formulas was purposely avoided, and the question of various actual levels of mortality is discussed by Mr. Jeffery. The situations used were intended to be merely illustrative. Further investigation of various other situations using the techniques set forth in the paper is certainly necessary for answers to the practical group problems faced by actuaries.

The compromise suggested by Mr. Porto of allowing small deficits to be carried forward is discussed at length by Mr. Jeffery. The theoretical attractiveness of an experience-rating plan with a stop-loss pool at the premium margin for claims is obvious. The marketability of such a plan certainly cannot be taken for granted. It may well be that the policyholder will find attractive whatever the insurance industry offers, if the package and the alternatives are properly understood by both the insurer and the policyholder.

Mr. Bailey explains the use of the convolution approach to solve the problems postulated by the paper. The major advantage of a convolution technique is that it produces the expected distribution of data. Simulation techniques can produce only an actual set of data drawn from the expected distribution. The major disadvantage of a convolution approach is the calculation difficulties encountered in testing a multitude of experience-rating plans or using a diverse portfolio of cases as input. Any actuary who wishes to do further research should study carefully each approach and determine for himself which is more suitable for solving his specific research question.

It is interesting to note that the mean and variance of the risk charge vary by portfolio size. This is shown in the accompanying tabulation,

No. Cases in Portfolio	Mean Risk Charge	Standard Deviation of Risk Charge
1	6.44%	14.79%
100	3.89	0.46
∞	3.89	0

drawn from Mr. Bailey's calculations of the risk charge over a ten-year period. Although these results assume a predictable termination rate, they do leave the impression that a small group portfolio will have more of a termination risk than a large group portfolio. I believe that this theoretical conclusion would be borne out in practice.

Individual comments on Mr. Bailey's list of general remarks are given below.

1. As discussed in my reply to Mr. Angle, the interest earned on accumulated funds should not enter directly into the calculation of a risk charge. But Mr. Bailey is correct that the risk charge should be increased for at least commission and premium tax expenses. To the extent that percent-of-premium expenses are artificially allocated by the actuary to recover internal operating expenses, no increase in the risk charge need follow.
2. The "deficit" referred to in the profit analysis section is the cumulative deficit.
3. Forms that a risk charge may take are limited only by the imagination of the actuary and the marketability of the end product. A risk charge which recovers the annual increase in active deficits and canceled deficits should not be used with a deficit carry-forward plan. To do so double-charges active cases for their deficits. If no deficit carry-forward is used, the risk charge is then equivalent to a stop-loss pool at the premium margin for claims.
4. The predictability of any risk charge is not based on sound insurance principles, because of the withdrawal antiselection problem. Any level of risk charge less than the charge for a stop-loss premium of the premium margin for claims may be inadequate.
5. The only deficit management techniques that will eliminate the need for deficit recovery are a full retroactive premium (self-insurance) or a stop-loss pool at the premium margin for claims (pure insurance). Other combinations of deficit management techniques will require an adequate risk charge/deficit carry-forward scheme or a termination charge. I prefer the latter.
6. Mr. Bailey's suggested analysis of the present value of group profits is akin to an Anderson's method analysis of life insurance.

7. The random number generator used is of the congruence type. This generator will produce integers uniformly distributed over the interval $(0, 2^{32} - 1)$. If N_0 is the initial value entered,

$$N_{i+1} = (N_i \times L) \text{ modulo } (2^{31} - 1) ,$$

$$N = N_{i+1}/(2^{32} - 1)$$

where $L = 455{,}470{,}314$ and R is uniformly distributed in the interval $(0, 1)$.
8. An updated version of the simulation program used in the paper incorporates Mr. Bailey's suggestion.
9. The legal question of when a deficit becomes a liability to the policyholder can probably be resolved by competent legal counsel.
10. The question of the predictability of a frequency distribution of case cancellations is discussed in my general comments.
11. Mr. Jeffery presents a model using different levels of actual claims rather than a single level of actual claims as used in the paper. Over an extended period of time it may be necessary to incorporate mortality improvement into the model as well as various actual mortality levels. The mortality model used in an analysis of an actual group portfolio should probably include each of these refinements. A stationary distribution of claims around an expectation of 100 per cent of the 1960 CSG Basic Table is probably adequate for illustrative purposes only.
12. The situations presented in the paper are merely illustrative. Any thorough investigation of actual dividend formulas would require testing a large number of variables not included in the paper.
13. Credibility formulas will be discussed in a paper being prepared for the 1974 Research Conference on Credibility Theory.

In addition to his excellent discussion, Mr. Bailey used his computer resources to independently verify the data in the paper. I am indebted to him for both of these efforts.

The most important idea presented by Drs. Jones and Gerber in the steps leading to a sound group insurance contract is the specific duration of the contract and the determination of premiums only after definition of the dividend. Used properly, these two bases will result in a sound contract. This pricing sequence would add sound theoretical consideration to group insurance dividend formulas.

The following are comments on the specific technical points raised by the discussants.

1. The fact that group health coverage does not follow a well-behaved mathematical model, and the consequences for determination of the risk charge, are discussed in my reply to Mr. Margolin.

2 and 3. I would like to thank Drs. Jones and Gerber for these two points, which simplify some of the calculation problems of the model.

4. I agree that the results of the model should be recognized as only an illustration. It was not my intent to calculate risk charges or to solve all possible group insurance problems. My intent was merely to illustrate the behavior of a group life portfolio and to identify the basic tenets upon which group insurance is underwritten today. To solve practical group problems for an actual insurance portfolio requires either a true simulation model or a convolution model such as the one presented by Mr. Bailey.

I would like to thank Drs. Jones and Gerber for their laudatory remarks concerning the potential value of the paper in the Society's education program.

Mr. Jeffery has presented a useful practical example of the theoretical framework presented in the paper. The author would like to thank him for taking the time to present his valuable work.

Three characteristics of Mr. Jeffery's model are particularly impressive. First, the model uses London Life's actual group portfolio as test data. This test portfolio will obviously produce the most relevant results for London Life. Second, the model uses a dispersion of expected claims around an over-all 100 per cent of tabular claims. Testing credibility formulas requires this approach, and it will also add a realistic dimension to testing dividend formulas. Third, the model uses a mixture of new business and existing business. Tracing one year's new business for fifty policy years, as the paper does, is interesting for illustrative purposes. Since an actual group portfolio has a mixture of new business and existing business, for realistic results testing must recognize this fact.

The approach suggested by Mr. Jeffery of deciding on a risk charge and premium margin and testing for an over-all stop-loss pool level has practical merit. By using a low stop-loss level, active deficits may be effectively controlled. If active cases are not allowed to run substantial deficits, withdrawal antiselection can be limited substantially by the real cost of rewriting a group plan with a new carrier.

With a low stop-loss pooling level, however, the additional cost of eliminating deficit carry-forwards may not be too great. The improvement in predictability obtained in pricing a stop-loss pool at the premium margin for claims to eliminate the insurer's exposure to canceled deficits over the predictability of a risk charge/deficit carry-forward scheme with only a small exposure to deficits may justify using a stop-loss pool at the premium margin for claims.

As Mr. Jeffery notes, individual life insurance does present the insurer with the risk of a termination loss when the asset share is less than the cash value. There is, however, a fundamental difference between the loss on individual life insurance and the loss on group insurance. Upon withdrawal in early durations from an individual life policy, the insured as well as the insurer will lose money. The individual will lose because he has paid more in premiums than he receives in cash value. Contrary to the individual situation, a group termination will result in a gain to the policyholder at the expense of the insurer. Therefore, while group termination is like playing a two-player zero-sum game between policyholder and insurer, individual terminations do not follow the same format. Group and individual terminations would be in a nearly identical position if insurance companies offered early cash values in excess of paid premiums.

I can see only one way to interpret the results of Mr. Jeffery's simulation. At a stop-loss level of about 117 per cent of expected claims, two important results occur: (1) cash dividends paid are equal to the 5 per cent premium margin for claims, and (2) deficits written off are equal to the $1\frac{1}{2}$ per cent risk charge. At stop-loss levels above 117 per cent of expected claims, the risk charge is no longer adequate over the ten-year observation period.

To answer the specific questions raised at the end of the discussion:

1. Despite the fact that credibility theory and its application to group insurance are important questions, adding a lengthy section on credibility formulas to the paper would have been difficult in view of the length of the paper. My thoughts on credibility will be presented at the 1974 Research Conference sponsored by the Committee on Research.
2. An updated version of the simulation program used in the paper incorporates the dispersion of actual claims as Mr. Jeffery suggests.
3. Pooling claims at an additional premium does increase the premium margin on experience-rated claims by lowering the expected claims of the nonpooled exposure. The calculation of the margins for each situation provided by Mr. Garrison and myself should allow proper consideration of the differences in margin caused by my methodology.

I again would like to thank each of the eight discussants. The scope of their comments has added significantly to the value of the paper. I hope that group actuaries find the paper and the discussion both stimulating and practical.