

THE TAXATION OF INSURANCE IN CANADA. II

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ABSTRACT

This paper is a sequel to the author's paper "The Taxation of Insurance in Canada," which was published in 1970 in Volume XXII of the *Transactions*. The first paper covered the history of insurance taxation in Canada through 1969, including a description of the radical changes that became effective that year as an early phase of "tax reform," following the 1967 recommendations of the Royal Commission on Taxation.

Further general tax reforms became effective in 1972, including the introduction of taxes on capital gains; a number of clarifications and refinements were also enacted in 1973. The present paper outlines these general changes, with particular emphasis on their effect on insurance.

Finally, a completely revised mathematical presentation of the taxation of insurance corporations is given, together with formulas and examples of the marginal taxation rates on various kinds of income and disbursement of such corporations.

I. CANADIAN TAX REFORM, 1972

A BRIEF summary of the main recommendations contained in the 1967 report of the Royal Commission on Taxation (the Carter Commission) is given in Section F of my paper "The Taxation of Insurance in Canada."¹ Many of these recommendations proved to be quite controversial, particularly the proposal to introduce taxes on capital gains and to tax such gains as ordinary income. A vigorous public debate arose, and many briefs and letters were submitted to the government during 1967 and 1968.

Some of the proposals in the report which were not politically controversial were implemented in 1969. Among these were radical changes in the taxation of life insurance.²

Late in 1969, in an unusual departure from the customary Canadian practice of proposing tax changes to Parliament and the country in the form of nearly immutable budget resolutions, the Minister of Finance published a white paper³ outlining the remaining tax reforms which the

¹ *TSA*, XXII, 100.

² *Ibid.*, p. 103.

³ Minister of Finance, *Proposals for Tax Reform* (Ottawa, 1969).

government considered desirable. The white paper was referred immediately to standing committees of the House of Commons and the Senate for consideration, and further public debate and representations were invited.

The most significant proposals in the white paper were those made with respect to the integration of corporation and personal taxes and the treatment of capital gains as ordinary income. In the case of "closely-held" corporations, the objective was to put them as nearly as possible in the same tax position as self-employed persons and partnerships; it was proposed that full credit be given to the individual shareholders for income tax paid by the corporation and that realized capital gains (less losses) on disposition of shares of such a corporation be taxed as ordinary income. In the case of "widely-held" corporations, where it was considered more likely that part of the corporation's income tax is ultimately borne by customers in the prices charged for products and services, it was proposed that credit be given to shareholders for only half of the corporation tax and that the shareholder take into account only one-half of capital gains and losses calculated on changes in the market value of his shares every five years and on any intervening dispositions.

It was also proposed to eliminate the existing preferential tax rate on the first \$35,000 of corporation taxable income in favor of a flat corporation tax rate of 50 per cent.

The proposal to broaden the income tax base through inclusion of capital gains was tempered by a proposal to reduce personal taxation rates; it was proposed that the top federal marginal personal tax rate of about 60 per cent be reduced to 40 per cent, and the hope was expressed that the additional provincial income taxes (with top rates varying from 22.4 per cent of taxable income in British Columbia, Ontario, and Nova Scotia to 31.2 per cent in Manitoba) would be reduced proportionately.⁴

To dampen the effect of including large capital gains in personal income in a particular taxation year, the Carter Commission had suggested that the individual be allowed to deduct from taxable income the amount of deposits made in a non-interest-bearing government account and to pay taxes only when withdrawals were made from the account. The white paper proposed instead that the existing limited income-averaging provisions be replaced by a general averaging calculation to be performed automatically by the Department of National Revenue's computer whenever an individual's taxable income for a particular year exceeded

⁴ These rates did not apply in Quebec, where special federal-provincial arrangements are in effect; prior to the removal of the surtax in 1971, the top marginal personal tax rates in Quebec were 42.4 per cent federal plus 42.4 per cent provincial.

some arbitrary threshold, such as four-thirds of his average taxable income for the preceding four years.

With respect to public insurance programs, the following proposals were made:

1. That employee premiums under the federal Unemployment Insurance Act be deductible and benefits be included in determining personal taxable income.
2. That contributions to provincial hospital and medical care plans paid by an employer on behalf of an employee be treated as a taxable benefit to the employee.
3. That the separate personal and corporation income taxes for Old Age Security⁵ be merged into the general income tax rates.

With respect to private medical expense insurance, it was proposed that medical expenses reimbursable by such insurance no longer be considered as "medical expenses" in determining personal taxable income but that in the future premiums for such insurance be so considered.

With respect to registered pension and retirement savings plans, the white paper concurred with the existing principle of imposing limits on the degree of tax deferral available through these mechanisms but acknowledged the difficulty of establishing equitable limits on the various types of plans in existence. Some limitations were suggested with respect to the following:

1. "Top-hat" plans, that is, those designed primarily for the benefit of shareholders.
2. The investments of those pension plans which are not subject to regulation under any of the "pension benefits standards" laws which have been enacted in recent years in some of the provinces (and federally with respect to certain federally regulated corporations).
3. The investment of Canadian pension fund assets in foreign investments.

Renewed public discussion of tax reform followed publication of the white paper. Many additional briefs were directed to the government and to the two parliamentary committees. Briefs submitted by the Canadian Life Insurance Association included the following recommendations:

1. That the proposed general averaging provisions be *in addition to*, and not a *replacement of*, the existing averaging provisions applicable to income arising out of lump sum maturities and surrenders of insurance policies and annuity contracts, and lump sum pension withdrawals.
2. That the existing dollar limits on contributions to registered pension plans and retirement savings plans be increased to reflect changes in wages and prices that had occurred since the limits were enacted.

⁵ TSA, XXII, 89.

3. That further study be given to the joint impact of estate tax and the proposed income tax on capital gains deemed to be realized at the death of the taxpayer.
4. That relief be given to life insurance companies from the effect of the pro-rating of share dividends received from taxable Canadian corporations, which was stated to be discriminatory vis-à-vis other savings institutions.⁶
5. That the proposal to tax unrealized capital gains on shares of widely held corporations quinquennially not be adopted.
6. That foreign tax credits be allowed Canadian life insurance companies with respect to foreign investments related to their Canadian insurance operations.⁷
7. That in order to help stimulate personal savings—a key source of capital—some form of preferred status be extended to life insurance premiums and that consideration also be given to introducing other forms of savings-incentive programs.

Many of the representations made by other organizations and individuals expressed apprehension over the proposal to tax as ordinary income (that is, at full income tax rates) any net capital gains realized by individuals and particularly by the owners of small businesses (whether self-employed, partnerships, or closely-held corporations). Both parliamentary committees were impressed by the fears expressed, and in their reports, published in the fall of 1970, both recommended that the tax rate on all net realized capital gains be one-half that on ordinary income. Both rejected the proposal to revalue shares quinquennially for tax purposes. The Senate committee also rejected the proposals to integrate corporation and personal taxes. Both committees recommended that some form of tax incentive for small businesses be retained.

A draft tax reform bill, the culmination of nearly a decade of study, was laid before Parliament by the Minister of Finance following his budget of June 18, 1971. Following normal parliamentary debate and numerous minor amendments, the bill was enacted on December 23, 1971, and most of the new provisions became effective January 1, 1972. A number of clarifications have been enacted subsequently.

The revised tax reform legislation embraced many of the concepts in the Carter Commission report and the white paper. However, the proposals in the white paper concerning the integration of personal and corporation taxes, the differentiation between closely-held and widely-held corporations, and the taxation of most types of capital gains as ordinary income were modified along the lines recommended by the parliamentary committees.

The provisions in the previous Income Tax Act which were not changed were carried forward and renumbered, so that the act was, in effect, completely recodified.

⁶ *Ibid.*, p. 119.

⁷ *Ibid.*, p. 108.

Capital Gains

The most important element of the 1971 tax reform was the introduction of tax on capital gains.⁸ The new general rules are as follows:

1. One-half of capital gains is to be included in income and taxed at normal personal or corporation tax rates.
2. One-half of capital losses may be deducted from the one-half of capital gains, plus, in the case of individuals but not corporations, up to \$1,000 of other types of income. Capital loss deductions may be made in the current year or the preceding year or may be carried forward indefinitely.
3. Gains are generally taxable and losses deductible when a taxpayer sells an asset, when he makes a gift of an asset, or at his death or emigration. However, taxes are deferred on gifts or bequests to one's spouse.
4. Deductions are not allowed on depreciable property. Businesses, however, may continue to deduct "capital cost allowances" on depreciable property, up to prescribed limits, in computing "income."⁹

Most depreciable property is, by its very nature, unlikely to result in a value on sale exceeding its purchase price. A notable exception is real estate, and a numerical example will serve to illustrate the tax treatment on sale:

Suppose that a building is purchased by a business for \$100,000 after January 1, 1972, and during the period of ownership the business deducts a total of \$20,000 in capital cost allowance for tax purposes. Thus the undepreciated capital cost at time of sale is \$80,000.

1. If the sale price is \$75,000, the business may deduct \$5,000 as a "terminal loss."
2. If the sale price is \$90,000, the business must bring \$10,000 into "income," that is, \$90,000 less \$80,000.
3. If the sale price is \$150,000, the business must bring \$45,000 into "income," that is, \$100,000 less \$80,000 as "income from business" and one-half of \$150,000 less \$100,000 as "taxable capital gain."

Transitional rules were established to avoid retroactive taxation of capital gains accrued on property owned when the new law became effective. In general, gains or losses on the disposition of assets held on January 1, 1972, are determined by comparing the value at the time of disposition with the median of original cost, Valuation Day fair market value, and value at time of disposition. Individuals (but not corporations) are allowed the alternative of simply comparing the value at the time of disposition with the Valuation Day value, provided that the method is used for *all* property held on December 31, 1971. Valuation Day was

⁸ *Ibid.*, p. 109.

⁹ *Ibid.*

established as December 22, 1971, in the case of publicly traded securities and December 31, 1971, in the case of other property.

The examples shown in the accompanying tabulation will illustrate the

| | A | B | C | D | E | F |
|---------------------------|-------|-------|-------|-------|-------|-------|
| Original cost..... | \$100 | \$100 | \$100 | \$100 | \$100 | \$100 |
| Valuation Day value..... | 110 | 110 | 110 | 90 | 90 | 90 |
| Value at disposition..... | 120 | 105 | 90 | 80 | 95 | 110 |
| Median-Rule method: | | | | | | |
| Capital gain..... | 10 | 0 | 0 | 0 | 0 | 10 |
| Capital loss..... | 0 | 0 | 10 | 10 | 0 | 0 |
| Valuation Day method: | | | | | | |
| Capital gain..... | 10 | 0 | 0 | 0 | 5 | 20 |
| Capital loss..... | 0 | 5 | 20 | 10 | 0 | 0 |

above two methods of determining capital gains or losses. The Median-Rule method is also known as the Tax-Free Zone method because use of this method results in neither capital gain nor capital loss if the value at disposition falls in the zone between original cost and Valuation Day value.

Life insurance policies *other than annuity contracts* are specifically excluded from the definition of property subject to capital gain or loss. However, the earlier provisions¹⁰ which bring into "income" any gains on disposition (other than at death) of a life insurance policy or an annuity contract (other than the segregated fund portion of such policy or contract) were carried forward into the new act.

The inclusion of annuity contracts in the scope of capital gains and losses apparently was intended to apply to third-party contracts, although it is difficult to understand why "capital gains" treatment should apply here and "interest" treatment in the case of a contract owned by the annuitant.

Notwithstanding the general exclusion of life insurance policies from capital gains treatment, the act elsewhere provides that the insurance company may allocate each year's taxable capital gains (one-half of realized capital gains) in its segregated funds to policyholders. Such allocations are then treated as taxable capital gains in the hands of the policyholders (or tax-deferred in the case of registered plans) and are not considered as capital gains of the insurance company. If the company fails to make such an allocation, it is taxed on such capital gains, with no apparent provision for refunds or carry-forward if the allocation is

¹⁰ *Ibid.*, p. 105.

made in a different year. Except for such allocations of the insurance company's realized capital gains, gains on disposition of segregated fund policies (other than registered plans) are exempt from capital gains tax. On the other hand, all amounts paid out under registered plans are taxed in the recipient's hands as ordinary income, even though part of these benefits may have resulted from capital gains.

Apart from the exclusion of those segregated fund capital gains which are allocated to policyholders, an insurance company must bring one-half of its net capital gains into its "income," as in the case of other taxpayers.

Capital Cost Allowance

Two important rules were changed in the regulations with respect to capital cost allowances on buildings used principally for the purpose of producing rental income:

1. The total capital cost allowance on all such buildings owned by a business may not exceed the total rental income derived from them. In the past, capital cost allowance on rental buildings could be used to reduce other types of business income, but this is no longer permitted unless the holding, development, or sale of real estate is the principal business being conducted.
2. Each such rental building costing \$50,000 or more constitutes a separate capital cost allowance class. In other words, it cannot be pooled with similar properties, and upon sale there is an immediate terminal loss or recapture of prior capital cost allowances in excess of any loss on sale.

These rules apply only to rental buildings. Capital cost allowance on buildings used for other purposes may be applied, up to the normal limits, against nonrental business income.

Inheritance and Gift Taxes

Federal estate and gift taxes had been liberalized effective in 1969¹¹ and, with the introduction of capital gains tax, were abolished completely effective January 1, 1972. It is important to note that, whereas previously the entire amount of a gift or bequest was taxable (subject to generous exemptions), under the new system only the portion representing a net capital gain is taxable. Also, whereas formerly life insurance owned by a decedent could form part of his taxable estate, under the new system death benefits from life insurance are completely exempt from federal taxes. In the case of life insurance policies owned by partnerships and corporations, provision is made that the excess of death benefits over premiums paid under the policy does not result in an increase in the taxable capital value of the partnership or corporation.

¹¹ *Ibid.*, p. 103.

In recent years British Columbia, Ontario, and Quebec have been imposing provincial succession duties, and lower federal estate tax rates have been applicable in these provinces than in the other seven.¹² However, under federal-provincial agreements in effect in each of the other seven provinces, 75 per cent of the federal estate tax collected from estates in each province was turned over to it. In recent years Alberta and Saskatchewan have in turn refunded their entire share to the estates from which they arose. However, the other five provinces have apparently considered this share of federal tax an essential source of provincial revenue, and, when the federal government announced its abolition, they expressed considerable concern, since the new capital gains tax will not produce very large replacement revenues in its early years. Accordingly, they requested the federal government to continue the existing system for at least one year. The federal government did not agree but instead offered to enter into a three-year collection agreement with any province that enacted model succession duty and gift tax acts. Manitoba, New Brunswick, Newfoundland, Nova Scotia, Prince Edward Island, and Saskatchewan thereupon enacted both the uniform Succession Duty Act and the uniform Gift Tax Act. British Columbia, Ontario, and Quebec also enacted new gift tax legislation, leaving Alberta the only province with no succession duties or gift tax. However, Prince Edward Island subsequently rescinded both acts, and they will be applied only for limited periods in New Brunswick, Nova Scotia, and Newfoundland.

Income Averaging

Two distinct types of income averaging have been included in the federal tax reform legislation and replace most of the limited options available previously. These new provisions are applicable only to individuals and not to corporations or trusts.

"General averaging" applies automatically along the lines proposed in the white paper, except that a more generous threshold has been provided. The calculation is made by the Department of National Revenue and provides for averaging when a taxpayer's "income" in a year is more than 120 per cent of the average for the preceding four years (ignoring years before 1972) and more than 110 per cent of that for the immediately preceding year.

"Forward averaging" is available on an optional basis with respect to certain unusual receipts such as capital gains; income from literary or artistic works; income of athletes and entertainers; a lump sum payment from a pension plan, a deferred profit sharing plan, or a stock option

¹² *Ibid.*, p. 98.

plan; a lump sum retirement allowance; a death benefit from an employer or from a registered retirement savings plan; and proceeds from selling depreciable property, business inventory, or "goodwill." Forward averaging is accomplished through the purchase of an "income-averaging annuity" from a life insurance company or a trust company. The purchase price of the annuity is deductible in computing taxable income, and the annuity payments are taxable in full when received. The annuity may be an annuity certain or a life annuity with or without a period certain; the first payment must commence within ten months of purchase, and the period certain may not extend beyond fifteen years, or age 85 if earlier. It is interesting to note that the use of income-averaging annuities for tax deferral appears to have been conceived by the government and not by the life insurance companies or the actuarial profession!

With respect to losses, the former provisions for a five-year carry-forward and a one-year carry-back have been retained for non-capital losses, and further provision has been made in the reform legislation for an indefinite carry-forward and a one-year carry-back of net capital losses. Actually, a business may be able to carry forward non-capital losses for more than five years by claiming less than full capital cost allowances and/or, in the case of insurance corporations, less than maximum tax actuarial reserves.¹³

Tax Rates

As proposed in the white paper, the separate income taxes for Old Age Security have been discontinued and are now merged in the general income tax rates.

The personal tax rate scale provided in the revised federal Income Tax Act for 1972 is a progressive scale ranging from 17 to 47 per cent (plus a 30 per cent surtax on any income not "earned in a province"—for example, that of residents of the federal territories). Provision is made for subsequent annual reductions at the lower end of the scale until a scale ranging from 6 to 47 per cent is reached in 1976.

The additional provincial income taxes on individuals in all provinces except Quebec are defined in the various provincial income tax acts as a flat percentage of the basic federal income *tax* (ranging, for 1973, from 30.5 per cent in British Columbia and Ontario to 42.5 per cent in Manitoba). Quebec defines its personal income tax as progressive percentages of *taxable income* (as defined in the federal act); the combined federal-provincial tax rates in Quebec fall within the range of those in the other provinces.

¹³ *Ibid.*, p. 110.

The corporation income tax rate in the revised federal Income Tax Act is 50 per cent for 1972, reducing 1 per cent annually to 46 per cent in 1976, less 10 per cent abatement on taxable income "earned in a province." (For economic reasons the basic rate was reduced from 50 to 46.5 per cent for 1972 only.) For 1974 the net federal rate is therefore 38 per cent of taxable income earned in a province and 48 per cent of taxable income earned elsewhere (for example, in the federal territories or abroad).

The additional provincial corporation income tax rates vary from 10 per cent of taxable income in New Brunswick, Nova Scotia, and Prince Edward Island to 13 per cent in Manitoba and Newfoundland.

The combined federal-provincial corporation income tax rates applicable to a particular insurance company depend upon the distribution of the company's business by province (the basis of allocation being premium income less policy dividends). For the industry as a whole, the average provincial-territorial rate is about 11.9 per cent. Thus the average combined federal-provincial rate for 1974 is about 49.9 per cent.

The revised federal act provides for an increase in most taxes on the income of non-residents from 15 to 25 per cent, to be effective in 1976. Where a tax treaty with another country stipulates a different rate, however, the treaty rate prevails.

Corporations and Shareholders

The white paper proposal for an integrated system of taxing corporation and shareholder income was not adopted. Instead, the existing system was modified with respect to dividend tax credits and "private corporations."

Private Corporations

Most Canadian corporations whose shares are listed on a Canadian stock exchange are designated "public corporations," and those whose shares are not listed are designated "private corporations." However, a corporation whose shares are listed may elect to be designated a private corporation if its shares are owned primarily by "insiders" and are not widely traded. Similarly, a corporation whose shares are not listed may elect to be designated a public corporation if its shares meet certain tests as to eligibility for distribution and as to number held by persons who are not insiders. The federal act specifies that resident life insurance corporations are deemed to be public corporations.

A form of tax incentive for small businesses has been retained in the revised act but is applicable only to Canadian-controlled private corporations. This consists essentially of a reduction in the corporation tax rate

to 25 per cent, applicable to the first \$50,000 of business income each year until a total of \$400,000 of business income has been accumulated. Special treatment is also given to the investment income and capital gains of private corporations, which attempts to achieve tax neutrality between individuals who choose to make investments through a private corporation and those who do so directly.

Dividend Tax Credit

The dividend tax credit system was first introduced in 1949. Under the 1949 system an individual recipient of share dividends from a taxable Canadian corporation was required to include such dividends, net of expenses and carrying charges, in his income but was given a tax credit of 10 per cent of the net dividend, which more or less offset the 10 per cent corporation tax then payable by smaller corporations and partially offset the higher level of taxes payable by larger corporations. The 10 per cent dividend tax credit was increased to 20 per cent in 1953. Effective in 1972, the credit has *theoretically*¹⁴ been increased to 33 $\frac{1}{3}$ per cent of the gross dividend, but the taxpayer must also include the 33 $\frac{1}{3}$ per cent as a "gross-up" in his "income."

Share dividends received by public corporations continue, in general, to be exempt from income tax. Under the prorating formula applicable to life insurance corporations,¹⁵ however, such dividends do attract tax. While the tax reform legislation retained the prorating concept, four significant changes were made:

1. Prorating in the other-than-life insurance branch (presumably an oversight in 1969) was eliminated, and full deduction is allowed.
2. The 3 per cent deduction for assumed investment expenses was eliminated.
3. The "50 per cent general expense deduction" is no longer included in the denominator; that is, the denominator is now simply "net Canadian life investment income."
4. The tax credit allowed against a life insurance company's investment income tax is now 28 $\frac{1}{3}$ per cent of the "policyholder share" of share dividends received, rather than 19.4 per cent. This rate differs from the rate of dividend tax credit for individuals mentioned above to reflect the fact that the individual must gross up his dividend by one-third before calculating tax, whereas the insurance company does not. Since the investment income tax rate is 15 per cent, the 28 $\frac{1}{3}$ per cent dividend tax credit produces results

¹⁴ The dividend tax credit enacted in federal law is actually 80 per cent of the 33 $\frac{1}{3}$ per cent "gross-up," that is, 26 $\frac{2}{3}$ per cent; the corresponding reduction in provincial tax payable varies from 8.1 to 11.3 per cent, so that the combined tax credit actually varies from 34.8 to 38 per cent rather than the theoretical 33 $\frac{1}{3}$ per cent.

¹⁵ TSA, XXII, 119.

equivalent to those that would be produced by the theoretical 33½ per cent credit mentioned above, applied to an individual at the same 15 per cent income tax rate (see the accompanying tabulation).

| | Insurance Company | Individual |
|-------------------------|-------------------|---------------|
| Dividend..... | \$300 | \$300 |
| Gross-up..... | | 100 |
| Tax base..... | \$300 | \$400 |
| Tax at 15%..... | \$ 45 | \$ 60 |
| Tax credit..... | (at 28½%) 85 | (at 33½%) 100 |
| After-tax dividend..... | \$340 | \$340 |

During parliamentary consideration of the tax reform bill, the Canadian Life Insurance Association again urged that the prorating formula applicable to share dividends received by life insurance companies be abandoned and that insurance corporations be treated the same as other corporations in this regard. In reporting out the bill, the Standing Senate Committee on Banking, Trade, and Commerce supported this view. However, in view of the urgency to enact the reform legislation prior to the end of 1971, consideration of this recommendation was deferred, and the prorating concept, with the above four modifications, was retained in the new act.

Tax-free Dividends

Provision was made in the new act whereby Canadian corporations might elect to pay a special 15 per cent tax on their 1971 undistributed earned surplus and then to distribute the remaining tax-paid undistributed surplus, and any 1971 capital surplus on hand, in the form of special tax-free dividends to shareholders. This avoids what might otherwise amount to retroactive taxation of pre-1972 capital gains. Special definitions were included to reflect the unique status of shareholders of life insurance corporations.

Public Insurance Programs

The white paper proposals mentioned above were approved by the parliamentary committees and were incorporated in the new Income Tax Act.

A completely revised Unemployment Insurance Act was enacted by Parliament in June, 1971. One of the revisions provided for the inclusion

of benefits for interruption of earnings because of accident, sickness, or pregnancy. Employers are allowed to use private plans for accident and sickness benefits as an alternative to the public plan, provided that equivalent benefits are provided in the private plan; if this alternative is followed, a reduction is made in the employer and employee contributions for the government plan.

Private Accident and Sickness Insurance

The white paper proposal mentioned above concerning medical expense insurance was incorporated in the reform legislation.

The white paper was silent on disability income insurance.¹⁶ The reform legislation provides that benefits are taxable if the taxpayer's employer contributes to the premium. An exemption is provided to the extent that benefits received after 1971 do not exceed the employee's contributions after 1967. A further exemption is provided for *all* benefits paid out with respect to disabilities occurring before 1974 from a plan which was established before June 19, 1971 (the date the tax reform bill was introduced), *and which is continued unaltered* except as required to qualify under the revised Unemployment Insurance Act.

Registered Pension Plans, Retirement Savings Plans, and Deferred Profit Sharing Plans

The old law provided that a pension plan or a profit sharing plan would lose its tax-free status if more than 10 per cent of its *income* came from foreign sources. The revised act requires 90 per cent of the *assets* of all three types of registered plans to be Canadian. Certain other restrictions are also imposed on the investments of such plans.

The old law provided for deductions, in computing personal income, of the taxpayer's contributions to such plans, up to \$1,500 per year for a member of a registered pension plan or profit sharing plan or the smaller of \$2,500 and 20 per cent of earned income if the individual was covered only by a retirement savings plan. These dollar limits were increased to \$2,500 and \$4,000, respectively, in the new act.

Mutual Funds and Investment Corporations

There are two forms of mutual funds: trusts and corporations. Although they are taxed differently, the intent is that, provided that certain criteria are satisfied, they both be treated as "conduits" between their shareholder-investors and the sources from which their income is derived.

¹⁶ *Ibid.*, p. 102.

In both cases, any gains realized on redemption by unit-holders or shareholders are taxed in their hands as capital gains (compare segregated fund insurance policies discussed earlier).

Trusts in general are taxed only on their net income (including half of net capital gains) after deducting amounts payable to beneficiaries. The tax rate is the same as the scale applicable to individuals (without the benefit of personal deductions), subject to a minimum of 39 per cent federal plus the corresponding provincial rate (11.9-16.6 per cent) of taxable income. In practice, most trusts generally contain covenants requiring the distribution of ordinary income, but these would not normally include capital gains. Thus, many ordinary trusts which have heretofore allocated all their ordinary income to beneficiaries (and thus had no taxable income) may now have taxable income in the form of taxable capital gains.

"Mutual fund trusts" are Canadian trusts which issue units redeemable on demand and meet various conditions as to investments, number of unit-holders, dispersal of units, and public trading of units. The tax rate on undistributed ordinary income is the same as for ordinary trusts, but is simply the minimum rate of 39 per cent noted above on one-half of undistributed net realized capital gains. The share of the fund's capital gains (realized and unrealized) allocable to terminating unit-holders is approximated by formula and is designated "capital gains redemptions." To the extent that the fund's net realized capital gains have not been formally distributed but have been retained and taxed, they are deemed to have been used as "capital gains redemptions" (taxable in the unit-holder's hands), and the federal act allows a "capital gains refund" of 20 per cent of this amount, that is, about the same as the federal tax of 39 per cent on one-half the net capital gains.

"Mutual fund corporations" are Canadian corporations which have as their only undertaking the investment of their funds and which issue shares redeemable on demand. If the mutual fund corporation satisfies certain other criteria (similar to those applicable to mutual fund trusts), it is also designated as an "investment corporation." A mutual fund corporation which does not qualify as an investment corporation is taxed at the same rates as a private corporation. An investment corporation is taxed on its ordinary income at normal corporation tax rates less 25 per cent (that is, in 1974 at 13 per cent federally plus 10-13 per cent provincially). The remaining 75 per cent (approximately) of after-tax ordinary income will then normally be distributed as an ordinary dividend, which is taxable and eligible for dividend tax credit in the shareholder's hands. An investment corporation is also taxed on one-half of

its net realized capital gains at normal corporation tax rates (that is, in 1974 at 38 per cent federally plus 10-13 per cent provincially). The remaining 75 per cent (approximately) of after-tax net capital gains is deemed first of all to be applied toward "capital gains redemptions" determined by formula (as described above for mutual fund trusts); any balance can then be distributed to shareholders as a "capital gains dividend," which is treated in the shareholder's hands as a realized capital gain. The mutual fund corporation is allowed a federal "capital gains refund" at 20 per cent of the sum of its "capital gains redemptions" and "capital gains dividends."

Investment corporations which are not mutual fund corporations are taxed as above but are not eligible for capital gains refunds.

Several provinces have also enacted provisions to refund capital gains taxes collected from mutual fund trusts and mutual fund corporations.

II. TAXATION OF INSURANCE COMPANIES

Income Tax

The basis for determining an insurance corporation's income tax was changed through the introduction of capital gains tax and through the change mentioned above in the treatment of share dividends received from taxable Canadian corporations.

Investment Income Tax

Reference already has been made to the change in determining dividend tax credit. In addition, the basis for determining taxable Canadian life investment income was changed in the following ways:

1. By allowing as a deduction the "interest element" for deferred profit-sharing plans (in addition to the deductions previously allowed for registered pension and retirement savings plans).
2. By reducing the deduction of "income" by the amount of non-capital losses carried forward (or back) from other years. This compensates for the fact that a non-capital loss (that is, negative "income") does not affect investment income tax in the year in which it is incurred.

Mathematical Presentation

As a result of these changes, the formulas given in my earlier paper and discussion are now obsolete.¹⁷ A revised mathematical presentation follows. All symbols are defined below, and none is necessarily related to the earlier presentation. Table 1 identifies the source data entering the tax calculations.

¹⁷ *Ibid.*, pp. 122-25, 386-90.

TABLE 1
SOURCE DATA

| | LIFE | | OTHER THAN LIFE |
|--|-----------------|-----------------|-----------------|
| | Nonsegregated | Segregated | |
| Income: | | | |
| 1. Premiums* subject to premium tax considered in computing investment income tax..... | x ₁ | y ₁ | |
| 2. Premiums* subject to premium tax not considered in computing investment income tax..... | x ₂ | y ₂ | z ₂ |
| 3. Premiums* and considerations* for annuities (including settlement annuities), not subject to premium tax..... | x ₃ | y ₃ | z ₃ |
| 4. Policy reserve beginning of year (BOY)..... | x ₄ | y ₄ | z ₄ |
| 5. Group life additional reserve BOY..... | x ₅ | | |
| 6. Policy dividends reserve BOY..... | x ₆ | y ₆ | z ₆ |
| 7. Interest, rents, royalties..... | x ₇ | y ₇ | z ₇ |
| 8. Taxable dividends received from taxable Canadian corporations..... | x ₈ | y ₈ | z ₈ |
| 9. Taxable dividends received from other corporations..... | x ₉ | y ₉ | z ₉ |
| 10. Investment reserve BOY..... | x ₁₀ | | |
| 11. Mortgage reserve BOY..... | | | z ₁₁ |
| 12. Accrual of discount and profit on sale of "Canada securities"..... | x ₁₂ | | |
| 13. Capital gains on disposition of other property..... | x ₁₃ | y ₁₃ | z ₁₃ |
| 14. Capital gains dividends..... | x ₁₄ | y ₁₄ | z ₁₄ |
| 15. Miscellaneous income, including inter-branch transfers (positive or negative)..... | x ₁₅ | y ₁₅ | z ₁₅ |
| Deductions (up to limits prescribed for income tax): | | | |
| 21. Policy payments..... | x ₂₁ | y ₂₁ | z ₂₁ |
| 22. Policy dividends deductible re premium tax considered re investment income tax..... | x ₂₂ | y ₂₂ | |
| 23. Policy dividends deductible re premium tax not considered re investment income tax..... | x ₂₃ | y ₂₃ | z ₂₃ |
| 24. Policy dividends not deductible re premium tax..... | x ₂₄ | y ₂₄ | z ₂₄ |
| 25. Segregated fund allocations of revenue..... | | y ₂₅ | |
| 26. Segregated fund allocations of capital gains..... | | y ₂₆ | |
| 27. Policy reserve end of year (EOY)..... | x ₂₇ | y ₂₇ | z ₂₇ |
| 28. Group life additional reserve EOY..... | x ₂₈ | | |
| 29. Policy dividends reserve EOY..... | x ₂₉ | y ₂₉ | z ₂₉ |
| 30. Interest paid or credited..... | x ₃₀ | y ₃₀ | z ₃₀ |
| 31. Investment reserve EOY..... | x ₃₁ | | |
| 32. Mortgage reserve EOY..... | | | z ₃₂ |
| 33. Amortization of premium and loss on sale of "Canada securities"..... | x ₃₃ | | |
| 34. Capital loss on disposition of other property..... | x ₃₄ | y ₃₄ | z ₃₄ |
| 35. Capital cost allowance and allowance for other capital expenditures—investment..... | x ₃₅ | y ₃₅ | z ₃₅ |

* Net of premium returns and experience-rating refunds.

TABLE 1—Continued

| | LIFE | | OTHER THAN LIFE |
|---|---------------|------------|-----------------|
| | Nonsegregated | Segregated | |
| Deductions—Continued: | | | |
| 36. Capital cost allowance and allowance for other capital expenditures—group life... | x_{36} | | |
| 37. Capital cost allowance and allowance for other capital expenditures—other..... | x_{37} | y_{37} | z_{37} |
| 38. Expenses including licenses, fees and taxes other than premium taxes, investment income tax, and income taxes—investment..... | x_{38} | y_{38} | z_{38} |
| 39. Expenses including licenses, fees and taxes other than premium taxes, investment income tax, and income taxes—group life..... | x_{39} | | |
| 40. Expenses including licenses, fees and taxes other than premium taxes, investment income tax, and income taxes—other..... | x_{40} | y_{40} | z_{40} |
| 41. Bad debts—investment (other than defaulted loans)..... | x_{41} | y_{41} | z_{41} |
| 42. Bad debts—group life..... | x_{42} | | |
| 43. Bad debts—other..... | x_{43} | y_{43} | z_{43} |
| Other items affecting taxes other than income tax: | | | |
| 51. Policy dividends not deductible re income tax but deductible re premium tax considered re investment income tax..... | x_{51} | | |
| 52. Policy dividends not deductible re income tax but deductible re premium tax not considered re investment income tax.... | x_{52} | | |

OTHER SOURCE DATA

- F = Policyholder gains;
- L = Deductible non-capital losses from other years;
- N = Deductible gifts to charitable organizations, etc.;
- R = "Interest element" on "existing fixed-premium" and registered plans;
- W = Deductible net capital losses from other years;
- Z = "Excess" dividends paid to shareholders;
- p = Premium tax rate;
- r = Corporation income tax rate (federal and provincial rates combined).

Let

X = Premium taxes considered in computing investment income tax

$$= (p \leq 0.02)(x_1 - x_{22} - x_{51} + y_1 - y_{22}) ;$$

Y = Other premium taxes

$$= (p - 0.02 \geq 0)(x_1 - x_{22} - x_{51} + y_1 - y_{22}) + p(x_2 - x_{23} - x_{52} + y_2 - y_{23} + z_2 - z_{23}) ;$$

G = Business income from life insurance operations, before deducting investment income tax

$$\begin{aligned} &= \sum_1^{12} x_n + x_{15} - \sum_{21}^{33} x_n - \sum_{35}^{43} x_n + \sum_1^9 y_n + y_{15} - \sum_{21}^{25} y_n \\ &\quad - \sum_{27}^{30} y_n - \sum_{35}^{43} y_n - p(x_1 + x_2 - x_{22} - x_{23} - x_{51} - x_{52} \\ &\quad\quad\quad + y_1 + y_2 - y_{22} - y_{23}) \end{aligned}$$

H = Business income from other operations

$$= \sum_2^{11} z_n + z_{15} - \sum_{21}^{32} z_n - \sum_{35}^{43} z_n - p(z_2 - z_{23});$$

U = Taxable capital gains = $\frac{1}{2}$ Capital gains

$$= \frac{1}{2}(x_{13} + x_{14} + y_{13} + y_{14} - y_{26} + z_{13} + z_{14}) \geq 0;$$

V = Allowable capital losses = $\frac{1}{2}$ Capital losses

$$= \frac{1}{2}(x_{34} + y_{34} + z_{34}) \geq 0;$$

D = Taxable dividends received from taxable Canadian corporations, non-segregated life

$$= x_8;$$

E = Taxable dividends received from taxable Canadian corporations, other than life

$$= z_8;$$

S = Net Canadian life investment income

$$= \sum_7^{12} x_n - \sum_{30}^{33} x_n - (x_{35} + x_{38} + x_{41}) - \frac{1}{2}(x_{37} + x_{40} + x_{43}).$$

Also, let

A = Premium taxes ;

B = Investment income tax ;

C = Income taxes ;

$T = A + B + C$.

The terminology of the Income Tax Act may then be defined algebraically:

Income from life insurance business = $P = G - B \geq 0$;

Loss from life insurance business = $B - G \geq 0$;

Income from business other than life insurance = $H \geq 0$;

Loss from business other than life insurance = $-H \geq 0$;

Taxable capital gains = U ;

Allowable capital losses = V ;

Income = $G - B + H + (U - V \geq 0) \geq 0$;

Non-capital loss = $-(G - B + H) - (U - V \geq 0) \geq 0$;

$$\text{Net capital loss} = V - U \geq 0;$$

$$\text{Deductible gifts} = N \leq 0.2[G - B + H + (U - V \geq 0)];$$

$$\text{Taxable income} = \{G - B + H + (U - V - W \geq 0) - L - N - D[\text{Smaller of } P/S \text{ and } (S - R)/S \geq 0] - E \geq 0\} + 2Z;$$

$$\text{Gross Canadian life investment income} = \sum_{12}^{12} x_n - \sum_{31}^{33} x_n \\ = S + (x_{30} + x_{35} + x_{38} + x_{41}) + \frac{1}{2}(x_{37} + x_{40} + x_{43});$$

$$\text{Net Canadian life investment income} = S;$$

$$\text{Taxable Canadian life investment income} = I \\ = S - F - R - (P - L \geq 0) \geq 0.$$

If $I > 0$ and $L = 0$, it is useful to consider the equation $S = P + Q + R$, where $Q = I + F$. Taxable dividends received from taxable Canadian corporations (D) form part of S and are deemed to be prorated among P , Q , and R :

1. The portion PD/S is deemed to be included in the company's income and is therefore allowed to be deducted in computing taxable income.
2. The portion QD/S is deemed partly (FD/S) to have been included in policyholder gains taxed in the policyholder's hands and partly (ID/S) to be included in the company's taxable investment income; in both events a dividend tax credit is appropriate, and therefore $0.283QD/S$ is deducted in computing investment income tax.
3. The portion R/S is deemed to be included in tax-free and tax-deferred items, and no further tax deduction or credit is allowed.

If $I = 0$ and $L = 0$, there is no investment income tax against which to allow a dividend tax credit. The portion PD/S is still allowed as a deduction in calculating taxable income unless $S - R < P$, in which case only $(S - R)D/S$ is allowed.

If $L > 0$, that is, if there is a carry over of non-capital losses from other years, this has no effect on the proration of the current year's dividends.

It is convenient to introduce the following items:

$$f = D/S; \quad g = 0.15 - 0.283f; \quad J = G - L;$$

$$K = g(S - R) - 0.15F - 0.5X,$$

$$M = H + (U - V - W \geq 0) - E - N.$$

Then

$$A = X + Y;$$

$$B = 0.15I - 0.283D(I + F)/S - 0.5X \geq 0$$

$$= gI - 0.283fF - 0.5X \geq 0$$

$$= g\{S - F - R - [(G - B \geq 0) - L \geq 0]\} - 0.283fF - 0.5X \geq 0;$$

$$\begin{aligned}
 C &= r\{[G - B - L - f(\text{Smaller of } P \text{ and } S - R \geq 0) + M \geq 0] + 2Z\} \\
 &= r\{[G - B - L - f(\text{Smaller of } G - B \geq 0 \text{ and} \\
 &\quad S - R \geq 0) + M \geq 0] + 2Z\}.
 \end{aligned}$$

These expressions for B and C are interdependent and therefore cannot be used immediately to calculate values. However, each can be stated in terms of independent variables. Several alternative solutions arise, depending on the limits imposed by the inequalities contained in the above expressions, and these are set forth in Table 2.

From Table 2 we can deduce that B can be expressed in the general form

$$B = K - \frac{g}{1-g}(J - K \geq 0) \geq 0.$$

Situations VIB, VIIB, VIIIA, and VIIIB are unlikely to occur, since the condition $K \leq 0$ occurs only if

$$g \leq h = (0.15F + 0.5X)/(S - R), \text{ i.e., if } f \geq (0.15 - h)/0.28\dot{3}.$$

This will occur if $S - R$ is relatively small (or negative), which is unlikely. It can also occur if taxable dividends received from taxable Canadian corporations should constitute a sufficiently large proportion of net Canadian life investment income, thereby giving rise to sufficient dividend tax credit to eliminate investment income tax. This is also unlikely to occur, because of investment practices and legal restrictions on investments in common stocks.

The usual situation for an established company with positive earnings from its life insurance operations is Situation IV. In this case, assuming

$$M \geq \frac{1-fg}{1-g}L - \frac{1-f}{1-g}(G - K),$$

we have

$$B + C = qG + (1 - q)K - (q + rf)L + r(M + 2Z),$$

where

$$q = \frac{r(1-f) - g}{1-g}.$$

If $L = 0$, Situations II, III, and V become superfluous because Situation II then becomes the boundary ($K = G$) common to Situations I and IV, and Situations III and V both degenerate into the boundary point ($K = G = S - R = 0$) theoretically common to all situations but impossible unless $F = X = 0$.

TABLE 2

| Situation | Conditions | B | C |
|------------|---|------------------------|---|
| I..... | $K \geq 0$ $S-R \geq 0 \geq G-K$ | K | $r\{(G-K-L+M \geq 0)+2Z\}$ |
| II..... | $K \geq 0$ $G \geq K \geq J$ $S-R \geq G-K \geq 0$ | K | $r\{[(1-f)(G-K)-L+M \geq 0]+2Z\}$ |
| III..... | $K \geq 0$ $G \geq K \geq J$ $G-K \geq S-R \geq 0$ | K | $r\{ G-K-f(S-R)-L+M \geq 0\}+2Z\}$ |
| IV..... | $J \geq K \geq gJ \geq 0$ $S-R \geq \frac{G-K-gL}{1-g} \geq 0$ | $\frac{K-g(G-L)}{1-g}$ | $r\left\{\left[\frac{1-f}{1-g}(G-K)-\frac{1-fg}{1-g}L+M \geq 0\right]+2Z\right\}$ |
| V..... | $J \geq K \geq gJ \geq 0$ $\frac{G-K-gL}{1-g} \geq S-R \geq 0$ | $\frac{K-g(G-L)}{1-g}$ | $r\left\{\left[\frac{G-K-L}{1-g}-f(S-R)+M \geq 0\right]+2Z\right\}$ |
| VIA..... | $gJ \geq K \geq 0$ $S-R \geq G \geq 0$ | 0 | $r\{[(1-f)G-L+M \geq 0]+2Z\}$ |
| VIB..... | $K \leq 0$ $S-R \geq G \geq 0$ | | |
| VIIA..... | $gJ \geq K \geq 0$ $G \geq S-R \geq 0$ | 0 | $r\{ G-f(S-R)-L+M \geq 0\}+2Z\}$ |
| VII B..... | $K \leq 0$ $G \geq S-R \geq 0$ | | |
| VIIIA..... | $K \leq 0$ $S-R \geq 0 \geq G$ | 0 | $r\{(G-L+M \geq 0)+2Z\}$ |
| VIIIB..... | $S-R \leq 0$ | | |

A very simplified but useful example occurs if we let

$$D = E = H = L = N = U = V = W = Z = 0.$$

In this case,

$$f = M = 0, \quad g = 0.15, \quad q = \frac{r - 0.15}{0.85}, \quad J = G,$$

and the relationships in Table 2 become those in Table 3.

TABLE 3

| Situation | Conditions | B | C | B+C |
|-----------|--|--------------------|-------------------|-----------------|
| I..... | $K \geq 0$ $S - R \geq 0 \geq G - K$ | K | 0 | K |
| II..... | $K = G \geq 0$ $S - R \geq 0$ | G | 0 | G |
| IV..... | $G \geq K \geq 0.15G \geq 0$ $S - R \geq 0$ | $(K - 0.15G)/0.85$ | $[r(G - K)]/0.85$ | $qG + (1 - q)K$ |
| VI, VII. | $0.15G \geq K$ $S - R \geq 0$ $G \geq 0$ | 0 | rG | rG |
| VIIIA.. | $K \leq 0$ $S - R \geq 0 \geq G$ | 0 | 0 | 0 |
| VIIIB.. | $S - R \leq 0$ | 0 | $r(G \geq 0)$ | $r(G \geq 0)$ |

III. MARGINAL TAXES

From the foregoing formulas we can derive marginal taxation rates, that is, the rates of additional taxes that arise from each additional dollar of revenue or that are "saved" by each additional dollar of expenditure or tax deduction. The marginal tax rate associated with any income or disbursement item x_n is simply the partial derivative $\partial T / \partial x_n$. From our previous equations we have

$$\partial T = \partial A + \partial B + \partial C,$$

$$\partial A = \partial X + \partial Y,$$

and ∂B and ∂C are as shown in Table 4. The partial derivatives of the functions $D, E, G, H, S, U, V, X,$ and Y with respect to any particular variable $x_n, y_n,$ or z_n can be determined readily by using the algebraic definitions of these functions. Next, the partial derivatives of the functions $f, g, K,$ and M with respect to any of the variables $x_n, y_n, z_n, F, L, N,$ or R can be determined by using the following relationships:

$$\partial f = (\partial D - f\partial S)/S ;$$

$$\partial g = -0.28\dot{3}\partial f ;$$

$$\partial K = g\partial S - g\partial R + (S - R)\partial g - 0.15\partial F - 0.5\partial X ;$$

$$\partial M = \partial H - \partial E - \partial N + \partial U - \partial V - \partial W .$$

Finally, by substitution in Table 4, the partial derivatives ∂B and ∂C with respect to any of the above variables can be determined.

TABLE 4

| Situation | ∂B | ∂C |
|-----------|--|---|
| I..... | ∂K | $r(\partial G - \partial B - \partial L + \partial M + 2\partial Z)$ |
| II..... | ∂K | $r\{(1-f)(\partial G - \partial B) - (G-K)\partial f - \partial L + \partial M + 2\partial Z\}$ |
| III..... | ∂K | $r\{\partial G - \partial B - (S-R)\partial f - f(\partial S - \partial R) - \partial L + \partial M + 2\partial Z\}$ |
| IV..... | $\{(1-g)[\partial K - g(\partial G - \partial L)] - (G-L-K)\partial g\} / (1-g)^2$ | $r\{(1-f)(\partial G - \partial B) - (G-K-gL)\partial f / (1-g) - \partial L + \partial M + 2\partial Z\}$ |
| V..... | $\{(1-g)[\partial K - g(\partial G - \partial L)] - (G-L-K)\partial g\} / (1-g)^2$ | $r\{\partial G - \partial B - (S-R)\partial f - f(\partial S - \partial R) - \partial L + \partial M + 2\partial Z\}$ |
| VI..... | 0 | $r\{(1-f)\partial G - G\partial f - \partial L + \partial M + 2\partial Z\}$ |
| VII..... | 0 | $r\{\partial G - (S-R)\partial f - f(\partial S - \partial R) - \partial L + \partial M + 2\partial Z\}$ |
| VIII..... | 0 | $r(\partial G - \partial L + \partial M + 2\partial Z)$ |

In order to produce concise formulas for the partial derivatives, it is convenient to introduce the following additional variables:

$$a = R/S \quad b = G/S ; \quad c = (G - K)/S ; \quad d = \frac{G - K - gL}{(1 - g)S} ;$$

$$e = \frac{0.28\dot{3}}{1 - g} \left[1 - \frac{R}{S} - \frac{G - K - L}{(1 - g)S} \right] ;$$

$$j = g + 0.28\dot{3}f(1 - a) = 0.15 - 0.28\dot{3}fa ;$$

$$k = -g + 0.28\dot{3}(1 - f)(1 - a) = -0.28\dot{3}(1 - f)a + 0.1\dot{3} ;$$

$$s = r(1 - f) ; \quad t = \frac{r - g}{1 - g} ;$$

$$q = \frac{s - g}{1 - g} = \frac{r(1 - f) - g}{1 - g}.$$

As an example of the calculation of partial derivatives, consider taxable dividends received from taxable Canadian corporations in the non-segregated life account (x_8). We have

$$\begin{aligned} \frac{\partial E}{\partial x_8} = \frac{\partial F}{\partial x_8} = \frac{\partial H}{\partial x_8} = \frac{\partial M}{\partial x_8} = \frac{\partial N}{\partial x_8} = \frac{\partial R}{\partial x_8} = \frac{\partial U}{\partial x_8} = \frac{\partial V}{\partial x_8} = \frac{\partial W}{\partial x_8} = \frac{\partial X}{\partial x_8} \\ = \frac{\partial Y}{\partial x_8} = \frac{\partial Z}{\partial x_8} = 0; \end{aligned}$$

$$\frac{\partial D}{\partial x_8} = \frac{\partial G}{\partial x_8} = \frac{\partial S}{\partial x_8} = 1.$$

Hence

$$\frac{\partial f}{\partial x_8} = \frac{1 - f}{S};$$

$$\frac{\partial g}{\partial x_8} = -0.283(1 - f)/S;$$

$$\frac{\partial K}{\partial x_8} = g - 0.283(1 - f)(S - R)/S.$$

In Situation IV we then have

$$\frac{\partial A}{\partial x_8} = 0,$$

$$\begin{aligned} \frac{\partial B}{\partial x_8} &= [(1 - g)\{g - 0.283(1 - f)[(S - R)/S] - g\} \\ &\quad + (G - L - K)(0.283)[(1 - f)/S]]/(1 - g)^2 \\ &= -(1 - f)e, \end{aligned}$$

$$\begin{aligned} \frac{\partial C}{\partial x_8} &= r\left[(1 - f)\left(1 - \frac{\partial B}{\partial x_8}\right) - \frac{G - K - gL}{1 - g} \frac{1 - f}{S}\right] \\ &= r(1 - f)\left(1 - d - \frac{\partial B}{\partial x_8}\right) = r(1 - f)(1 - d) + s(1 - f)e, \end{aligned}$$

whence

$$\begin{aligned} \frac{\partial T}{\partial x_8} &= r(1 - f)(1 - d) - (1 - s)(1 - f)e \\ &= (1 - f)[r(1 - d) - (1 - s)e]. \end{aligned}$$

Using the same technique, formulas for marginal tax rates with respect to any other variable may be derived.

In Table 5 we show, for each of the eight situations, the formulas for marginal taxes for most of the common transactions and also some illustrative numerical results. The numerical results shown in Table 5 result from applying the formulas to the arbitrary illustrative values displayed in Table 6 and are not necessarily typical of any company. Indeed, the marginal tax rates applicable to any particular company depend very much upon its own data.

In all cases the value $r = 0.499$ has been used. This consists of the 1974 statutory federal corporation tax rate of 48 per cent less 10 per cent abatement plus an average provincial-territorial corporation tax rate of 11.9 per cent. The value $p = 0.02$ has also been used; this is the provincial premium tax rate applied uniformly to life insurance (other than annuities) and other-than-life insurance (except automobile insurance and fire insurance).¹⁸

The marginal tax rate formulas and examples given in Table 5 for disbursements, losses, and other deduction items are all based on the assumption that the item in question may be used as a deduction in computing the company's over-all taxable income for the particular taxation year; that is, that there is sufficient income (or capital gains) against which the deductions (or losses) may be taken.

Where this is not the case (that is, where the company's taxable income for the year is zero), then the marginal tax rate for further disbursements or losses for the *current* year is zero. However, such a disbursement or loss will generally create a "non-capital loss" or a "net capital loss" for the current year, which may be claimed as a deduction in *another* taxation year at the marginal tax rates shown in Tables 5.21 and 5.22.

In using marginal tax rates, it should be remembered that some transactions, such as interest income, most general expense, and so on, can be viewed marginally, independent of other transactions. Others, notably premium income, really cannot be viewed independently, since a marginal dollar of premium income will always have some marginal deductions associated with it in the form of expenses, claims, and reserve increases. Thus the marginal tax rates shown in the table for premiums would have to be combined with the marginal rates on expenses, policy payments, reserve increases, and so on, each multiplied by the proportion of the premium that is automatically expended in that manner. These proportions will, of course, vary with the plan of insurance and with the age of the policy.

¹⁸ *Ibid.*, p. 126.

TABLE 5

MARGINAL TAX RATE FORMULAS AND ILLUSTRATIVE VALUES

5.1

Life insurance premiums, net of premium returns and experience-rating refunds, subject to premium tax considered in computing investment income tax (x_1, y_1)

| | |
|---|-------|
| I. $r + (1 - r)$ (greater of $\frac{1}{2}p, p - 0.01$) | 50.4% |
| II. $s + (1 - s)$ (greater of $\frac{1}{2}p, p - 0.01$) | 47.8 |
| III. $r + (1 - r)$ (greater of $\frac{1}{2}p, p - 0.01$) | 50.4 |
| IV. $q + (1 - q)$ (greater of $\frac{1}{2}p, p - 0.01$) | 39.6 |
| V. $t + (1 - t)$ (greater of $\frac{1}{2}p, p - 0.01$) | 42.7 |
| VI. $s + (1 - s)p$ | 48.3 |
| VII. $r + (1 - r)p$ | 50.9 |
| VIII. $r + (1 - r)p$ | 50.9 |

5.2

Life insurance premiums, net of premium returns and experience-rating refunds, subject to premium tax not considered in computing investment income tax (x_2, y_2)

| | |
|--------------------------------|-------|
| I. $r + (1 - r)p$ | 50.9% |
| II. $s + (1 - s)p$ | 48.3 |
| III. $r + (1 - r)p$ | 50.9 |
| IV. $q + (1 - q)p$ | 40.2 |
| V. $t + (1 - t)p$ | 43.2 |
| VI. $s + (1 - s)p$ | 48.3 |
| VII. $r + (1 - r)p$ | 50.9 |
| VIII. $r + (1 - r)p$ | 50.9 |

5.3

Life insurance premiums and considerations for annuities (including settlement annuities), not subject to premium tax (x_3, y_3)

Interest, rents, royalties, taxable dividends—segregated life (y_7, y_8, y_9)

Miscellaneous income, including interbranch transfers (positive or negative)—life (x_{15}, y_{15})

| | |
|---------------------|-------|
| I. r | 49.9% |
| II. s | 47.3 |
| III. r | 49.9 |
| IV. q | 39.0 |
| V. t | 42.1 |
| VI. s | 47.3 |
| VII. r | 49.9 |
| VIII. r | 49.9 |

TABLE 5—Continued

5.4

Interest, rents, royalties—non-segregated life (x_1)
 Taxable dividends received from corporations other than taxable
 Canadian corporations—non-segregated life (x_8)
 Accrual of discount and profit on sale of "Canada securities" (x_{12})

| | |
|----------------------------------|-------|
| I. $r + (1 - r)j$ | 57.1% |
| II. $s + (1 - s)j + cfr$ | 54.9 |
| III. $r + (1 - r)j - afr$ | 56.1 |
| IV. $s + (1 - s)ef + dfr$ | 47.9 |
| V. $r(1 - af) + (1 - r)ef$ | 49.4 |
| VIA. $r[1 - f(1 - b)]$ | 48.8 |
| VIB. $r[1 - f(1 - b)]$ | 47.3 |
| VIIA. $r(1 - af)$ | 48.8 |
| VIIIB. $r(1 - af)$ | 47.3 |
| VIII. r | 49.9 |

5.5

Taxable dividends received from taxable Canadian corporations—
 non-segregated life (x_8)

| | |
|--|-------|
| I. $r - (1 - r)k$ | 48.6% |
| II. $s(1 - c) - (1 - s)k$ | 44.5 |
| III. $as - (1 - r)k$ | 17.6 |
| IV. $(1 - f)[r(1 - d) - (1 - s)e]$ | 36.1 |
| V. $(1 - f)[ar - (1 - r)e]$ | 9.8 |
| VIA. $s(1 - b)$ | 19.4 |
| VIB. $s(1 - b)$ | 46.6 |
| VIIA. as | 18.9 |
| VIIIB. as | 46.1 |
| VIII. r | 49.9 |

5.6

Other-than-life insurance premiums subject to premium tax (z_2)

| | |
|----------------------|-------|
| $r + (1 - r)p$ | 50.9% |
|----------------------|-------|

5.7

Other-than-life insurance premiums not subject to premium tax
 (z_3)

Interest, rents, royalties, and taxable dividends received from
 corporations other than taxable Canadian corporations—other
 than life (z_7, z_8)

Miscellaneous income, including interbranch transfers (positive
 or negative)—other than life (z_{15})

| | |
|-----------|-------|
| r | 49.9% |
|-----------|-------|

TABLE 5—Continued

5.8

Taxable dividends received from taxable Canadian corporations—
other than life (z_8)

0..... 0%

5.9

Capital gains on disposition of property other than “Canada
securities” (x_{13} , y_{13} , z_{13})

Capital gains dividends (x_{14} , y_{14} , z_{14})

$\frac{1}{2}r$ 25.0%

5.10

Life insurance policy payments (x_{21} , y_{21})

Life insurance policy dividends deductible re income tax but not
re premium tax (x_{24} , y_{24})

Segregated fund allocations of revenue (y_{25})

Increase in life insurance policy reserve ($x_{27} - x_4$, $y_{27} - y_4$)

Increase in group life additional reserve ($x_{28} - x_5$)

Increase in life insurance policy dividends reserve ($x_{29} - x_6$,
 $y_{29} - y_6$)

Interest paid or credited—segregated life (y_{30})

Expenses, including licenses, fees, and taxes other than premium
taxes and income taxes; capital cost allowance and allowance
for other capital expenditures; bad debts—group life (nonin-
vestment) and segregated life (x_{36} , x_{39} , x_{42} , y_{35} , y_{37} , y_{38} , y_{40} , y_{41} , y_{43})

Negatives of formulas in Table 5.3

5.11

Life insurance policy dividends deductible re income tax and re
premium tax considered in computing investment income tax
(x_{22} , y_{22})

Negatives of formulas in Table 5.1

5.12

Life insurance policy dividends deductible re income tax and re
premium tax not considered in computing investment income
tax (x_{23} , y_{23})

Negatives of formulas in Table 5.2

TABLE 5—Continued

5.13

Life insurance policy dividends not deductible re income tax but deductible re premium tax considered in computing investment income tax (x_{31})

| | | |
|-------|--|-------|
| I. | $-(1 - r)$ (greater of $\frac{1}{2}p$, $p - 0.01$) | -0.5% |
| II. | $-(1 - s)$ (greater of $\frac{1}{2}p$, $p - 0.01$) | -0.5 |
| III. | $-(1 - r)$ (greater of $\frac{1}{2}p$, $p - 0.01$) | -0.5 |
| IV. | $-(1 - q)$ (greater of $\frac{1}{2}p$, $p - 0.01$) | -0.6 |
| V. | $-(1 - t)$ (greater of $\frac{1}{2}p$, $p - 0.01$) | -0.6 |
| VI. | $-(1 - s)p$ | -1.1 |
| VII. | $-(1 - r)p$ | -1.0 |
| VIII. | $-(1 - r)p$ | -1.0 |

5.14

Life insurance policy dividends not deductible re income tax but deductible re premium tax not considered in computing investment income tax (x_{32})

| | | |
|-------|-------------|-------|
| I. | $-(1 - r)p$ | -1.0% |
| II. | $-(1 - s)p$ | -1.1 |
| III. | $-(1 - r)p$ | -1.0 |
| IV. | $-(1 - q)p$ | -1.2 |
| V. | $-(1 - t)p$ | -1.2 |
| VI. | $-(1 - s)p$ | -1.1 |
| VII. | $-(1 - r)p$ | -1.0 |
| VIII. | $-(1 - r)p$ | -1.0 |

5.15

Interest paid or credited—non-segregated life (x_{30})

Increase in investment reserve ($x_{31} - x_{10}$)

Amortization of premium and loss on sale of "Canada securities" (x_{32})

Investment expenses, including licenses, fees, and taxes other than investment income tax and income taxes; capital cost allowance and allowance for other capital expenditures on investment property; bad debts re investments (other than defaulted loans)—non-segregated life (x_{35} , x_{38} , x_{41})

Negatives of formulas in Table 5.4

TABLE 5—Continued

5.16

General expenses, including licenses, fees and taxes other than premium taxes, investment income tax, and income taxes; capital cost allowance and allowance for other capital expenditures; bad debts—non-segregated life other than group life and investment (x_{37} , x_{40} , x_{42})

| | | |
|--------|--|--------|
| I. | $-r - \frac{1}{2}(1-r)j$ | -53.5% |
| II. | $-s - \frac{1}{2}(1-s)j - \frac{1}{2}cfr$ | -51.1 |
| III. | $-r(1 - \frac{1}{2}af) - \frac{1}{2}(1-r)j$ | -53.0 |
| IV. | $-s - \frac{1}{2}(1-s)cf - \frac{1}{2}dfr + \frac{1}{2}(1-q)g$ | -43.3 |
| V. | $-r(1 - \frac{1}{2}af) - \frac{1}{2}(1-r)ef + \frac{1}{2}(1-t)g$ | -45.7 |
| VIA. | $-r[1 - f(1 - \frac{1}{2}b)]$ | -48.0 |
| VIB. | $-r[1 - f(1 - \frac{1}{2}b)]$ | -47.3 |
| VIIA. | $-r(1 - \frac{1}{2}af)$ | -49.4 |
| VII B. | $-r(1 - \frac{1}{2}af)$ | -48.6 |
| VIII. | $-r$ | -49.9 |

5.17

Other-than-life insurance policy payments (z_{21})
 Other-than-life insurance policy dividends not deductible re premium tax (z_{24})
 Increase in other-than-life insurance policy reserve ($z_{27} - z_4$)
 Increase in other-than-life insurance policy dividends reserve ($z_{29} - z_6$)
 Interest paid or credited—other than life (z_{30})
 Increase in mortgage reserve—other than life ($z_{32} - z_{11}$)
 Expenses, including licenses, fees, and taxes other than premium taxes and income taxes; capital cost allowance and allowance for other capital expenditures; bad debts—other than life (z_{35} , z_{37} , z_{39} , z_{40} , z_{41} , z_{43})

| | |
|------|--------|
| $-r$ | -49.9% |
|------|--------|

5.18

Other-than-life policy dividends deductible re premium tax (z_{23})

| | |
|---------------|--------|
| $-r - (1-r)p$ | -50.9% |
|---------------|--------|

5.19

Segregated fund allocations of capital gains (y_{26})
 Capital losses on disposition of property other than "Canada securities" (x_{34} , y_{34} , z_{34})

| | |
|-----------------|--------|
| $-\frac{1}{2}r$ | -25.0% |
|-----------------|--------|

TABLE 5—Continued

5.20

Policyholder gains (*F*)

| | | |
|----------------|----------------|-------|
| I. | $-0.15(1 - r)$ | -7.5% |
| II. | $-0.15(1 - s)$ | -7.9 |
| III. | $-0.15(1 - r)$ | -7.5 |
| IV. | $-0.15(1 - q)$ | -9.1 |
| V. | $-0.15(1 - t)$ | -8.7 |
| VI, VII, VIII. | 0 | 0 |

5.21

"Interest element" on "existing fixed-premium" and registered plans (*R*)

| | | |
|-------|------------------|-------|
| I. | $-(1 - r)g$ | -6.8% |
| II. | $-(1 - s)g$ | -7.1 |
| III. | $-(1 - r)g + fr$ | -4.1 |
| IV. | $-(1 - q)g$ | -5.3 |
| V. | $-(1 - t)g + fr$ | -5.2 |
| VI. | 0 | 0 |
| VII. | fr | +2.6 |
| VIII. | 0 | 0 |

5.22

Deductible non-capital losses from other years (*L*)

| | | |
|----------------|-----------|--------|
| I, II, III. | $-r$ | -49.9% |
| IV. | $-q - fr$ | -41.7 |
| V. | $-t$ | -42.1 |
| VI, VII, VIII. | $-r$ | -49.9 |

5.23

Deductible net capital losses from other years (*W*)Deductible gifts to charitable organizations, etc. (*N*)

| | |
|------|--------|
| $-r$ | -49.9% |
|------|--------|

5.24

"Excess" dividends to shareholders (*Z*)

| | |
|------|-------|
| $2r$ | 99.8% |
|------|-------|

TABLE 6

ILLUSTRATIVE VALUES

Data Used in All Situations:

| | | |
|-------------|----------------|-------------|
| $S = 1,000$ | $M + 2Z = 100$ | $r = 0.499$ |
| $D = 53$ | $f = 0.053$ | $s = 0.473$ |
| $F = 20$ | $g = 0.135$ | $t = 0.421$ |
| $X = 16$ | $p = 0.02$ | $q = 0.390$ |

| | SITUATION | | | | | | | | | | |
|------------------------------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | I | II | III | IV | V | VIA | VIB | VIIA | VIIIB | VIIIA | VIIIB |
| R | 400 | 400 | 400 | 400 | 400 | 400 | 975 | 400 | 975 | 975 | 1025 |
| $S-R$ | 600 | 600 | 600 | 600 | 600 | 600 | 25 | 600 | 25 | 25 | -25 |
| K | 70 | 70 | 70 | 70 | 70 | 70 | -7.6 | 70 | -7.6 | -7.6 | -14.4 |
| G | 50 | 100 | 700 | 100 | 700 | 590 | 15 | 700 | 50 | -10 | 100 |
| L | 30 | 80 | 680 | 20 | 620 | 10 | 10 | 20 | 30 | 0 | 20 |
| J | 20 | 20 | 20 | 80 | 80 | 580 | 5 | 680 | 20 | -10 | 80 |
| gJ | | | | 10.8 | 10.8 | 78.3 | | 91.8 | | | |
| gL | | | | 2.7 | 83.7 | | | | | | |
| $G-K$ | -20 | 30 | 630 | 30 | 630 | | | | | | |
| $(G-K-gL)/$ $(1-g)$ | | | | 31.6 | 631.6 | | | | | | |
| $G-K-L$ | | | | 10 | 10 | | | | | | |
| $P-L$ | -50 | -50 | -50 | 11.6 | 11.6 | 580 | 5 | 680 | 20 | -10 | 80 |
| I | 580 | 580 | 580 | 568.4 | 568.4 | 0 | 0 | 0 | 0 | 5 | 0 |
| B | 70 | 70 | 70 | 68.4 | 68.4 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 25.0 | 24.2 | 9.1 | 54.8 | 39.8 | 323.7 | 52.0 | 373.4 | 59.2 | 44.9 | 89.8 |
| a | | | 0.400 | | 0.400 | | | 0.400 | 0.975 | | |
| b | | | | | | 0.590 | 0.015 | | | | |
| c | | 0.030 | | | | | | | | | |
| d | | | | 0.032 | | | | | | | |
| e | | | | 0.193 | 0.193 | | | | | | |
| j | 0.144 | 0.144 | 0.144 | 0.144 | 0.144 | | | | | | |
| k | 0.026 | 0.026 | 0.026 | | | | | | | | |

While the marginal tax rates applicable to any particular company depend on its own data, they will tend to remain reasonably stable as long as the company remains in the same tax situation. Thus, if the company remains in the same situation from year to year, then the marginal tax rates should prove useful in premium and dividend determination by indicating the effective after-tax investment results and after-tax "cost" of various forms of disbursement.

If a company's tax position is not stable but shifts from one situation to another from year to year, the formulas highlight the effect of the shift on after-tax results. It should be noted particularly that, whereas T is a continuous function that is uniquely determined at each boundary point as a company shifts from one situation to another, the associated marginal tax rates are generally discontinuous at the boundaries of each situation.

IV. DISCLAIMER

This paper has been prepared primarily as a reference for the use of Fellowship students and other members of the Society of Actuaries and is not intended as a legal or accounting interpretation of the provisions of the tax laws and regulations.

V. ACKNOWLEDGMENTS

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DISCUSSION OF PRECEDING PAPER

BRIAN R. NEWTON:

Actuaries responsible for the affairs of Canadian life insurers will be greatly indebted to Mr. Whaley for the provision of a fine body of work and formulas on which they may draw. To analyze the multiplicity of taxation phases rigorously and impose order on the presentation is no mean feat.

To illustrate the possibilities for actuaries to explore, I have manipulated some of Mr. Whaley's formulas presented in an earlier paper to produce a graphical representation of the taxation structure affecting life insurance policyholders' funds in Canada. The graphical approach and tentative conclusions are discussed in detail in a paper that I presented to the Canadian Institute of Actuaries on October 17, 1974. For convenience I have summarized the basic material using the notation of Mr. Whaley's latest paper. To produce a simplified structure, assumptions similar to those in Table 3 of Mr. Whaley's paper were made, except that D and subsequently f were retained as variables.

Allowing business income, G , to increase steadily, the following formulas were used. (Numbers correspond to paragraphs in my own paper.)

If $G \leq K$, where K is positive, Part XII tax eliminates liability to business tax:

$$C = 0, \quad B = K. \quad (2.2)$$

If business income $G > K$ but $G \leq K/g$,

$$C = r(1 - f) \frac{G - K}{1 - g}, \quad B = K - g \frac{G - K}{1 - g}. \quad (2.3)$$

If $G \geq K/g$, business income is large enough to eliminate Part XII tax, but relief in respect of taxable dividends from taxable Canadian corporations is available if $G < S - R$:

$$C = r(1 - f)G = r(G - fG), \quad B = 0. \quad (2.4)$$

If $G > S - R$, relief in respect of taxable dividends from taxable Canadian corporations is restricted:

$$C = r[G - f(S - R)], \quad B = 0. \quad (2.5)$$

A generalized form would be

$$C \doteq r \left[\underset{(A)}{G} - \underset{(B)}{fG} - \frac{\underset{(C)}{K} - \underset{(D)}{gG}}{1 - \underset{(E)}{g}} (1 - f) \right], \quad (2.6)$$

$$B = \underset{(D)}{K} - \underset{(E)}{g} \frac{\underset{(C)}{G} - \underset{(D)}{K}}{1 - \underset{(E)}{g}}.$$

In term (B), G is limited to a maximum of $S - R$; term (C) = $B(1 - f)$; and term (E) has a maximum of K and a minimum of 0.

If K is negative, the situation degenerates:

$$C = r \left(\underset{(A)}{G} - \underset{(B)}{fG} \right), \quad (2.7)$$

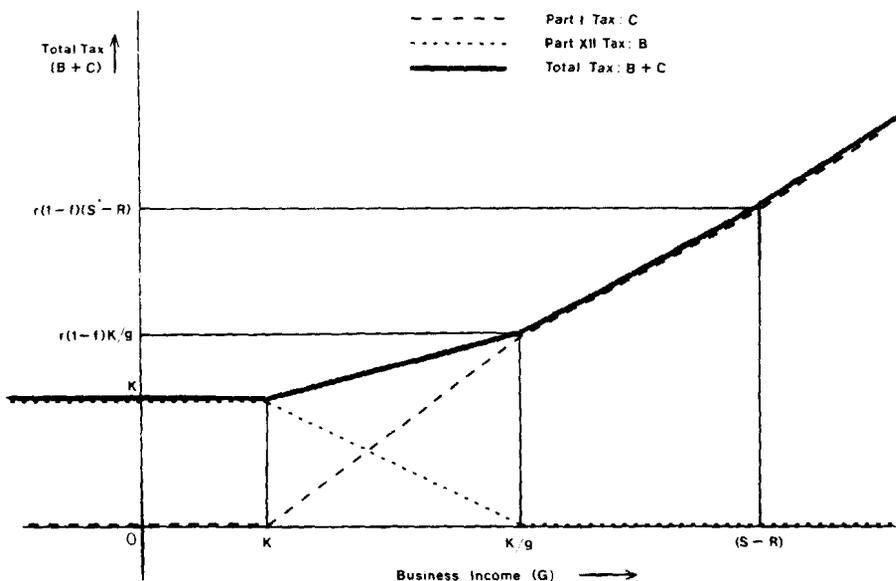
with G limited to a maximum of $S - R$ in term (B), and

$$B = 0.$$

Chart I is developed from the preceding formulas.

Through correspondence with Mr. Whaley he has related his formulas to mine. I agree with his relating of formula (2.2) with Situations I, II, and III; (2.3) with IV; (2.4) with VI; (2.5) with VII; and (2.7) with VIB and VIIB. Situations V and VIII appear not to be specifically covered. However, juggling with the boundary conditions for Situation V

CHART I



(if the ordering in the graph of $S - R > K > 0$ is accepted), I believe that only one value of G , namely, $G = K/g$, exists to satisfy the conditions, and thus Situation V degenerates. Situation V may exist if the ordering in the graph is ignored. Situation VIII is covered by formula (2.7) by adding restrictions for $G \leq 0$ and term (B) when $S - R$ is negative, which are easily visualized from the graph.

On the basis of the graphic presentation, it was postulated that in certain circumstances the taxation of policyholder funds could be regressive or threaten the solvency of companies. Also inherent in the system is considerable risk of double taxation, because, as tax liability is passed on to policyholders, the insurance funds do not necessarily receive credits to offset completely the taxes levied on the funds.

Chart I also indicates that the prorating procedure in respect of dividends from taxable Canadian corporations can place life insurance funds in a disadvantageous position compared to banks and trust companies.

To form a basis for discussion, amendments to the income tax structure were suggested, in keeping with a flow-through philosophy that the insurance company redistributes capital among claimants and that annuitants and policyholders surrendering contracts are taxed on the interest element of proceeds. The suggested amendments included removal of Part XII taxation and the limitation on deductions in respect of policyholder dividends, if the government could not bring itself to revert to the pre-1969 basis for taxation at the company level.

It should be noted that in the discussion of my paper some controversy arose in relation to the flow-through concept, the perception of the relative level of taxation, and the implications of formula (2.4). These issues indicate that further study of the Canadian tax system is needed by all actuaries concerned. Mr. Whaley has laid an excellent foundation for that study.

GRAHAM R. MC DONALD:

Once again, Mr. Whaley has published a very important paper. He is to be commended especially for his fine mathematical analysis—particularly the development and discussion of the various tax situations and the treatment of marginal tax rates. In fact, this paper should be required reading for anyone daring to venture into the subject of the Canadian income taxation of insurance companies.

Unfortunately, the development of the various tax situations is a very complex matter and Mr. Whaley has given the readers only the results. For example, those attempting to prove that the situations given in Table 2 are mutually exclusive may have a very difficult time unless they

follow a methodology similar to Mr. Whaley's. Basically, it appears that he took the basic formula for the Part XII tax,

$$B = K - g[(G - B \geq 0) - L \geq 0] \geq 0,$$

and defined all the possible conditions and the resulting B 's, as shown below:

| Situation | Condition | B |
|-----------|--|------------------------------|
| B1. | $K \geq 0, G - B \leq 0$ | $K \geq 0$ |
| B2. | $K \geq 0, G - B - L \leq 0 \leq G - B$ | $K \geq 0$ |
| B3. | $G - L \geq K \geq g(G - L) \geq 0^*$ | $\frac{K - g(G - L)}{1 - g}$ |
| B4. | $g(G - L) \geq K \geq 0^\dagger$ | 0 |
| B5. | $K \leq 0$ | 0 |
| B6. | $S - R \leq 0$ | 0 |

* This is a combined form of $(G - B \geq 0) - L \geq 0$ and $K \geq g(G - L) \geq 0$.

† This is a combined form of $(G - B \geq 0) - L \geq 0$ and $g(G - L) \geq K \geq 0$.

The same was done with the basic Part I tax formula, $C = r\{[G - B - L - f \min(G - B \geq 0, S - R \geq 0) + M \geq 0] + 2Z\}$:

| Situation | Condition | C |
|-----------|--------------------------------|---|
| C1. | $S - R \geq G - B \geq 0$ | $r\{(1 - f)(G - B) - L + M \geq 0\} + 2Z\}$ |
| C2. | $S - R \geq 0 \geq G - B$ | $r\{[G - B - L + M \geq 0] + 2Z\}$ |
| C3. | $G - B \geq S - R \geq 0$ | $r\{[G - B - L - f(S - R) + M \geq 0] + 2Z\}$ |
| C4. | $S - R \leq 0$ | $r\{[G - B - L + M \geq 0] + 2Z\}$ |

The situations given in Table 2 of Mr. Whaley's paper are then derived by matching each of situations B1, B2, B3, B4, B5, and B6 with C1, C2, C3, and C4. For example, the combination B2/C2 leads to Whaley's Situation III. The difficulty in proving "mutual exclusiveness" from Table 2 arises in part because certain of the B/C combinations are impossible and have been omitted from the table (e.g., B1/C1, B3/C4).

In using the above approach, I agree with all of Mr. Whaley's situations if the following inequality holds:

$$L > \frac{0.15F + 0.5X}{g} > 0.$$

This is unlikely, since most companies in a loss position will find it advantageous to underclaim their actuarial reserves in order to effect an indefinite loss carry-forward rather than be subject to the "one year back, five years forward" limitation.

If $0 \leq L \leq (0.15F + 0.5X)/g$, then I believe that Situation V (i.e., B3/C3) is impossible.

Consider the conditions for B3, that is,

$$(G - B \geq 0) - L \geq 0, \quad (a)$$

$$K \geq g(G - L) \geq 0. \quad (b)$$

Then, since the condition for C3 is $G - B \geq S - R \geq 0$, in order to show that B3/C3 is not always valid, it is only necessary to prove that

$$B3 \Rightarrow S - R \geq G - B \geq 0$$

in certain cases.

From inequalities (a) and (b) it follows that

$$B = \frac{K - g(G - L)}{1 - g}. \quad (c)$$

Therefore,

$$\begin{aligned} G - B &= G - \frac{K - g(G - L)}{1 - g} \\ &= \frac{G - K - gL}{1 - g}. \end{aligned} \quad (d)$$

But $K = g(S - R) - 0.15F - 0.5X$, as defined by Mr. Whaley, or

$$\begin{aligned} S - R &= \frac{K + 0.15F + 0.5X}{g} \\ &= \frac{K + Y}{g}, \end{aligned} \quad (e)$$

where $Y = 0.15F + 0.5X \geq 0$.

Now, given inequalities (a) and (b), assume C3, that is,

$$G - B \geq S - R \geq 0. \quad (f)$$

Combining inequalities (d), (e), and (f), we have

$$\frac{G - K - gL}{1 - g} \geq \frac{K + Y}{g} \geq 0 \quad (g)$$

$$\Rightarrow g(G - K - gL) \geq (1 - g)(K + Y) \geq 0$$

$$\Rightarrow gG \geq K + g^2L + (1 - g)Y$$

$$\Rightarrow g(G - L) \geq K + (1 - g)(Y - gL) \quad (h)$$

$$\Rightarrow g(G - L) > K \quad (i)$$

$$\text{if } (1 - g)(Y - gL) > 0 \quad \text{or } Y - gL > 0.$$

But inequality (i) is inconsistent with inequality (b). Therefore, B3/C3 is impossible if $Y - gL > 0$, that is,

$$L < \frac{Y}{g} = \frac{0.15F + 0.5X}{g}.$$

Hence, to be completely accurate, Situation V should be split into two situations: (i) a new Situation V with the conditions given for Situation V in Table 2, plus the additional condition

$$0.15F + 0.5X - gL \leq 0$$

giving the tax formula as shown for Whaley's Situation V, and (ii) Situation VA with conditions as given in Table 2, plus

$$0.15F + 0.5X - gL > 0, \quad (c')$$

which is an impossible situation and therefore should be omitted from Table 2.

The above analyses can be simplified by assuming $L = 0$. In this case, inequality (h) above becomes

$$\begin{aligned} gG &\geq K + (1 - g)Y \\ &\Rightarrow gG \geq K, \end{aligned} \quad (i')$$

and Whaley's Situation V becomes impossible, since $Y \geq 0$ and inequality (i') is inconsistent with inequality (b) unless $Y = 0$.

At Great-West Life we developed five basic tax situations depending on the values of B , G , and $S - R$ (ignoring "deductible noncapital losses from other years" [L], since, if the situation arises, we believe it would be advantageous to underclaim actuarial reserves to force C to equal zero rather than to declare a loss). The conditions, shown in the accompanying tabulation, give a formula in each case for the direct calculation of B

| Situation | Condition |
|-----------|---------------------------------|
| 1..... | $B > 0, G \geq B$ |
| 2..... | $B > 0, G < B$ |
| 3..... | $B = 0, G \geq 0, S - R > G$ |
| 4..... | $B = 0, G \geq 0, S - R \leq G$ |
| 5..... | $B = 0, G < 0$ |

and a formula for C as a function of B . We analyzed our ultimate situations and found them to correspond exactly with Mr. Whaley's when $L = 0$ (given that his Situation V is impossible). In order to analyze which situation a particular company was in, we developed a simple

“truth table” based on the various inequalities involved. Mr. Whaley does the same thing in much more compact form.

The other technical aspect of Mr. Whaley’s paper I would like to discuss is marginal tax rates. Having developed marginal rates for our company, I can well appreciate the great deal of time and effort involved in deriving the formulas and calculating the rates for the eleven illustrative cases.

A marginal tax rate is simply the “change in tax” for a very small change in one of the variables—in calculus terms, $\partial T/\partial x_n$, where $T = f(x_1, x_2, \dots, x_k)$. Whether or not the marginal rate can be used to calculate accurately the ΔT for a discrete Δx_i depends on the mathematical properties of the tax formula for T .

A function $f(x_1, x_2, \dots, x_k)$ is a homogeneous function of degree n if, for all values of x_1, x_2, \dots, x_k and for each real number m ,

$$f(mx_1, mx_2, \dots, mx_k) = m^n f(x_1, \dots, x_k).$$

A homogeneous function f of degree n has the property that¹

$$\sum_{i=1}^k x_i \frac{\partial f}{\partial x_i} = n f. \quad (1)$$

If the equation is of the first degree, then

$$\sum_{i=1}^k x_i \frac{\partial f}{\partial x_i} = f. \quad (2)$$

Analysis of the function T , the total tax, shows that it is indeed a homogeneous function of degree 1, since, if the values of x_1, x_2, \dots, x_k are all multiplied by a constant m , then the value of T is also multiplied by m . (In fact, both B and C are homogeneous functions of degree 1). Therefore, the total tax can be expressed as a linear compound function of the tax variables x_1, \dots, x_k , that is,

$$T(x_1, x_2, \dots, x_k) = x_1 \frac{\partial T}{\partial x_1} + x_2 \frac{\partial T}{\partial x_2} + \dots + x_k \frac{\partial T}{\partial x_k}. \quad (3)$$

Let $T_i(x_1, x_2, \dots, x_k) = \partial T(x_1, x_2, \dots, x_k)/\partial x_i$. Let $\Delta T = T(x'_1, x'_2, \dots, x'_k) - T(x_1, x_2, \dots, x_k)$, where x'_1, x'_2, \dots, x'_k represent the new values and x_1, x_2, \dots, x_k represent the previous values of the variables.

¹ This type of analysis is described for United States federal income tax in a discussion by Cecil J. Nesbitt and Donald A. Jones of John C. Fraser’s classic paper “Mathematical Analysis of Phase 1 and Phase 2 of the ‘Life Insurance Income Tax Act of 1959’” (*TSA*, XIV, 130).

In order for marginal tax analysis to be completely accurate, the following equation must hold:

$$\begin{aligned}\Delta T &= T(x'_1, \dots, x'_k) - T(x_1, \dots, x_k) \\ &= \sum_{i=1}^k (x'_i - x_i) T_i(x_1, x_2, \dots, x_k),\end{aligned}\quad (4)$$

where $x'_i - x_i = \Delta x_i$. If equation (4) holds, then ΔT can be computed exactly by multiplying the *change* in variables by the respective marginal tax rates (T_1, T_2, \dots, T_k) and adding the products. However, from equation (3),

$$T(x_1, x_2, \dots, x_k) = \sum_{i=1}^k x_i T_i(x_1, x_2, \dots, x_k), \quad (5)$$

$$T(x'_1, x'_2, \dots, x'_k) = \sum_{i=1}^k x'_i T_i(x'_1, x'_2, \dots, x'_k). \quad (6)$$

But

$$\begin{aligned}x'_i T_i(x'_1, x'_2, \dots, x'_k) - x_i T_i(x_1, x_2, \dots, x_k) \\ \equiv (x'_i - x_i) T_i(x_1, x_2, \dots, x_k) \\ + x'_i [T_i(x'_1, x'_2, \dots, x'_k) - T_i(x_1, x_2, \dots, x_k)].\end{aligned}\quad (7)$$

Subtracting equation (5) from equation (6) and using equation (7), we obtain

$$\begin{aligned}T(x'_1, x'_2, \dots, x'_k) - T(x_1, x_2, \dots, x_k) \\ = \sum_{i=1}^k (x'_i - x_i) T_i(x_1, x_2, \dots, x_k) \\ + \sum_{i=1}^k x'_i [T_i(x'_1, x'_2, \dots, x'_k) - T_i(x_1, x_2, \dots, x_k)].\end{aligned}\quad (8)$$

Therefore, equation (4) holds only if

$$\sum_{i=1}^k x'_i [T_i(x'_1, x'_2, \dots, x'_k) - T_i(x_1, x_2, \dots, x_k)] = 0. \quad (9)$$

In most cases, equation (9) will hold only if

$$T_i(x'_1, x'_2, \dots, x'_k) = T_i(x_1, x_2, \dots, x_k) \quad \text{for } i = 1, 2, \dots, k. \quad (10)$$

In general, equation (10) will not hold; that is, the marginal rates for all variables will not remain constant from one period to another. Nevertheless, most companies will find the rates surprisingly stable, and for most

practical purposes equation (10) and, hence, equation (4) can be assumed to be true. Each company should, however, test the sensitivity of its marginal rates to changes in the variables. Some are extremely stable, even with major shifts in the relative and absolute values of the variables resulting from changes in type of business written and so on, as long as the company remains in the same tax situation. Marginal tax analysis becomes much more complex for companies that shift frequently from one situation to another.

In our company we developed marginal tax rates in much the same way as Mr. Whaley, except that we calculated the Part I and Part XII components of the marginal tax rates as well as the total rates. In addition, we computed marginal tax rates only for the variables listed in the accompanying tabulation. We have found that virtually all transactions

| | Whaley's Notation |
|---|---|
| 1. Life gain before Part XII tax..... | <i>G</i> |
| 2. Health gain..... | <i>H</i> |
| 3. Charitable donations..... | <i>N</i> |
| 4. Gross Canadian life investment income..... | $\sum_7^{12} x_n - \sum_{31}^{33} x$ |
| 5. Stock dividends in the life account..... | <i>D</i> |
| 6. Stock dividends in the health account..... | <i>E</i> |
| 7. Life investment expenses..... | x_{38} |
| 8. Life "interest paid"..... | x_{30} |
| 9. Life capital cost deduction (Part XII)..... | x_{35} |
| 10. 50% of allowable general expenses..... | $\frac{1}{2}(x_{37} + x_{40} + x_{43})$ |
| 11. Interest element of registered and "fixed" plans..... | <i>R</i> |
| 12. Amounts taxable to policyholders..... | <i>F</i> |
| 13. Premium tax deduction under Part XII..... | 0.5 <i>X</i> |

can be analyzed with these marginal tax rates. However, as Mr. Whaley correctly points out, the use of marginal tax rates can be hazardous; it is extremely important to make sure that *all* the elements of the transaction being studied are considered. Some items, such as \$1 of life investment income, will affect both *G* and gross Canadian life investment income. Premiums, on the other hand, affect *G* or *H* and *X* and are usually accompanied by other items which will be simultaneously affected (such as reserves, expenses, and the like).

The purpose of splitting the marginal rates into the Part XII portion and the Part I portion is that these elements can be helpful in analyzing tax implications in situations where a company has underclaimed actuarial reserves in order to avoid taking a business loss.

Once again, I would like to congratulate Mr. Whaley for giving us such a fine contribution to the actuarial literature. I am sure that it will stand the test of time and will be a major reference for tax practitioners in life insurance companies subject to Canadian income tax.

(AUTHOR'S REVIEW OF DISCUSSION)

RAYMOND L. WHALEY:

A picture is worth a thousand words. Mr. Newton's graph of the life insurance segment of an insurance corporation's total income tax (Chart I) is a great help in demonstrating the relationship between income tax (referred to as "Part I tax" in his discussion) and investment income tax (referred to as "Part XII tax").

For his illustration he has chosen an example slightly less simplified than that given in Table 3 of my paper, by letting $E = H = L = N = U = V = W = Z = 0$. It follows that $M = 0$, $J = G$, and the values of B and C are those shown in Table 1 of this discussion. For his graph Mr. Newton has selected the usual conditions $g > 0$ and $0 < K < S - R$, and then, with G as his independent variable, has illustrated the four possible situations, which are listed here in Table 2.

There are two other possibilities which Mr. Newton did not graph. One is the condition $K \leq 0 < S - R$, that is, where we let K in his graph go to zero or become negative. This gives rise to the situations listed in Table 3. The other possibility is the condition where $S - R \leq 0$, that is, where we also let $S - R$ go to zero or become negative. In

TABLE 1

| Situation | Conditions | B | C | B + C |
|-----------------|---|------------------------|---------------------------|------------------|
| I..... | $K \geq 0$ $S - R \geq 0 \geq G - K$ | K | 0 | K |
| IV..... | $G \geq K \geq gG \geq 0$ $S - R \geq 0$ | $\frac{K - gG}{1 - g}$ | $\frac{r(1-f)(G-K)}{1-g}$ | $qG + (1-q)K$ |
| VIA, VIB..... | $gG \geq K$ $S - R \geq G \geq 0$ | 0 | $r(1-f)G = sG$ | sG |
| VIIA, VIIB..... | $gG \geq K$ $G \geq S - R \geq 0$ | 0 | $r\{G - f(S - R)\}$ | $rG - rf(S - R)$ |
| VIIIA..... | $K \leq 0$ $S - R \geq 0 \geq G$ | 0 | 0 | 0 |
| VIIIB..... | $S - R \leq 0$ | 0 | $rG \geq 0$ | $rG \geq 0$ |

this case, only Situation VIII B is possible, and we have $B = 0$, whence $B + C = C = rG \geq 0$.

The graphs of the above three alternatives are shown as Charts II, III, and IV. In Chart II, which is similar to Mr. Newton's, it will be noted that the slopes of the four linear segments of the function $B + C$ are 0, q , s , and r , indicating a progressively higher rate of tax (up to the full corporation tax rate r) on business income G as an increasing ratio of G relative to K moves a company from one tax situation to another. These tax rates correspond to those given in Table 5.3 of my paper for Situations⁸

TABLE 2

| Situation | Conditions | B | C | B + C |
|-----------|-----------------------------|------------------------|---------------------------|------------------|
| I..... | $G \leq K$ | K | 0 | K |
| IV..... | $0 < K \leq G \leq K/g$ | $\frac{K - gG}{1 - g}$ | $\frac{r(1-f)(G-K)}{1-g}$ | $qG + (1-q)K$ |
| VIA..... | $0 < K/g \leq G \leq S - R$ | 0 | $r(1-f)G = sG$ | sG |
| VIIA..... | $0 < S - R \leq G$ | 0 | $r[G - f(S - R)]$ | $rG - rf(S - R)$ |

TABLE 3

| Situation | Conditions | B | C = B + C |
|------------|-----------------------|---|-------------------|
| VIIIA..... | $G \leq 0 < S - R$ | 0 | 0 |
| VIB..... | $0 \leq G \leq S - R$ | 0 | $r(1-f)G = sG$ |
| VIIIB..... | $0 < S - R \leq G$ | 0 | $r[G - f(S - R)]$ |

IV, VI, and VII. As noted in the paper, however, the marginal tax rate formulas given in Table 5 do not apply when $C = 0$ and hence do not apply in Situation I in the simplified example given here.

There is, therefore, some danger in using these graphs that one might conclude that there is no tax on life insurance business income if the company is in Situation I. This would be true if the company conducted only a life insurance operation. However, if there is income from other than life insurance operations, then a tax on life insurance transactions can arise even in Situation I. This is shown algebraically in my paper and becomes evident geometrically if we choose a more general case than the one used by Mr. Newton. The completely general case, for which the algebra is given in Table 2 of my paper, is rather formidable to graph.

CHART II

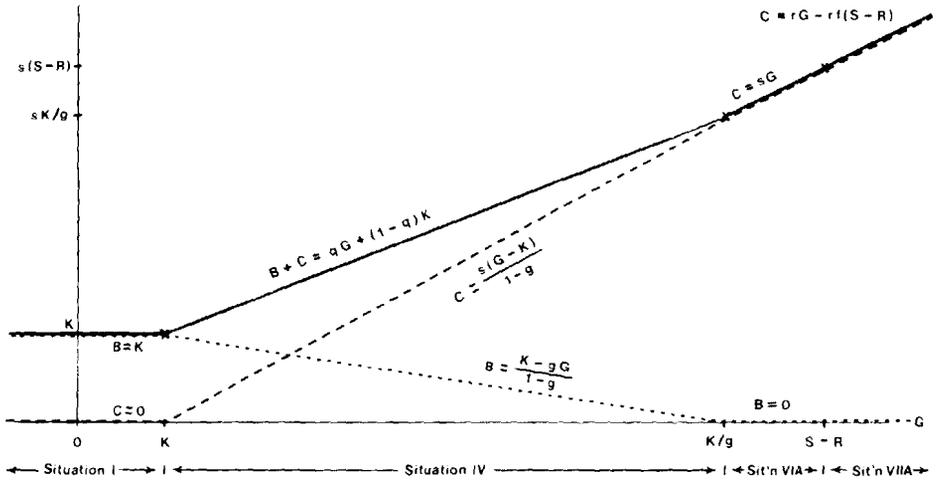


CHART III

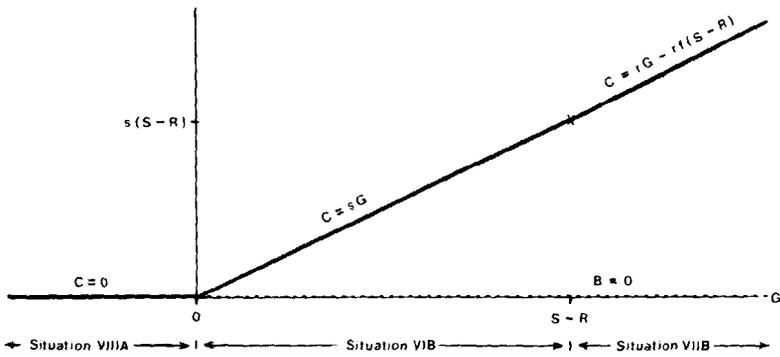
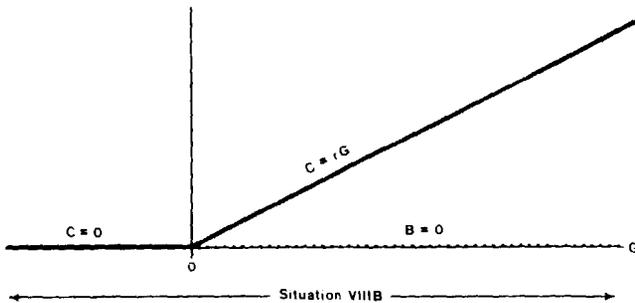


CHART IV



However, the geometry becomes manageable if we simplify the algebra slightly by letting $L = Z = 0$. If we again take G as our independent variable, we again have three graphs, depending on whether K and/or $S - R$ are positive or negative.

If $0 < K < S - R$, there are five possible situations, and these are listed in Table 4. If $K \leq 0 < S - R$, there are four possible situations, which are listed in Table 5. Finally, if $S - R \leq 0$, only Situation VIII B

TABLE 4

| Situation | Conditions | B | C | $B - C$ |
|-----------|-------------------------|------------------------|--|-----------------------|
| IA..... | $G \leq K - M$ | K | 0 | K |
| IB..... | $K - M \leq G \leq K$ | K | $r(G - K + M)$ | $rG + (1 - r)K + rM$ |
| IV..... | $K \leq G \leq K/g$ | $\frac{K - gG}{1 - g}$ | $r \left[\frac{1 - f}{1 - g} (G - K) + M \right]$ | $qG + (1 - q)K + rM$ |
| VIA..... | $K/g \leq G \leq S - R$ | 0 | $r[(1 - f)G + M]$ | $sG + rM$ |
| VIIA.... | $S - R \leq G$ | 0 | $r[G - f(S - R) + M]$ | $rG - rf(S - R) + rM$ |

TABLE 5

| Situation | Conditions | B | $C = B + C$ |
|----------------|-----------------------|-----|-----------------------|
| VIIIA(i)..... | $G \leq -M$ | 0 | 0 |
| VIIIA(ii)..... | $-M \leq G \leq 0$ | 0 | $r(G + M)$ |
| VIB..... | $0 \leq G \leq S - R$ | 0 | $r[(1 - f)G + M]$ |
| VII B..... | $0 < S - R \leq G$ | 0 | $r[G - f(S - R) + M]$ |

is possible, and we have $B = 0$ and $B + C = C = r(G + M) \geq 0$. Graphs of the above three alternatives are shown as Charts V, VI, and VII. These graphs become the same as those in Charts II, III, and IV, respectively, if $M = 0$.

Chart V illustrates the condition $-(1 - r)K/r < K - M < 0 < K$. An essentially similar graph would illustrate the condition $0 < K - M < K$. However, if $K - M < -(1 - r)K/r$, that is, if $M > K/r$, then the line B would intersect the line C in Situation I rather than in Situation IV. Under any of these conditions, the graph serves to demonstrate that in Situation I there can be an effective tax on income arising from life insurance operations if $M > 0$.

Mr. Newton and Mr. McDonald have both observed that Situation V degenerates if $L = 0$. In the interest of clarity, I have accordingly added

CHART V

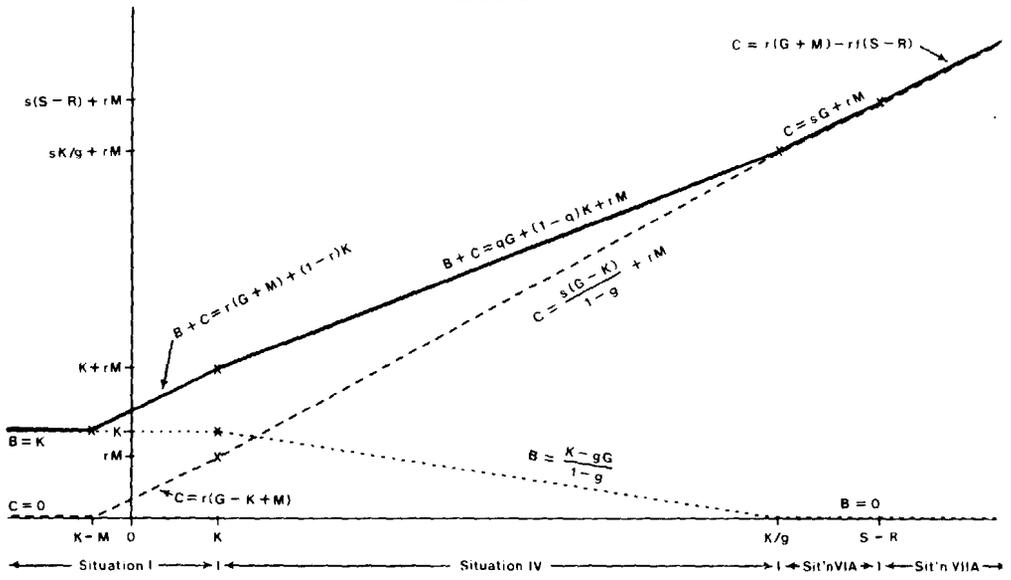


CHART VI

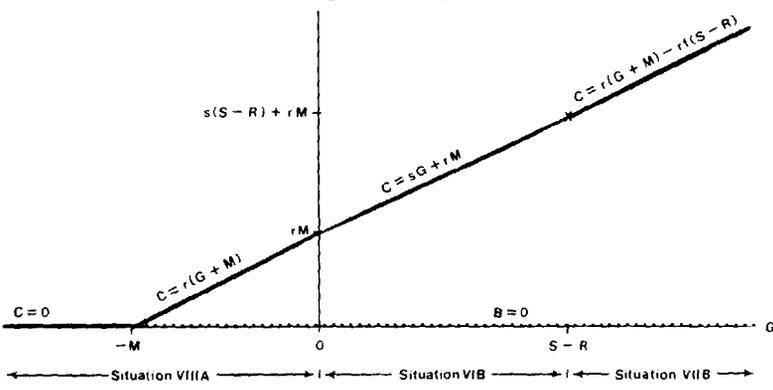
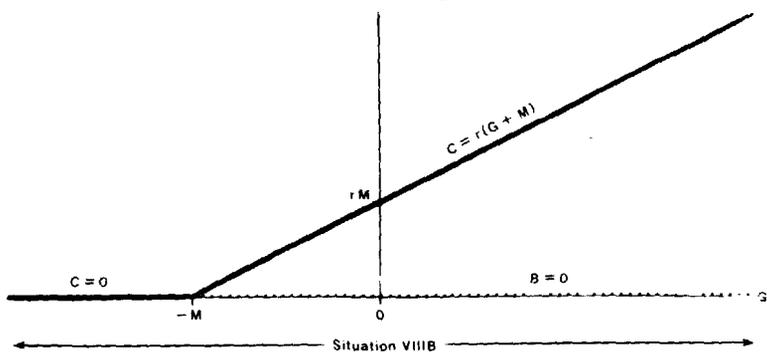


CHART VII



a sentence to the text of my paper to the effect that Situations II, III, and V all become superfluous when $L = 0$.

Mr. McDonald has demonstrated neatly how the conditions which define the various situations in Table 2 of my paper can be derived. He has listed correctly the set of possible conditions {B1, B2, . . . , B6} for the formula for B and the corresponding set {C1, C2, C3, C4} for the formula for C . He has then noted that certain combinations of these conditions are impossible¹ and has deduced that only those combinations which are possible were listed in Table 2. A complete tabulation of the relationship between Mr. McDonald's sets of conditions and those in Table 2 is shown here in Table 6; impossible combinations are denoted by an asterisk.

TABLE 6

| FORMULA C CONDITIONS | FORMULA B CONDITIONS | | | | | |
|-------------------------|----------------------|-----|----|------|-------|-------|
| | B1 | B2 | B3 | B4 | B5 | B6 |
| C1..... | * | II | IV | VIA | VIB | * |
| C2..... | I | * | * | * | VIIIA | * |
| C3..... | * | III | V | VIIA | VIIIB | * |
| C4..... | * | * | * | * | * | VIIIB |

* Impossible combination.

Mr. McDonald has proved that my Situation V (his combination B3/C3) is impossible not only if $L = 0$ but also if $L < (0.15F + 0.5X)/g$. He goes on to suggest, first, that accordingly my Situation V should be subject to the condition $gL \geq 0.15F + 0.5X$ (in addition to the conditions shown in Table 2) and, secondly, that an additional Situation VA be defined to cover the impossible situation when $gL < 0.15F + 0.5X$.

The condition $gL \geq 0.15F + 0.5X$ also can be stated as $gL \geq g(S - R) - K$. It follows that the second condition given in Table 2 for Situation V could be expanded as follows:

$$\frac{G - g(S - R)}{1 - g} \geq \frac{G - K - gL}{1 - g} \geq S - R \geq 0.$$

However, the first inequality is redundant because the condition that Mr. McDonald suggests, while perfectly valid, is not really an additional constraint but simply a "condition subsequent" which results directly, as he has proved, from the "conditions precedent" already included in

¹ All of the "impossible" combinations are theoretically possible if $L = K = G = S - R = 0$.

Table 2. Consequently, I do not think it is necessary to amend the conditions for Situation V.

Mr. McDonald has included in his discussion a careful analysis of the conditions under which the formulas given in Table 5 of my paper are completely valid and those under which they must be viewed as approximations. I believe that the constraint implied by his formula (10) can be stated simply to be that the partial derivative $\partial T/\partial x_n$ is valid as a marginal tax rate formula for the variable x_n only if it is independent of x_n ; otherwise it can be taken as valid only with respect to an infinitesimal increment ∂x_n .

It follows that the formulas given in Tables 5.1, 5.2, 5.3, 5.6, 5.7, 5.8, 5.9, 5.10, 5.11, 5.12, 5.13, 5.14, 5.17, 5.18, 5.19, 5.20, 5.22, 5.23, and 5.24 are always valid regardless of the size of the increment in the variable. On the other hand, most of the formulas given in Tables 5.4, 5.5, 5.15, 5.16, and 5.21 are functions of the variable under consideration and consequently are valid only with respect to relatively small increments in that variable. Nevertheless, as Mr. McDonald notes, it will be found in practice that most of these functions are reasonably stable and that my formulas can be used with reasonable accuracy.

In preparing my paper, I did derive the formulas for the partial derivatives $\partial A/\partial x_n$, $\partial B/\partial x_n$, and $\partial C/\partial x_n$ separately but added them together to obtain the formulas for $\partial T/\partial x_n$ shown in Table 5. It did not occur to me that the component formulas had any particular intrinsic use. Mr. McDonald has indicated that the component formulas derived in his company have in fact proved useful. While I still doubt that mine are of sufficient general interest to warrant inclusion in this review, I shall be glad to make them available to any member of the Society who requests them.

I am most grateful to Mr. Newton and Mr. McDonald for submitting their very useful discussions. Mr. Newton's has provided the stimulus for further development of his graphic analysis of the tax formulas. Mr. McDonald has introduced some helpful theoretical considerations not dealt with in the original paper. Both have added greatly to its scope and have, I am sure, helped to make the subject more understandable.