TRANSACTIONS OF SOCIETY OF ACTUARIES
1979 VOL. 31

A CONCEPTUAL ANALYSIS OF NONPARTICIPATING LIFE INSURANCE GROSS PREMIUM AND PROFIT FORMULAS

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ABSTRACT

This paper is divided into two sections. The first section begins with an investigation of the conceptual profit model represented by the approach developed by James Anderson, which is referred to in this paper as Anderson's method. This model then is compared with two other profit models developed in the paper: the first representing policy-year earnings as the increase in the asset share less the increase in statutory reserves; and the second, in its most general form, representing policy-year earnings in a manner analogous to Anderson's method, with the generalization that the funds associated with the policy may be other than statutory reserves. Simplified formulas are utilized to emphasize concepts rather than mathematics. Applications of each of the formulas are suggested as the formulas are developed.

The second section of the paper contains an illustration of a method of incorporating, by use of a marginal tax approach, federal income tax directly as a cash outflow item in each of the profit formulas discussed in Section I. The reasons for any differences are discussed and analyzed with the use of examples.

The principal purposes of the paper are (1) to provide insight into the conceptual setting represented by the various profit and earnings formulas, (2) to suggest possible applications of each of the formulas discussed, and (3) to demonstrate how the direct calculation of federal income tax as a cash outflow item in each of the models will depend on the assumptions implied by the model.

I. DEVELOPMENT OF FORMULAS

A. *Introduction*

During the last two decades, Anderson's method[1] has become a widely used method of pricing nonparticipating individual life insurance products. The purpose of this section of the paper is to develop insight into the conceptual model represented by Anderson's formula. This will be achieved by comparing Anderson's formula with two alternative profit formulas that are developed in this paper. This analysis is intended to cultivate a better understanding of the assumptions implied by Anderson's approach as well as the assumptions implied by these alternative methods. Some applications and limitations of each method also are discussed. For simplicity, all formulas presented assume an annual premium whole life policy. No explicit consideration is given to agency development costs or federal income tax. However, the concepts presented are applicable to most insurance and annuity products.

For most plans of insurance and issue ages, in the year of issue the premiums collected plus the investment income attributed to the policy are insufficient to cover first-year commissions and other expenses, benefit payments, and the statutory policy reserve needed at the end of the policy year. Therefore, a company's statutory surplus generally will decline when a policy is issued. Normally, in most ensuing policy years, the premiums plus investment income are in excess of expenses, benefits, and the increase in statutory reserves. Therefore, issue of the policy results in an increase in the company's surplus in most years after the first. Anderson's approach treats the initial year's book loss as an investment of shareholders' surplus in the product. The subsequent years' book profits are considered to be returns of this investment with interest. Anderson's present value of profit index is the sum of book profits and book losses for all policy durations, discounted to issue at a predetermined interest rate reflecting the prevailing investment yields and the nature of the risk assumed by issuing the product. The "internal rate of return" index[2] is the interest rate at which the present value of book profits for each policy year equals the present value of the initial surplus depletion. It can be considered to be the rate of interest earned on the surplus invested in the product during the initial policy year.

The notation used in the formulas is summarized below. All formulas are with respect to an insured age x at issue.

[1] Developed in James C. H. Anderson, "Gross Premium Calculations and Profit Measurement for Nonparticipating Insurance," *TSA*, XI, 357.

[2] Donald R. Sondergeld, "Earnings and the Internal Rate of Return Measurement of Profit," *TSA*, XXVI, 617.

${}_tBP_x$ = Statutory book profit generated in policy year t, valued at the beginning of the policy year, per thousand dollars of insurance originally issued.

${}^1_tAP_x, {}^2_tAP_x$ = Adjusted profits in policy year t. These terms are used to measure policy-year earnings generated by formulas (3) and (7), respectively. They are similar to ${}_tBP_x$ in that they measure earnings on a policy-year basis.

${}_tGP_x$ = Annual gross premium, including policy fee, for policy year t.

E_t^p = Percent of premium expense for policy year t.

E_t^m = Per thousand expense for policy year t (includes premium collection and other per policy expense divided by average policy size).

$q^d_{[x]+t-1}$ = Probability of dying in policy year t.

$q^w_{[x]+t-1}$ = Probability of lapsing in policy year t.

${}_{t-1}p^T_{[x]}$ = Probability of persisting from issue to the beginning of policy year t.

E^d = Death benefit expense, expressed as expense per claim divided by average policy size.

E^w = Surrender expense, expressed as expense per surrender divided by average policy size.

${}_tCV$ = Cash value at the end of policy year t.

i_t^A = Interest earnings rate for invested assets in policy year t.

i_t^D = Interest rate at which book profits are discounted.

i_t^S = Interest earnings rate for generated surplus and contingency reserves, as described in the discussion preceding formula (8).

$v_t^A = 1/(1 + i_t^A); v_t^D = 1/(1 + i_t^D); d_t^A = i_t^A/(1 + i_t^A)$.

${}_tV_x$ = Terminal reserve at the end of policy year t, per thousand dollars of insurance in force.

${}_tRV_x$ = Asset share at the end of policy year t, per thousand dollars of insurance in force.

${}_tS_x$ = Accumulated surplus generated at the end of policy year t, per thousand dollars of insurance in force.

B. *Anderson's Method*

Formula (1) defines, using Anderson's method, the book profit in policy year t, valued at the beginning of the policy year. The formula assumes an annual premium whole life policy issued for $1,000 of insurance.

$$
\begin{aligned}
{}_tBP_x = {}_{t-1}p^T_{[x]}\{{}_tGP_x(1 - E^p_t) - E^m_t - (v^A_t)^{1/2} q^d_{[x]+t-1}(1{,}000 + E^d) \\
- v^A_t q^w_{[x]+t-1}({}_tCV + E^w) \qquad (1) \\
- [v^A_t(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1})\ {}_tV_x - {}_{t-1}V_x]\} .
\end{aligned}
$$

Rearranging the final term, this formula may be rewritten as follows:

$$
\begin{aligned}
{}_tBP_x = {}_{t-1}p^T_{[x]}\{{}_tGP_x(1 - E^p_t) - E^m_t - (v^A_t)^{1/2} q^d_{[x]+t-1}(1{,}000 + E^d) \\
- v^A_t q^w_{[x]+t-1}({}_tCV + E^w) \qquad (2) \\
- v^A_t[(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1})\ {}_tV_x - {}_{t-1}V_x] + d^A_t\ {}_{t-1}V_x\} .
\end{aligned}
$$

The present value of book profits discounted to the time of issue is given by

$$\sum_{t=1}^{\infty} {}_tBP_x \prod_{s=1}^{t-1} v^D_s ,$$

where

$$\prod_{s=1}^{0} v^D_s$$

is defined to equal 1. The internal rate of return, i_{IRR}, is the interest rate that satisfies the equation

$$\sum_{t=1}^{\infty} {}_tBP_x \frac{1}{(1 + i_{\text{IRR}})^{t-1}} = 0 .$$

An internal rate of return based on the formulas discussed above is appropriate for answering the question, "What will be the rate of return on shareholder surplus that is invested in the product?"

One question that arises concerns the amount of surplus required to be invested in a new product. According to the above formula, the accumulating fund associated with the product is equal to the terminal reserve. The fund on hand at the beginning of the tth policy year is ${}_{t-1}p^T_{[x]}\ {}_{t-1}V_x$, interest earnings on ${}_{t-1}p^T_{[x]}\ {}_{t-1}V_x$ during the year provide increment to ${}_tBP_x$, and the fund on hand at the end of the tth policy year is ${}_tp^T_{[x]}\ {}_tV_x$. Therefore, the formula is consistent with the concept of investing, during the year of issue, funds from the company's surplus that are sufficient to cover first-year policy benefits, first-year expenses, and the reserve at the end of the first policy year. The positive book profits that develop in ensuing policy years are returned to surplus at the end of each policy year. This stream of book profits represents the return on the company's investment made during the initial policy year.

In the early policy years, the asset share associated with the product usually will be smaller than the policy reserve. However, the book profits are calculated in such a way that the fund associated with the product is equal to the policy reserve. In the later policy years, the asset share eventually will exceed the reserve. When this occurs, the shareholders will have recovered their investment and the net increase in company surplus generated by the policy will go from negative to positive. Once again, however, the book profits are calculated in such a way that the fund associated with the product equals the reserve. The excess of the asset share over the reserve is no longer associated with the product but is a part of company surplus, where, at the discretion of the stockholders, it may be invested in additional new business; invested in stocks, bonds, or other interest-bearing assets; or distributed as dividends to stockholders.

C. *Development of* ${}^1_tAP_x$

It is instructive to examine formula (3) as a basis for determining policy-year earnings.

$$
\begin{aligned}
{}^1_tAP_x = {}_{t-1}p^T_{[x]}\{{}_tGP_x(1 - E^p_t) - E^m_t - (v^A_t)^{1/2}q^d_{[x]+t-1}(1{,}000 + E^d) \\
- v^A q^w_{[x]+t-1}({}_tCV + E^w) \qquad (3) \\
- [v^A_t(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1})\ {}_tV_x - {}_{t-1}V_x] + d^A_t\ {}_{t-1}S_x\}\,.
\end{aligned}
$$

Note that by making use of the relationship ${}_{t-1}RV_x = {}_{t-1}V_x + {}_{t-1}S_x$ it is possible to write the formula as follows:

$$
\begin{aligned}
{}^1_tAP_x = {}_{t-1}p^T_{[x]}\{{}_tGP_x(1 - E^p_t) - E^m_t - (v^A_t)^{1/2}q^d_{[x]+t-1}(1{,}000 + E^d) \\
- v^A_t q^w_{[x]+t-1}({}_tCV + E^w) \qquad (4) \\
- v^A_t[(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1})\ {}_tV_x - {}_{t-1}V_x] + d^A_t\ {}_{t-1}RV_x\}\,.
\end{aligned}
$$

The only difference from formula (2) is the use of $d^A_t\ {}_{t-1}RV_x$ instead of $d^A_t\ {}_{t-1}V_x$.

In all policy years, ${}^1_tAP_x$ will be affected by interest earnings on ${}_{t-1}S_x$ as well as on ${}_{t-1}V_x$. The interest earnings on ${}_{t-1}S_x$ will be negative in the early years, because ${}_{t-1}S_x$ will be negative until the initial surplus investment is recovered. This "investment loss" reflects the fact that surplus invested in new business would be earning interest at the asset earnings rate if it had been invested instead in an interest-bearing asset. In later years when ${}_{t-1}S_x$ is positive, interest earnings on the entire asset share, rather than on the policy reserve, contribute to the product's earnings for that year.

Neither formula (3) nor formula (4) normally would be used to determine a yield rate on surplus invested in new business. Instead, the formulas may be used to measure the difference in earnings resulting from the choice of one course of action over another, the first alternative being the investment of company surplus in a new product and the second alternative being the investment of the same amount of surplus in portfolio assets.

Suppose that the expenses utilized in the formulas represent only variable expenses, which are defined to include all expenses resulting from the development, sale, and maintenance of the product. These expenses contrast with fixed expenses and overhead, which will be incurred whether or not the product is marketed. The sum of the earnings, discounted to issue generated by formulas (3) and (4) may be considered as an indication of the increase in the economic value of the enterprise that may be achieved by investing in a unit of new business instead of investing in portfolio assets. This concept can be visualized by considering formula (3) in two segments. The first segment includes all terms except ${}_{t-1}p^T_{[x]}d^A_t\ {}_{t-1}S_x$. It is the same as formula (1) and generates book profits for each policy year. The second segment consists of ${}_{t-1}p^T_{[x]}d^A_t\ {}_{t-1}S_x$. When ${}_{t-1}S_x < 0$, this segment has the effect of deducting from policy-year profits the additional interest earnings the company would have enjoyed if the surplus invested in the product had been invested instead at the asset earnings rate. On the other hand, if ${}_{t-1}S_x > 0$, the second segment serves to increase policy-year profits by the additional interest earnings resulting from investment of the surplus in the product rather than in portfolio assets. As the discounted sum of the policy-year earnings increases, the liquidating value of the company increases. This is consistent with a vigorous sales philosophy. A poor earnings expectation, as measured by the profit index described above, could indicate that a company is not in any better financial position by issuing the product than by investing in portfolio assets the surplus designed to support the new product.

The above argument can be generalized to form a basis of comparison between the effects of issuing two different products. For example, suppose that management is considering whether to issue product A or product B. One way to determine which option would have a more positive effect on the company would be to measure the difference in earnings for the two products, where earnings are determined by formula (3). Assuming that both products experience the same persistency in each policy year, the following expression would be analyzed for policy year t:

$$
\begin{aligned}
{}^1_tAP_x(\text{product A}) - {}^1_tAP_x(\text{product B}) &= {}_tBP_x(\text{product A}) - {}_tBP_x(\text{product B}) \\
&+ {}_{t-1}p^T_{[x]}d^A_t[{}_{t-1}S_x(\text{product A}) - {}_{t-1}S_x(\text{product B}) \ .
\end{aligned}
$$

The sum of the terms of the above expression for all policy years discounted to issue can be viewed as the additional economic value, per unit of new business, obtained by issuing product A instead of product B. Note that it is necessary to reflect the interest on the accumulated surplus arising from each product in order to analyze which product will have the more positive effect on the company's earnings.

Another application of this approach may be visualized by considering the problem of choosing the most profitable reserve basis for a new product. Any analysis of the additional profits produced by choosing one reserve basis over another must reflect the fact that, if a more stringent reserve basis is chosen, the investment income associated with policy reserves will be greater but the investment income associated with company surplus will be correspondingly smaller. Formula (1) does not consider the investment income attributable to surplus and analyzes profits on a line-of-business basis only. Therefore, it does not exhibit adequately the effect of the choice of a reserve basis on the entire company's profit picture. The use of formula (3), adjusted to recognize federal income tax, removes this bias since it takes account of changes in investment earnings associated with changes in surplus arising from the use of different reserve bases.

Another basis of comparison between formula (1) and formula (3) is useful. Assuming that the surplus generated by a block of business earns interest at the asset earnings rate, formula (1) may be written as follows:

$$
{}_tBP_x = {}_{t-1}p^T_{[x]}[(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1}) \ {}_tS_xv^A_t - {}_{t-1}S_x]
$$

or (5)

$$
{}_tBP_x(1 + i^A_t) = {}_{t-1}p^T_{[x]}[(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1}) \ {}_tS_x - {}_{t-1}S_x(1 + i^A_t)] \ ,
$$

where ${}_tBP_x(1 + i^A_t)$ represents book profit for policy year t valued at the end of policy year t. When ${}_{t-1}S_x > 0$, the book profit as measured at the end of policy year t does not include interest earnings at the asset rate on the previous year's surplus. In other words, surplus that arises from interest earnings on that portion of the asset share in excess of the reserve is not included in book profits, since these funds have been shifted to shareholders' surplus and no longer are associated with the product. When ${}_{t-1}S_x < 0$, the book profit for year t includes interest earnings on the "negative accumulated surplus" as well as the asset share. Since this negative surplus represents the unrecovered portion of the shareholders' investment in the product, the book profit formula credits shareholders with interest on this investment at the asset rate.

Formula (3) also may be expressed in terms of ${}_tS_x$ and ${}_{t-1}S_x$ as follows:

$$\begin{aligned} {}^1_tAP_x &= {}_tBP_x + {}_{t-1}p^T_{[x]}d^A_t \; {}_{t-1}S_x \\ &= {}_{t-1}p^T_{[x]}[(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1}) \; {}_tS_xv^A_t - {}_{t-1}S_x + d^A_t \; {}_{t-1}S_x] \\ &= {}_{t-1}p^T_{[x]}[(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1}) \; {}_tS_xv^A_t - {}_{t-1}S_x(1 - d^A_t)] \\ &= v^A_t \; {}_{t-1}p^T_{[x]}[(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1}) \; {}_tS_x - {}_{t-1}S_x] \, . \end{aligned} \tag{6}$$

Therefore, the earnings in any policy year, ${}^1_tAP_x$, equals the change in surplus generated from one year to the next. All increments to surplus are included in the profit formula, including interest earnings on the previous year's surplus (that is, interest on the portion of the asset share in excess of the reserve). When ${}_{t-1}S_x < 0$, the formula excludes interest on the unrecovered portion of the shareholders' investment, since the fund earning interest as of the beginning of policy year t is assumed to contain only ${}_{t-1}p^T_{[x]} \; {}_{t-1}RV_x$ rather than ${}_{t-1}p^T_{[x]} \; {}_{t-1}V_x$.

D. *Development of* ${}^2_tAP_x$

A third profit formula, which is illustrated for comparative purposes, can be written as follows:

$$\begin{aligned} {}^2_tAP_x = {}_{t-1}p^T_{[x]}\{&{}_tGP_x(1 - E^p_t) - E^m_t - (v^A_t)^{1/2}q^d_{[x]+t-1}(1{,}000 + E^d) \\ &- v^A_tq^w_{[x]+t-1}({}_tCV + E^w) \\ &- [v^A_t(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1}) \; {}_tV_x - {}_{t-1}V_x]\} \\ &+ d^A_t\sum_{s=2}^{t-1} {}^2_sAP_x \, . \end{aligned} \tag{7}$$

The expression on the right-hand side of this formula is the same as that in formula (1) except for the last term. Like the ${}_tBP_x$'s in formula (1), the stream of ${}^{2}_{t}A\,P_x$'s can be utilized to determine a rate of return on shareholders' surplus that has been invested in the product, where the rate of return reflects a one-time return from the investment upon termination of the block of business rather than a return of profits to surplus each policy year.

The fund associated with the product at the beginning of policy year t includes both ${}_{t-1}p^{T}_{[x]}\ {}_{t-1}V_x$ and $\Sigma_{s=2}^{t-1}\ {}^{2}_{s}A\,P_x$. The shareholders are assumed to provide the same surplus investment as in formula (1), that is, ${}^{2}_{1}A\,P_x = {}_{1}B\,P_x$. ${}^{2}_{1}A\,P_x$ then represents the amount of shareholder funds that must be invested in the product to cover all first-year benefits and expenses and provide for the fund associated with the product at the beginning of the second year (the terminal reserve at the end of policy year 1). However, the ensuing years' profits are not returned to shareholders on a year-by-year basis but instead remain associated with the product to earn interest at the asset earnings rate. All profits are returned to shareholders on a one-time basis when the block of business terminates.

Since profits generated by the product remain in the fund associated with the product and are invested at the asset earnings rate, it is assumed that companies are not free to invest these profits in new business or distribute them to shareholders as dividends until the block of business terminates. Therefore, formula (7) could be practical only for companies that provide shareholder dividends and finance new business using only profits from capital and surplus retained from terminating blocks of business.

One additional analogy may be made between formula (1) and formula (7). Formula (1) provides returns that are comparable to investing in a bond with returns payable to the investors annually, the returns each year being the book profits. Formula (7) provides a return that is comparable to investing in an accumulation bond, with all returns received at maturity.

By additional refinements to formula (7), this model may be made more practical. For example, it might be assumed that p_t percent of the profits produced by the product in policy year t will be retained in the fund associated with the product and that $(100 - p_t)$ percent will be returned to "free surplus," where it may be distributed to shareholders or invested in new business. In addition, the profits retained by the product might be invested at a different earnings rate, i^{S}_{t}, which represents a portfolio earnings rate for retained profits or contingency funds. Under this scenario, a more general formula for ${}_{t}^{2}AP_x$ may be written as follows:

$$\begin{aligned} {}_{t}^{2}AP_x = {}_{t-1}p^T_{[x]}\{{}_tGP_x(1 - E^p_t) - E^m_t - (v^A_t)^{1/2}q^d_{[x]+t-1}(1{,}000 + E^d) \\ - v^A_t q^w_{[x]+t-1}({}_tCV + E^w) \\ - [v^A_t(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1})\ {}_tV_x - {}_{t-1}V_x]\} \\ + v^A_t i^S_t \sum_{s=2}^{t-1} {}_{s}^{2}AP_x(0.01p_s)\ . \end{aligned} \tag{8}$$

The utilization of a formula of this type to determine profits is consistent with the concept of retaining a contingency reserve, or an amount in excess of the statutory reserve, in order to provide for adverse fluctuations from expected experience. ${}_{t}^{2}AP_x$, the profit generated in policy year t, will be allocated between free surplus and the contingency reserve associated with the product. The manner in which the allocation takes place will depend on the choice of p_t. ${}_{t}^{2}AP_x$ is increased by interest earnings on the contingency reserve held at the beginning of the tth policy year. It generally is agreed that contingency reserves for a particular product should be related to the degree of the risk of adverse deviation from the expected values associated with that product. In the above conceptual setting, the contingency funds retained for a product can be made to vary with the profits expected to be generated by the product. Since the amount of expected profits will be related to the risk associated with a product, it follows that the size of the contingency funds also depends to some degree on the risk associated with the product. Note that, by the choice of appropriate values for each p_t, this generalized definition of ${}_{t}^{2}AP_x$ can include any Anderson-type formula where the reserves held are other than statutory reserves. Formula (1) may be viewed as a special case of the above formula where $p_t = 0$ for all t.

A rate of return calculation can be visualized in the following manner. Each year the product generates profits in the amount of ${}_{t}^{2}AP_x$. Of these profits, p_t percent will be retained by the product, representing an increase (or decrease, if p_t is negative) in the contingency funds associated with the product as of the end of policy year t over the contingency funds associated with the product as of the end of policy year $t - 1$. The remaining $(100 - p_t)$ percent of the profits will be returned to free surplus, where free surplus in this context refers to company surplus not associated with any particular product. Note that, by making p_t negative and releasing contingency funds, the return in policy year t can be made greater than ${}_{t}^{2}AP_x$. Consequently, $(100 - p_t)\ (0.01)\ {}_{t}^{2}AP_x$ represents the excess for policy year t of the cash flow and interest earnings over the funds required to provide the reserve increase and the increase in contingency funds. Thus, a rate of return may be calculated by using an initial investment in the product of free surplus of ${}^{2}_{1}A\,P_x = {}_1BP_x$ and returns on this investment of $(100 - p_t)(0.01)\;{}^{2}_{t}A\,P_x$ for each policy year after the first, where the returns on investment represent returns to free surplus in excess of returns that are utilized as contingency funds for the product.

The fund associated with the product at the beginning of policy year t consists of two components, the first being the statutory reserve, ${}_{t-1}p^{T}_{[x]}\;{}_{t-1}V_x$, and the second being the contingency funds that have been generated through policy year $t - 1$. In determining profits for policy year t, the reserves are credited with interest earnings at the asset rate, i^{A}_{t}, while the contingency funds are credited with interest earnings at the rate i^{S}_{t}.

II. DEVELOPMENT OF FEDERAL INCOME TAX COSTS

In this section of the paper, consideration will be given to the modifications required in each of the formulas discussed in the previous section in order to include a direct analysis of federal income tax in the profitability analysis. Once again, the formulas used assume an annual premium whole life policy. The results may be generalized easily so as to be applicable to most other insurance and annuity products.

The purposes of this section are (1) to demonstrate one method of incorporating federal income tax directly into an Anderson-type book profit formula and (2) to illustrate the distinctions in the federal income tax effect on policy-year profits that result from employing each of the different profit formulas discussed in Section I. The insight provided by investigating these distinctions may help some actuaries currently using an Anderson-type formula to avoid inconsistencies when adjusting their analysis to handle federal income tax costs directly.

The general procedure will be based on marginal tax rates as discussed by John Fraser.[3] It is assumed that there are available, for each item contributing to the tax, the marginal tax rates for the current policy year and the projected marginal tax rates for each future policy year being considered. The formulas are designed to apply to any company regardless of its tax situation. Different tax situations may be represented by utilizing the appropriate marginal tax rates.

The formulas developed in Section I have been modified to reflect earnings at the end of each policy year. This avoids the problem of choosing an after-tax interest rate for discounting cash flows to the beginning of the policy year.

[3] John C. Fraser, "Mathematical Analysis of Phase 1 and Phase 2 of 'The Life Insurance Company Tax Act of 1959,'" *TSA*, XIV, 51.

Some additional assumptions not mentioned in the preceding paragraphs are as follows:

1. Only one marginal tax rate is used for investment income; it represents a weighted average of the marginal tax rates for different types of investments.
2. The reserves in policy year t for tax purposes are assumed to be the average of reserves held at the end of policy year $t - 1$ and reserves held at the end of policy year t, immediately before the receipt of the premium for the next policy year. For example, if the company holds net level premium reserves, the tax reserves for policy year t are midterminal reserves.
3. It is assumed that taxes are incurred continuously throughout the policy year and that taxes for each policy year are paid at the end of the policy year.
4. Certain items that affect the tax, such as the small business deduction, 3 percent nonparticipating insurance deduction, and 2 percent special deduction, are ignored in this discussion.
5. Also ignored are taxes incurred as a result of withdrawals from the policyholders' surplus account, capital gains and losses, loss carry-forwards, and foreign tax credits.

The notation will be the same as used in Section I, with the following additions:

$_t\mathrm{Tax}_x$, $^1_t\mathrm{Tax}_x$, $^2_t\mathrm{Tax}_x$ = Federal income tax component of policy-year profits after taxes represented by $_tBP^{AT}_x$, $^1_tAP^{AT}_x$, and $^2_tAP^{AT}_x$, respectively.

$^{\mathrm{tax}}_tG_x$ = Underwriting gain for tax purposes in policy year t. The items in $^{\mathrm{tax}}_tG_x$ include premiums, death benefits, surrender benefits, increase in reserves, maturity benefits, and expenses. Underwriting gain includes those items that flow directly into the tax formula for companies whose tax is based on gain from operations. For these companies each item in $^{\mathrm{tax}}_tG_x$ will have a marginal tax rate of 0.46 or -0.46 as of 1979, when the tax rate applied to taxable income will be reduced from 48 percent to 46 percent.

$^{\mathrm{tax}}_tMV_x$ = Taxable reserves for policy year t.

$_tMA_x$, $^1_tMA_x$, $^2_tMA_x$ = Mean assets for policy year t for use in the calculation of $_t\mathrm{Tax}_x$, $^1_t\mathrm{Tax}_x$, and $^2_t\mathrm{Tax}_x$, respectively.

$_tI_x$, $^1_tI_x$, $^2_tI_x$ = Investment income for policy year t for use in the calculation of $_t\mathrm{Tax}_x$, $^1_t\mathrm{Tax}_x$, and $^2_t\mathrm{Tax}_x$, respectively.

${}^{G}_{t}m$, ${}^{MV}_{t}m$, ${}^{MA}_{t}m$, ${}^{I}_{t}m$ = Marginal tax rates for underwriting gain, reserves, mean assets, and investment income, respectively, for policy year t.

${}^{G}_{t}\mathrm{Tax}_x$ = Contribution to tax from underwriting gain in policy year t.

${}^{MV}_{t}\mathrm{Tax}_x$ = Contribution to tax from reserves in policy year t.

${}_{t}\mathrm{Tax}_x^{MA}$, ${}^{1}_{t}\mathrm{Tax}_x^{MA}$, ${}^{2}_{t}\mathrm{Tax}_x^{MA}$ = Contribution to tax from assets in policy year t for use in the calculation of ${}_{t}\mathrm{Tax}_x$, ${}^{1}_{t}\mathrm{Tax}_x$, and ${}^{2}_{t}\mathrm{Tax}_x$, respectively.

${}_{t}\mathrm{Tax}_x^{I}$, ${}^{1}_{t}\mathrm{Tax}_x^{I}$, ${}^{2}_{t}\mathrm{Tax}_x^{I}$ = Contribution to tax from investment income in policy year t for use in the calculation of ${}_{t}\mathrm{Tax}_x$, ${}^{1}_{t}\mathrm{Tax}_x$, and ${}^{2}_{t}\mathrm{Tax}_x$, respectively.

${}_{t}RV_x^{AT}$ = After-tax asset share at the end of policy year t, per thousand dollars of insurance in force.

${}_{t}RV_x^{BT}$ = ${}_{t}RV_x^{AT} + {}^{1}_{t}\mathrm{Tax}_x / {}_{t-1}p_{[x]}^{T}$, that is, ${}_{t}RV_x^{BT}$ includes taxes in all policy years through $t-1$ but does not include taxes in policy year t.

A. *Restatement of Policy-Year Profit Formulas*

Shown below are the formulas developed in Section I, modified to reflect the direct inclusion of taxes and the calculation of profits as of the end of each policy year.

$$\begin{aligned} {}_{t}BP_x^{AT} = {}_{t-1}p_{[x]}^{T}\{[{}_{t}GP_x(1 - E_t^{p}) - E_t^{m}](1 + i_t^{A}) \\ - q_{[x]+t-1}^{d}(1{,}000 + E^{d})(1 + i_t^{A}/2) \\ - q_{[x]+t-1}^{w}({}_{t}CV + E^{w}) \\ - [(1 - q_{[x]+t-1}^{d} - q_{[x]+t-1}^{w})\ {}_{t}V_x - {}_{t-1}V_x] \\ + i_t^{A}\ {}_{t-1}V_x\} - {}_{t}\mathrm{Tax}_x\,. \end{aligned} \tag{9}$$

$$\begin{aligned} {}^{1}_{t}AP_x^{AT} = {}_{t-1}p_{[x]}^{T}\{[{}_{t}GP_x(1 - E_t^{p}) - E_t^{m}](1 + i_t^{A}) \\ - q_{[x]+t-1}^{d}(1{,}000 + E^{d})(1 + i_t^{A}/2) \\ - q_{[x]+t-1}^{w}({}_{t}CV + E^{w}) \\ - [(1 - q_{[x]+t-1}^{d} - q_{[x]+t-1}^{w})\ {}_{t}V_x - {}_{t-1}V_x] \\ + i_t^{A}\ {}_{t-1}RV_x^{AT}\} - {}^{1}_{t}\mathrm{Tax}_x\,. \end{aligned} \tag{10}$$

$$
\begin{aligned}
{}^2_tAP_x^{\mathrm{AT}} = {}_{t-1}p_{[x]}^T\{[{}_tGP_x(1 - E_t^p) - E_t^m](1 + i_t^A) \\
- q_{[x]+t-1}^d(1{,}000 + E^d)(1 + i_t^A/2) \\
- q_{[x]+t-1}^w({}_tCV + E^w) \qquad (11)\\
- [(1 - q_{[x]+t-1}^d - q_{[x]+t-1}^w)\ {}_tV_x - {}_{t-1}V_x] \\
+ i_t^A\ {}_{t-1}V_x\} + i_t^A\sum_{s=2}^{t-1}{}^2_sAP_x^{\mathrm{AT}} - {}^2_t\mathrm{Tax}_x\,.
\end{aligned}
$$

${}_tBP_x^{\mathrm{BT}}$, ${}^1_tAP_x^{\mathrm{BT}}$, and ${}^2_tAP_x^{\mathrm{BT}}$ are defined in a manner analogous to their after-tax counterparts, except that the term involving taxes is excluded in each before-tax formula.

Note that, if profits are valued at the end of the policy year, the internal rate of return using formula (9) may be interpreted as the interest rate that satisfies the equation

$$\sum_{t=1}^{\infty} {}_tBP_x \frac{1}{(1 + i_{\mathrm{IRR}})^t} = 0\,.$$

The tax liability in policy year t associated with the plan of insurance being investigated is now calculated for each of the three profit formulas. Each item affecting the amount of tax associated with the product is identified and multiplied by the marginal tax rate for that item, and the results are summed.

B. *Reserve Contribution to Tax*

The contribution to federal income tax made by reserves in policy year t is the same for each of the profit formulas discussed. This analysis assumes a whole life policy and a modified reserving method, with the company electing the approximate section 818(c) adjustment to net level premium reserves. The reserves for tax purposes in policy year t are

$$
\begin{aligned}
{}^{\mathrm{tax}}_tMV_x = \tfrac{1}{2}\,{}_{t-1}p_{[x]}^T\{{}_{t-1}V_x + 0.021(1{,}000 - {}_{t-1}V_x) \\
+ (1 - q_{[x]+t-1}^d - q_{[x]+t-1}^w)[{}_tV_x + 0.021(1{,}000 - {}_tV_x)]\}\ .
\end{aligned}
$$

The reserve contribution to tax then is expressed as ${}^{\mathrm{MV}}_t\mathrm{Tax}_x = {}^{\mathrm{tax}}_tMV_x\ {}^{\mathrm{MV}}_tm$.

C. *Underwriting Gain Contribution to Tax*

For each of the profit formulas discussed, the underwriting gain for policy year t is given by

$$
\begin{aligned}
{}^{\text{tax}}_{t}G_x = {}_{t-1}p^T_{[x]} \llbracket {}_tGP_x(1 - E^t_p) - E^m_t - q^d_{[x]+t-1}(1{,}000 + E^d) \\
- q^w_{[x]+t-1}({}_tCV + E^w) - \{(1 - q^d_{[x]+t-1} - q^w_{[x]+t-1}) \\
\times [{}_tV_x + 0.021(1{,}000 - {}_tV_x)] \\
- [{}_{t-1}V_x + 0.021(1{,}000 - {}_{t-1}V_x)]\} \rrbracket \, .
\end{aligned}
$$

The gain contribution to tax then is expressed as ${}^G_t\text{Tax}_x = {}^{\text{tax}}_{t}G_x \; {}^G_t m$, where, as of 1979,

${}^G_t m$ = 0 for a company whose tax is based on taxable investment income;
= 0.46 for a company whose tax is based on gain from operations;
= 0.23 for a company whose tax is based on $\frac{1}{2}$(taxable investment income + gain from operations).

D. *Asset Contribution to Tax*

The contribution to tax from assets will depend on which assets are associated with the product at the time the calculation is performed. This will vary for each of the three formulas being considered. In each case the assets for tax purposes must be consistent with the assets whose interest earnings contribute to policy-year profits.

1. ASSET CONTRIBUTION TO TAX, ${}_tBP^{\text{AT}}_x$

As discussed in Section I, the funds associated with the product at the beginning and the end of policy year t are the $(t - 1)$st- and tth-year terminal reserves, respectively. Therefore, assuming that mean assets are evaluated immediately before after-tax book profits are returned to free surplus, a reasonable expression for mean assets is

$$
\tfrac{1}{2}\{{}_{t-1}p^T_{[x]}[{}_{t-1}V_x + (1 - q^d_{[x]+t-1} - q^w_{[x]+t-1}) \; {}_tV_x] + {}_{t-1}BP^{\text{AT}}_x + {}_tBP^{\text{AT}}_x\} \, .
$$

However, since ${}_tBP^{\text{AT}}_x$ is unknown at this point, it is not included in the calculation of ${}_tMA_x$ but is handled by an adjustment in later calculations. This leaves as an expression for mean assets

$$
MA_x = \tfrac{1}{2}\{{}_{t-1}p^T_{[x]}[{}_{t-1}V_x + (1 - q^d_{[x]+t-1} - q^w_{[x]+t-1}) \; {}_tV_x] + {}_{t-1}BP^{\text{AT}}_x\}
$$

The asset contribution to tax is ${}_t\text{Tax}^{\text{MA}}_x = {}_tMA_x \; {}^{\text{MA}}_{t}m$.

A simplification in the asset calculations that avoids the need for an adjustment in later calculations can be derived by assuming that the assets include before-tax book profits rather than after-tax book profits. The more complex calculations are included in this paper because that approach is more consistent with the assumption that taxes are incurred continuously. Since ${}^{\text{MA}}_{t}m$ usually is quite small, the numerical results will be similar regardless of which approach is taken.

2. ASSET CONTRIBUTION TO TAX, ${}^{1}_{t}AP^{\mathrm{AT}}_{x}$

On the basis of the assumptions implied by ${}^{1}_{t}AP^{\mathrm{AT}}_{x}$, the fund associated with the product is not the reserve but the asset share. Therefore, a reasonable expression for mean assets is

$$ {}^{1}_{t}MA_{x} = \tfrac{1}{2}({}_{t-1}p^{T}_{[x]}\ {}_{t-1}RV^{\mathrm{AT}}_{x} + {}_{t}p^{T}_{[x]}\ {}_{t}RV^{\mathrm{AT}}_{x}) \, . $$

However, the value of ${}_{t}RV^{\mathrm{AT}}_{x}$ is unknown since it will depend on the value calculated for ${}^{1}_{t}\mathrm{Tax}_{x}$. Therefore, ${}_{t}RV^{\mathrm{BT}}_{x}$, the asset share prior to the deduction of federal income tax in policy year t, is used temporarily as the asset base, with an adjustment made later in the calculations to include the tax item. The formula for mean assets then is

$$ {}^{1}_{t}MA_{x} = \tfrac{1}{2}({}_{t-1}p^{T}_{[x]}\ {}_{t-1}RV^{\mathrm{AT}}_{x} + {}_{t}p^{T}_{[x]}\ {}_{t}RV^{\mathrm{BT}}_{x}) \, , $$

and the asset contribution to tax is ${}^{1}_{t}\mathrm{Tax}^{\mathrm{MA}}_{x} = {}^{1}_{t}MA_{x}\ {}^{\mathrm{MA}}_{t}m$.

3. ASSET CONTRIBUTION TO TAX, ${}^{2}_{t}AP^{\mathrm{AT}}_{x}$

This formula implies that profits earned during a policy year remain associated with the product until the block of business terminates. The fund associated with the product at the beginning of policy year t is given by

$$ {}_{t-1}p^{T}_{[x]}\ {}_{t-1}V_{x} + \sum_{s=2}^{t-1} {}^{2}_{s}AP^{\mathrm{AT}}_{x} \, , $$

and mean assets then may be expressed as

$$ \frac{1}{2}\Big\{ {}_{t-1}p^{T}_{[x]}[{}_{t-1}V_{x} + (1 - q^{d}_{[x]+t-1} - q^{w}_{[x]+t-1})\ {}_{t}V_{x}] + \sum_{s=2}^{t-1} {}^{2}_{s}AP^{\mathrm{AT}}_{x} + \sum_{s=2}^{t} {}^{2}_{s}AP^{\mathrm{AT}}_{x} \Big\} \, . $$

Since ${}^{2}_{t}AP^{\mathrm{AT}}_{x}$ is unknown at the time taxes are calculated, it is excluded from the calculation of ${}_{2}MA^{\mathrm{tax}}_{t}$ but is reflected in later calculations. This leaves as the expression for mean assets

$$ {}^{2}_{t}MA_{x} = \frac{1}{2}\Big\{ {}_{t-1}p^{T}_{[x]}[{}_{t-1}V_{x} + (1 - q^{d}_{[x]+t-1} - q^{w}_{[x]+t-1})\ {}_{t}V_{x}] + 2\sum_{s=2}^{t-1} {}^{2}_{s}AP^{\mathrm{AT}}_{x} \Big\} \, , $$

with the asset contribution to tax determined by ${}^{2}_{t}\mathrm{Tax}^{\mathrm{MA}}_{x} = {}^{2}_{t}MA_{x}\ {}^{\mathrm{MA}}_{t}m$. For companies whose tax is based on gain from operations, ${}^{\mathrm{MA}}_{t}m = 0$.

E. *Investment Income Contribution to Tax*

The contribution to tax from investment income is based on the investment earnings arising from funds associated with the product. Again, the contribution depends upon the profit formula designated, since investment income for tax purposes coincides with the investment earnings increment to policy-year profits.

1. INVESTMENT INCOME CONTRIBUTION TO TAX, ${}_tBP_x^{AT}$

The investment income consists of interest earned on the fund held at the beginning of the policy year plus interest earned on cash flow during the year. Since the fund at the beginning of the year equals the previous year's terminal reserve, the investment income may be expressed as

$$ {}_tI_x = {}_{t-1}p_{[x]}^T\{{}_{t-1}V_x\ i_t^A + [{}_tGP_x(1 - E_t^p) - E_t^m]i_t^A - q_{[x]+t-1}^d(1{,}000 + E^d)i_t^A/2\}\ , $$

and the investment income contribution to tax is ${}_t\text{Tax}_x^I = {}_tI_x\ {}_t^Im$.

2. INVESTMENT INCOME CONTRIBUTION TO TAX, ${}_t^1AP_x^{AT}$

The fund associated with the product is assumed to be the asset share. The investment income may be expressed as

$$ {}_t^1I_x = {}_{t-1}p_{[x]}^T\{{}_{t-1}RV_x^{AT}i_t^A + {}_tGP_x[(1 - E_t^p) - E_t^m]i_t^A - q_{[x]+t-1}^d(1{,}000 + E^d)i_t^A/2\}\ , $$

and the investment income contribution to tax represented by ${}_t^1\text{Tax}_x^I = {}_t^1I_x\ {}_t^Im$.

3. INVESTMENT INCOME CONTRIBUTION TO TAX, ${}_t^2AP_x^{AT}$

Again expressing investment income as interest on the fund held at the beginning of the year plus interest on cash flow during the year, the investment income may be represented by

$$ {}_t^2I_x = \left(\sum_{s=2}^{t-1} {}_s^2AP_x^{AT} + {}_{t-1}p_{[x]}^T\ {}_{t-1}V_x\right) i_t^A + {}_{t-1}p_{[x]}^T\{{}_tGP_x[(1 - E_t^p) - E_t^m]i_t^A - q_{[x]+t-1}^d(1{,}000 + E^d)i_t^A/2\}\ , $$

and the investment income contribution to tax by ${}_t^2\text{Tax}_x^I = {}_t^2I_x\ {}_t^Im$.

F. *Calculation of After-Tax Profits*

The after-tax profits are calculated from the before-tax profits by use of the formulas shown below. Note that the adjustments mentioned in the mean asset calculations are developed here.

Where policy-year profits are based on ${}_tBP_x^{AT}$, the calculation of after-tax book profits is as follows:

$$
\begin{aligned}
{}_tBP_x^{\mathrm{AT}} &= {}_tBP_x^{\mathrm{BT}} - {}^G_t\mathrm{Tax}_x - {}^{\mathrm{MV}}_t\mathrm{Tax}_x - {}_t\mathrm{Tax}_x^{\mathrm{MA}} - {}_t\mathrm{Tax}_x^I \\
&\quad - 0.5\,{}_tBP_x^{\mathrm{AT}}\ {}^{\mathrm{MA}}_tm \\
&= \frac{{}_tBP_x^{\mathrm{BT}} - {}^G_t\mathrm{Tax}_x - {}^{\mathrm{MV}}_t\mathrm{Tax}_x - {}_t\mathrm{Tax}_x^{\mathrm{MA}} - {}_t\mathrm{Tax}_x^I}{1 + 0.5\,{}^{\mathrm{MA}}_tm}
\end{aligned}
$$

and ${}_t\mathrm{Tax}_x = {}_tBP_x^{\mathrm{BT}} - {}_tBP_x^{\mathrm{AT}}$.

The formulas below apply where the profits in each policy year are based on ${}^2_tAP_x^{\mathrm{AT}}$. Note that the asset-base adjustment described earlier is included in the final term.

$$
\begin{aligned}
{}^1_tAP_x^{\mathrm{AT}} &= {}^1_tAP_x^{\mathrm{BT}} - {}^G_t\mathrm{Tax}_x - {}^{\mathrm{MV}}_t\mathrm{Tax}_x - {}^1_t\mathrm{Tax}_x^{\mathrm{MA}} - {}^1_t\mathrm{Tax}_x^I \\
&\quad - 0.5\,{}^1_t\mathrm{Tax}_x\ {}^{\mathrm{MA}}_tm \\
&= {}^1_tAP_x^{\mathrm{BT}} - {}^G_t\mathrm{Tax}_x - {}^{\mathrm{MV}}_t\mathrm{Tax}_x - {}^1_t\mathrm{Tax}_x^{\mathrm{MA}} - {}^1_t\mathrm{Tax}_x^I \\
&\quad - 0.5({}^1_tAP_x^{\mathrm{BT}} - {}^1_tAP_x^{\mathrm{AT}})\ {}^{\mathrm{MA}}_tm \\
&= \frac{{}^1_tAP_x^{\mathrm{BT}}(1 - 0.5\,{}^{\mathrm{MA}}_tm) - {}^G_t\mathrm{Tax}_x - {}^{\mathrm{MV}}_t\mathrm{Tax}_x - {}^1_t\mathrm{Tax}_x^{\mathrm{MA}} - {}^1_t\mathrm{Tax}_x^I}{1 - 0.5\,{}^{\mathrm{MA}}_tm}
\end{aligned}
$$

and ${}^1_t\mathrm{Tax}_x = {}^1_tAP_x^{\mathrm{BT}} - {}^1_tAP_x^{\mathrm{AT}}$.

Where profits are based on ${}^2_tAP_x^{\mathrm{AT}}$, the formulas are similar to those where profits are based on ${}_tBP_x^{\mathrm{AT}}$ and are as follows:

$$
\begin{aligned}
{}^2_tAP_x^{\mathrm{AT}} &= {}^2_tAP_s^{\mathrm{BP}} - {}^G_t\mathrm{Tax}_x - {}^{\mathrm{MV}}_t\mathrm{Tax}_x - {}^2_t\mathrm{Tax}_x^{\mathrm{MA}} - {}^2_t\mathrm{Tax}_x^I \\
&\quad - 0.5\,{}^2_tAP_x^{\mathrm{AT}}\ {}^{\mathrm{MA}}_tm \\
&= \frac{{}^2_tAP_x^{\mathrm{BT}} - {}^G_t\mathrm{Tax}_x - {}^{\mathrm{MV}}_t\mathrm{Tax}_x - {}^2_t\mathrm{Tax}_x^{\mathrm{MA}} - {}^2_t\mathrm{Tax}_x^I}{1 + 0.5\,{}^{\mathrm{MA}}_tm}
\end{aligned}
$$

and ${}^2_t\mathrm{Tax}_x = {}^2_tAP_x^{\mathrm{BT}} - {}^2_tAP_x^{\mathrm{AT}}$.

As demonstrated in the preceding discussion, the total effect of federal income tax on a product's profits will be different depending on which of the profit formulas is used. Formula (9) assumes that interest earned on surplus generated by the product in prior years is independent of the current year's book profits associated with the product. This idea is extended one step further in the tax calculations described above, which assume that any taxes incurred by the company that arise because of interest earnings on accumulated surplus also are independent of the product's book profits. In other words, once book profits have been returned to free surplus, future interest earnings and the federal income tax resulting from these interest earnings are associated with free surplus and do not affect future profits generated by the product.

G. *An Example*

The purpose of this example is to demonstrate the relationships among the after-tax profits obtained by using each of the three approaches discussed. It is assumed that the tax calculations described above already have been performed.

Suppose that the after-tax book profits for the first six policy years are −$15.00, $8.00, $6.00, $5.00, $4.00, and $4.00, respectively. Assume that the rate of return on invested assets is 8 percent in all policy years, and that the company pays 48 percent of its investment earnings in taxes. An "after-tax investment return" then may be defined as $8\% \times 0.52 = 4.16\%$. The derivation of the surplus generated by the product for the six policy years is illustrated in the accompanying tabulation.

Policy Year	Surplus at End of Previous Policy Year (1)	Interest minus Taxes on Surplus at End of Previous Policy Year (2)	After-Tax Book Profit for Policy Year (3)	Surplus at End of Policy Year [(1)+(2)+(3)] (4)
1..........	$ 0	$0	−$15.00	−$15.00
2..........	− 15.00	(−$1.20+$0.58)	8.00	− 7.62
3..........	− 7.62	(− 0.61+ 0.29)	6.00	− 1.94
4..........	− 1.94	(− 0.15+ 0.07)	5.00	2.98
5..........	2.98	(0.24− 0.12)	4.00	7.10
6..........	7.10	(0.57− 0.27)	4.00	11.40

The −$15.00 surplus at the end of policy year 1 represents the shareholders' initial surplus investment in the product. As indicated in the discussion following formula (5), Anderson's book profit formula credits shareholders with interest on this investment at the asset rate. Furthermore, it charges shareholders with federal income taxes incurred as a result of these interest earnings. Therefore, the interest and taxes of −$1.20 and $0.58, respectively, for the second policy year have no impact on the fund associated with the product and do not affect the product's book profit, which includes interest earnings on ${}_1V_x$ regardless of the size of the first-year investment. The interest and taxes affect only the amount of free surplus.

If formula (10) is used to measure profitability, the accumulated surplus generated by the product is included in the fund associated with the product. The interest earnings on generated surplus and the taxes incurred because of these interest earnings are included in the product's profits for a policy year. Thus, in this example, the fund associated with the product at the end of the first policy year is ${}_1V_x$ − \$15.00. The −\$1.20 of interest and \$0.58 of taxes on the \$15.00 investment are included in the profits for the second policy year, where profits are measured by ${}^1_2A\,P^{AT}_x$. Similarly, in the sixth policy year, the fund at the beginning of the year equals ${}_5V_x$ + \$7.10, and the \$0.57 of interest and −\$0.27 of taxes on the \$7.10 are included in the product's profits for the year. If the surplus earnings rate for the sixth policy year is changed, the earnings for the year also will change, since the interest earnings on the surplus associated with the product will be affected. This calculation is consistent with the calculation of ${}^1_tA\,P^{AT}_x$ and ${}^1_tI_x$, described earlier. In this example ${}^1_6A\,P^{AT}_x = {}_6BP_x + (0.0416)(7.10) = \$4.30$.

If formula (11) is used to measure profitability, the fund associated with the product for a given policy year can be found by employing the following relationships:

$$
\begin{aligned}
{}^2_1AP^{AT}_x &= {}_1BP^{AT}_x\,; \\
{}^2_2AP^{AT}_x &= {}_2BP^{AT}_x\,; \\
{}^2_3AP^{AT}_x &= {}_3BP^{AT}_x + i\ {}^2_2AP^{AT}_x - \Delta\,\text{Taxes}_3 \\
&= {}_3BP^{AT}_x + k\ {}_2BP^{AT}_x\,,
\end{aligned}
$$

where Δ Taxes$_t$ is the change in the tax liability associated with the product in policy year t arising from the change in the policy-year profit formula, and k is the after-tax asset rate described in the example above. In general, it can be verified easily that

$$
{}^2_tAP^{AT}_x = {}_tBP^{AT}_x + k\sum_{s=3}^{t}(1+k)^{t-s}\ {}_{s-1}BP^{AT}_x \qquad \text{for } t \geq 3\,.
$$

At the beginning of the second policy year, the fund associated with the product is ${}_1V_x$, and the effect of investment income and taxes on the profitability of the product is the same as for formula (9). However, at the beginning of policy year 6, the fund associated with the product equals ${}_5V_x + \Sigma^5_{s=2}\,{}^2_sA\,P^{AT}_x$. By substituting the given book profit values in each of the relationships exhibited above and solving for each value of ${}^2_tA\,P^{AT}_x$, the value of $\Sigma^5_{s=2}\,{}^2_sA\,P^{AT}_x$ can be shown to be \$24.76. The fund associated with the policy is ${}_5V_x$ + \$24.76. Therefore, the interest earnings on ${}_5V_x$ + \$24.76 and the taxes associated with these earnings will be reflected in the product's profits for policy year 6, and ${}^2_6A\,P^{AT}_x = {}_6BP_x + (0.0416)(\$24.76) = \$5.03$.

The change in the fund associated with the product at the beginning of the sixth policy year that arises from using formula (11) rather than formula (10) is ${}_5V_x + \$24.76 - ({}_5V_x + \$7.10) = \$17.66$. This difference represents the initial surplus investment accumulated to the end of the fifth policy year at the after-tax asset earnings rate.

The taxes based on the generalized version of ${}^2_tA\,P^{AT}_x$ described by formula (8) may be determined easily from the above calculations. The fund associated with the product at the beginning of policy year t is ${}_{t-1}p^T_{[x]}\ {}_{t-1}V_x + \Sigma_{s=2}^{t-1}\, 0.01p_s\ {}^2_sA\,P^{AT}_x$ for both the asset contribution to tax and the investment income contribution to tax, and the calculations are analogous to those discussed earlier.

III. CONCLUSION

In this paper, three methods of determining policy-year earnings have been examined. The underlying theory and some uses for each method also have been discussed.

Of course, a company's total investment income or total tax liability will not change because it uses one method instead of another. However, each method is explicit in the amount of investment income and tax that should be allocated to a particular product. For example, consider a company that uses formula (9) for a model-office profit analysis for the life insurance line of business. Once book profits have been returned to surplus, they no longer have any effect on the policy-year profits of in-force business. Any profit generated by free surplus is independent of the profit generated by the products comprising the model office. Theoretically, this is consistent with the concept of considering surplus as a separate line of business for the purposes of allocating investment income and taxes by line of business and examining profit by line of business.

I hope that this paper will encourage further discussion of the above concept and of others that have been suggested but not fully developed.

DISCUSSION OF PRECEDING PAPER

JAMES A. TILLEY:

I read Mr. Lee's paper with a great deal of enthusiasm, having worked on many of the same ideas myself. In particular, the second of his adjusted profit measures has proved very useful in pricing calculations.

Mr. Lee states properly that the original formulation of Anderson's method assumes that the assets associated with a block of business at any point in time are exactly equal to the statutory reserves. His paper then shows how the conventional equations can be modified to treat either free surplus or contingency reserves (or both) as part of the assets explicitly associated with the block of business. When the latter method is used, it is essential to include in the profit measure after-tax earnings on free surplus or contingency reserves, and the author addresses this point carefully. More important, he identifies the concepts underlying these modifications to Anderson's method.

Mr. Lee describes how to incorporate federal income tax in the asset share equations using marginal tax rates. He mentions the necessity of projecting these rates as many years into the future as policy-year profits need to be calculated. I have found projecting marginal tax rates to be a tricky matter at best, and prefer to bring tax into the pricing equations via the federal income tax formula. This technique avoids the computation and projection of marginal rates. Moreover, if unassigned surplus or contingency reserves have been included as part of the assets associated with the block of business, the federal income tax on the earnings on these assets arises naturally from the tax formulas.

Those interested in the material presented in Mr. Lee's paper may also wish to read certain sections of my paper "The Pricing of Nonparticipating Single Premium Immediate Annuities," also appearing in this volume of the *Transactions*. The paper develops profit formulas analogous to Mr. Lee's two adjusted profit measures, and the pricing examples utilize what is, in effect, a version of his second adjusted measure.

FRANK C. METZ:

I enjoyed reading Mr. Lee's interesting paper. It furthered my understanding of the relationship between various expressions for policy-year earnings, and it should prove to be a valuable reference for actuaries involved in product development work or preparation for the Society of Actuaries' pricing examination.

Formula (8), the general one for ${}^{2}_{t}AP_x$, was of particular interest to me. I believe this formula may be cast in an even more general form by using the concepts developed in the first part of the paper. The value of a product's earnings for a policy year at the beginning of that year can be viewed as the sum of discounted cash flows and the discounted investment earnings on the fund associated with the product, less the discounted increase in reserve. This may be expressed symbolically as

$$
\begin{aligned}
{}^{3}_{t}AP_x = {}_{t-1}p^{T}_{[x]}\{&{}_tGP_x(1 - E^{p}_{t}) - E^{m}_{t} - (v^{A}_{t})^{1/2} q^{d}_{[x]+t-1}(1{,}000 + E^{d}) \\
&- v^{A}_{t} q^{w}_{[x]+t-1}({}_tCV_x + E^{w}) \\
&- v^{A}_{t}[(1 - q^{d}_{[x]+t-1} - q^{w}_{[x]+t-1})\ {}_tV_x - {}_{t-1}V_x]\} \\
+ v^{A}_{t} i^{S'}_{t}\ {}_{t-1}F_x\,,&
\end{aligned}
\tag{8a}
$$

where ${}_{t-1}F_x$ is the fund associated with the product at duration $t - 1$, $i^{S'}_{t}$ is a weighted average interest rate defined by the equation

$$
i^{S'}_{t} = \frac{(i^{A}_{t} - i^{S}_{t})\ {}_{t-1}p^{T}_{[x]}\ {}_{t-1}V_x}{{}_{t-1}F_x} + i^{S}_{t}\,,
$$

and all other symbols are as defined in the paper.

By making the appropriate definitions for the fund associated with the product, we can derive Mr. Lee's formulas (2), (3), and (8). If we define ${}_{t-1}F_x$ as ${}_{t-1}p^{T}_{[x]}\ {}_{t-1}V_x$, formula (2) is the result. Similarly, by letting ${}_{t-1}F_x$ equal ${}_{t-1}p^{T}_{[x]}\ {}_{t-1}V_x + \Sigma^{t-1}_{s=1}\ {}^{3}_{s}AP_x$ and i^{S}_{t} equal i^{A}_{t}, formula (3) can be produced. Finally, if we let ${}_{t-1}F_x$ equal ${}_{t-1}p^{T}_{[x]}\ {}_{t-1}V_x + \Sigma^{t-1}_{s=2}\ {}^{3}_{s}AP_x(0.01)p_s$, formula (8) is produced.

STEVEN D. SOMMER AND ROBERT L. COLLETT:

We found Mr. Lee's paper absorbing and very timely. The interest of many of our clients in after-tax pricing is currently at a high level, no doubt arising in part from the intense competition existing in the marketplace today. The real world is an after-tax world, and it seems necessary to have the maximum understanding of the likely future profit picture in that real world.

Our work in the past several years in updating our tax tools has led us, as Mr. Lee's work appears to have led him, to reexamine pre-tax pricing calculations, particularly with regard to the disposition of emerging surplus or book profits. Mr. Lee's paper is a valuable contribution in this area. It sets the stage for more meaningful pricing on both pre-tax and after-tax bases.

Our response is divided into three sections. The first examines Mr.

Lee's four primary pre-tax formulas. The comments in our second section attempt to generalize further on his formulas. In the final section we comment on some of our own after-tax pricing work and how it compares with Mr. Lee's approach.

I. *Pre-Tax Formulas*

Mr. Lee presents four different formulas for policy-year profits. His formula (1) is the usual formula for book profits (Anderson's method). It assumes that the initial book investment comes from free surplus and that subsequent book profits are returned to free surplus at the end of each policy year. Interest is earned on the reserve.

In his formula (3) the initial investment and the subsequent profits are left "associated with the product" rather than transferred to and from free surplus. Interest is earned each year on the entire accumulated cash flow (i.e., the asset share).

In formula (7) the initial book investment comes from free surplus but the subsequent book profits remain associated with the product. Interest is earned on the accumulated cash flow, excluding the effect of the first-year book loss.

In formula (8) the initial book investment is also taken from free surplus; however, only a portion of each year's book profit is distributed to free surplus. The remainder is kept associated with the product. Interest is earned only on the portion of the book profits that has remained with the product.

A. COMPARISON OF FORMULAS (3) AND (7)

Formula (7) can be rewritten as

$$ {}_t^2AP_x = {}_tBP_x + d_t^A \sum_{s=2}^{t-1} {}_s^2AP_x \, . $$

Formula (3) can be written as

$$ {}_t^1AP_x = {}_tBP_x + {}_{t-1}p_{[x]}^T d_t^A \; {}_{t-1}S_x \, . $$

Substituting the formula for surplus, assuming that surplus earns interest at the asset earnings rate, formula (3) can be written as

$$ {}_t^1AP_x = {}_tBP_x + d_t^A \sum_{s=1}^{t-1} {}_sBP_x \prod_{r=s}^{t-1} (1 + i_r^A) \, . $$

It can be shown that

$$ \sum_{s=1}^{t} {}_sBP_x \prod_{r=s}^{t} (1 + i_r^A) = \sum_{s=1}^{t} {}_s^1AP_x(1 + i_s^A) $$

by rewriting the right-hand side of this expression as

$$\sum_{s=1}^{t}\left[{}_sBP_x(1+i_s^A)+i_s^A\sum_{j=1}^{s-1}{}_jBP_x\prod_{r=j}^{s-1}(1+i_r^A)\right]$$

and showing that the coefficient of ${}_sBP_x$ equals

$$(1+i_s^A)\prod_{r=s+1}^{t}(1+i_r^A)\,.$$

Then formula (3) can be rewritten as

$${}^1_tAP_x={}_tBP_x+d_t^A\sum_{s=1}^{t-1}{}^1_sAP_x(1+i_s^A)\,.$$

With formula (3) so expressed, its similarity to formula (7) is easily seen. There are two differences. In formula (7) the summation begins in the second year rather than in the first, reflecting the requirement of formula (7) that the initial investment come from free surplus. The second difference is that the restated formula (3) above contains the factor $(1+i_s^A)$ in the summation. We believe formula (7) makes more sense if this factor is included here (and in formula [8]).

While the necessity of the $(1+i_s^A)$ factor can be demonstrated mathematically, it can also be justified by a comparison with formula (4). Formulas (4) and (7) are similar except for the last term. Note, however, that in formula (4) the asset share is valued as of the end of policy year $t-1$, but in formula (7) each of the adjusted profits is valued as of the beginning of the policy year, making it necessary to bring their sum to the end of the year by multiplying by $(1+i_s^A)$.

Thus we propose to restate formula (7) as

$${}^2_tAP_x={}_tBP_x+d_t^A\sum_{s=2}^{t-1}{}^2_sAP_x(1+i_s^A)\,,$$

leaving the difference between formulas (3) and (7) as the accumulated interest on the initial book investment.

B. PRESENT VALUES AND ACCUMULATED VALUES

Mr. Lee's paper deals mainly with the yearly profits or emerging surplus, and leaves the exploration of the related present values and accumulated values to the reader. We found it enlightening to go on to compare the time-weighted values of the profits. It is particularly interesting to concentrate on the special case where the net investment earnings rates assumed by policy year are identical with the interest rates used to accumulate or discount surplus. It is perhaps too narrow to call this a special case, since it is a very commonly considered one.

As long as the rates are the same, the present value or accumulated value will be the same for all four pre-tax formulas, even though the year-by-year profits within the cell study will differ. This situation results because the formula for discounting or accumulating profits must complement and be consistent with the timing of the release of emerging surplus.

This equivalence may not be immediately evident when one examines the formulas, but it is fairly obvious on a conceptual level. Perhaps it can be seen most easily by thinking of each of the formulas for adjusted profit in terms of formula (1), but with a modified reserve basis. For example, the reserve basis for formula (3) is one that produces a book profit of zero in every year except the last, at which time all of the profit is released. Since a reserve basis does not affect the accumulated value of profits at surrender or maturity (as long as the accumulation rate equals the investment rate and federal income taxes are not considered), the accumulated profits must be the same under all the formulas.

Note that formulas (1) and (3) are two extremes in the sense that in formula (1) all profits and losses are returned to free surplus, while in formula (3) they all remain associated with the product. Formulas (7) and (8) lie somewhere between the two. It can be shown that the present value or accumulated value of the profits under those two formulas is the same as that under formulas (1) and (3).

In his review of the information provided by formula (3) as opposed to formula (1), Mr. Lee seems to suggest that formula (3) provides more useful information in the situation where a choice of possible actions exists. We have just shown that in the special case where the investment rate and the accumulation rate are the same, formula (3) contributes little new information that cannot also be gained from formula (1). Formula (3) shows yearly increases in the asset share, but that is all.

In the case where the two rates differ, we question how much additional information is provided by formula (3) to assist in choosing between new products. Formula (3) could even be misleading for products having internal rates of return in excess of the accumulation rate. In this case formula (3) will always tend to favor the one that ties up surplus the longest, even though it might have a slightly lower internal rate of return. In any case, one must be careful in accumulating or discounting the profits under formulas (3), (7), and (8) to be sure that the accumulated interest included in the annual adjusted profit is handled properly.

II. *Generalizing the Formulas*

We would like to suggest a further generalization of formula (8), namely, to begin the summation in the first year rather than the second:

$$ {}_t^2AP_x = {}_tBP_x + v_t^A i_t^S \sum_{s=1}^{t-1} {}_s^2AP_x(0.01p_s)(1 + i_s^A) \, . $$

This change has two advantages. First, with a suitable choice of the p_t's, the generalized formula encompasses all the other formulas. Second, the change removes the restriction that the initial investment in the product must be equal to the first-year book loss. The value of p_1 could be made negative if it were desired to hold contingency funds in addition to the first-year statutory reserve.

A nagging question for most actuaries trying to determine the return or yield on a product is, "Return on what?" Anderson's method, as expressed by Mr. Lee, can yield the internal rate of return on the drain associated with posting the required reserve. In fact, a company will choose or be required to retain not only a reserve but also surplus at some suitable level.

The required fund may be thought of as the reserve plus some retained surplus (possibly a negative amount). The retained surplus addition will be a function of a variety of issues and elements. It may reflect risk evaluation and provision for contingencies, as Mr. Lee mentions. It should also be a function of shareholder dividend requirements, management's desired progression of surplus in relation to reserves, deficiency reserves (if the basic reserves exclude them), statutory reserves (if the basic reserves are on a GAAP basis), Phase 3 tax considerations, and possibly other things as well. Formula (8), so generalized, can recognize the profit-study–cell-level adjustments intended to cope with these issues.

Of course, the most difficult part may be the selection of suitable proportions for surplus retention within the profit-study system. Our approach is still evolving. At present it begins with a modeling (projection) of the full company, exclusive of any major direction changes for new business, but including taxes and dividends. This projection can point the way toward the appropriate pattern for future surplus growth. Next, we consider, at the macro level, implications for surplus of new-business activities. Finally, we allocate the surplus requirements back to the policy cell level. Incidentally, all such efforts by us to date have been crude or hypothetical or both. However, it is our feeling that an understanding of this generalization and the underlying issues is helpful, even if they are recognized in the cell studies only very crudely or indeed not at all.

III. *After-Tax Formulas*

While Mr. Lee's modifications to the formulas to account for federal income tax seem complicated at first, on closer examination they are seen to be straightforward applications of the provisions of the tax law to the pre-tax profit formulas. One of the most important by-products of trying to adjust profit calculations for income tax is that it forces one to determine just what is being assumed in the pre-tax formulas with regard to the distribution of profits. Once that is done, the tax adjustments, while somewhat complex, follow without too much additional difficulty.

Our approach to after-tax pricing is similar to that described in the paper. Probably the most significant difference, other than the fact that our profit studies and formulas are on a calendar-year basis, is that we do not use the marginal tax rates. Instead, we use the actual tax rates (46, 23, or 0 percent, depending on the tax situation) and modify T and G for such items as the section 818(c) election, the 10-for-1 rule, and a portion of the special deductions. Thus, while we do not require the marginal tax rates, we do require other information not required by Mr. Lee's formulas.

Our treatment of the tax on the interest on retained surplus has evolved to be equivalent to our modification of formula (8). Initially we used an Anderson formula adjusted for the tax on the emerging profit, ignoring any tax on the interest on the surplus accumulations of prior years. This approach was tantamount to assuming that all surplus, once earned, was no longer the responsibility of, or associated with, the cell from which it came. This approach fits some actual situations, but it is insufficient for many others.

The other extreme, usually developed by an after-tax asset share, may fit some actual situations, but usually it is not realistic. This case implies that none of the surplus drained or earned by the cell would have been distributed or used in ways not resulting in the payment of taxes.

We believe the best solution is an after-tax version of formula (8) (with our modifications):

$$ {}_t^2AP_x^{\mathrm{AT}} = {}_tBP_x + i_t^A \sum_{s=1}^{t-1} {}_s^2AP_x^{\mathrm{AT}} - {}_t^2\mathrm{Tax}_x \, . $$

One may select how much emerging surplus is to be retained with the product, so that total surplus may grow as judged necessary. Then, only the interest on this apportioned surplus is reflected and taxed in the profit calculation.

DONALD R. SONDERGELD:

The paper contains three basic formulas: a book profit formula: Nos. (1), (2), (5), and (9); an earnings formula ${}^1_tAP$: Nos. (3), (4), (6), (10); and an earnings formula ${}^2_tAP$: Nos. (7), (8), (11). I will discuss the difference between profits and earnings later.

I have no trouble with the book profit formulas. Book profits are as of a point in time, and either the beginning or the end of a policy year can be used. However, I think the paper might have been easier to follow if book profits had been defined consistently. Formulas (1), (2), and (5) express book profits as of the beginning of the policy year, whereas formula (9) uses the end of the year.

The pre-tax (${}^1_tAP^B$) formulas (3), (4), and (6) describe something as of the *beginning* of the policy year, and the after-tax (${}^1_tAP^E$) formula (10) defines something at the *end* of the year. However, the values those formulas produce are unaffected by whether book profits are as of the beginning or as of the end of the policy year. The same comment applies to the ${}^2_tAP$ formulas, (7), (8), and (11).

Let me explain by expressing the formulas in terms of change in surplus. In what follows, ${}_tBP^B$ and ${}_tBP^E$ are defined as in the paper on a "per original issue" basis. Also, ${}_tS^E$ is defined on a "per original issue" basis rather than on a "per survivor" or "per in-force" basis as in the paper.

Beginning-of-Year Book Profit Formula like Formulas (3), (4), and (6)

$$({}_{t-1}S^E + {}_tBP^B)(1+i) = {}_tS^E .$$

$$\text{Thus, } {}_tBP^B = v\ {}_tS^E - {}_{t-1}S^E ;$$

$${}^1_tAP^B = {}_tBP^B + \frac{i}{1+i}\ {}_{t-1}S^E$$

$$= v({}_tS^E - {}_{t-1}S^E) .$$

End-of-Year Book Profit Formula like Formula (10)

$${}_{t-1}S^E(1+i) + {}_tBP^E = {}_tS^E .$$

$$\text{Thus, } {}_tBP^E = {}_tS^E - {}_{t-1}S^E(1+i) ;$$

$${}^1_tAP^E = {}_tBP^E + i\ {}_{t-1}S^E$$

$$= {}_tS^E - {}_{t-1}S^E .$$

As previously noted, it would not have made any difference if ${}^1_tAP^B$ had been defined in the paper in terms of ${}_tBP^B$ or ${}_tBP^E$. We would still have ${}^1_tAP^B = v({}_tS^E - {}_{t-1}S^E)$.

Obviously,

$${}_tBP^E = {}_tBP^B(1+i) ;$$

$${}^1_tAP^E = {}^1_tAP^B(1+i) ;$$

$${}^2_tAP^E = {}^2_tAP^B(1+i) .$$

I think that ${}_tBP^B$, ${}_tBP^E$, ${}^1_tAP^E$, and ${}^2_tAP^E$ are useful. Although ${}^1_tAP^B$ and ${}^2_tAP^B$ can be defined, I would not use them. The expression ${}^1_tAP^E$ represents the change in surplus for the year, or what I prefer to call earnings (not profit). Unlike book profits, which are expressed as of a point in time, earnings represent the change in the value of surplus from one point in time to another.

For example, assume that a fund earns 10 percent interest; that is, ${}_{t-1}S^E = \$100$ and ${}_tS^E = \$110$. The earnings for the year, ${}^1_tAP^E$, are \$10. There are no book profits. Although we can calculate \$10/1.10 = \$9.09, what purpose is served?

I agree that the earnings formula ${}^1_tAP^E$ should not be used to determine a yield rate. However, why does the paper say "not normally" rather than "never"?

The author mentions the use of contingency funds with earnings formula ${}^2_tAP$ at the end of Section I. I prefer to include something similar to this in my definition of ${}_tBP$. I modify Anderson's book profit formula by using $(1 + {}_tk)\ {}_tV$ instead of ${}_tV$, and $(1 + {}_{t-1}k)\ {}_{t-1}V$ instead of ${}_{t-1}V$. The item ${}_tk\ {}_tV$ represents "statutory benchmark surplus." To oversimplify, if benchmark surplus is 5 percent of reserves, then ${}_tk$ is 0.05. This factor can be whatever you choose it to be, and need not be a function of reserves. Since it is impossible to operate a life insurance company with no statutory surplus, I include what I think is needed when I calculate book profits and an internal rate of return on a block of policies.

Could more uses of earnings formula ${}^2_tAP^E$ be mentioned? One example was given of the one-time return from the termination of a block of business. I fail to see why the reinvestment of book profits at some rate of interest should be averaged in with the underlying internal rate of return to produce a composite rate. I wish the author would expand on this point and suggest other uses.

I thought that an expansion, as shown in Table 1 of this discussion, of the table shown in Section II, G, of the paper might be helpful to others, since I had trouble at first in understanding ${}^2_tAP^E$. It is derived from the book profits of that table and uses the after-tax interest rate of 4.16 percent used in the table. Surplus and other items shown here are all on a "per original issue" basis.

Let me interpret what the paper and my expanded table indicate. We can look at an isolated piece of free surplus of \$15. Free surplus is the excess of total statutory surplus over statutory benchmark surplus. If we invest that \$15 in a product having a −\$15.00 book profit at the end of the first year, the \$15 of free surplus is reduced immediately to zero.

However, if the book profits generated over the life of the product are reinvested at the 4.16 percent after-tax interest rate given, the initial \$15 of free surplus (${}^2_tS^E$) grows to \$29.79 by the end of the sixth year.

The expression ${}^1_tS^E$ represents the traditional asset share less the reserve on a "per original issue" basis associated with the product. At the end of the first year, it is −\$15.00. At the end of the sixth year it is \$11.40. The \$15 initially borrowed from free surplus has been repaid by the end of the fourth year with 4.16 percent interest. If the amounts

TABLE 1

Policy Year t	Book Profits at End of Year ${}_tBP^E$ (1)	${}^0_tS^E$ $[15\times(1.0416)^{t-1}]$ (2)	${}^0_tAP^E$ (3)	${}^1_tS^E$ (4)	${}^1_tAP^E$ (5)	${}^2_tS^E$ (6)	${}^2_tAP^E$ (7)
1.....	−\$15	\$15.00	\$0	−\$15.00	−\$15.00	\$ 0	−\$15.00
2.....	8	15.62	0.62	− 7.62	7.38	8.00	8.00
3.....	6	16.27	0.65	− 1.94	5.68	14.33	6.33
4.....	5	16.95	0.68	2.98	4.92	19.93	5.60
5.....	4	17.66	0.71	7.10	4.12	24.76	4.83
6.....	4	18.39	0.73	11.40	4.30	29.79	5.03

NOTES:

$$_t(3) = {}_t(2) - {}_{t-1}(2)\,, \qquad {}^0_tAP^E = {}^0_tS^E - {}^{\;\;0}_{t-1}S^E\,;$$

$$_t(4) = {}_{t-1}(4)(1.0416) + {}_t(1)\,, \qquad {}^1_tS^E = {}^{\;\;1}_{t-1}S^E(1.0416) + {}_tBP^E\,;$$

$$_t(5) = {}_t(4) - {}_{t-1}(4)\,, \qquad {}^1_tAP^E = {}^1_tS^E - {}^{\;\;1}_{t-1}S^E\,;$$

$$_t(6) = {}_t(2) + {}_t(4)\,, \qquad {}^2_tS^E = {}^0_tS^E + {}^1_tS^E\,;$$

$$_t(7) = {}_t(3) + {}_t(5)\,, \qquad {}^2_tAP^E = {}^0_tAP^E + {}^1_tAP^E\,.$$

repaid are not distributed to stockholders or reinvested in new business, then ${}^0_tS^E$ represents the accumulated value of the \$15 at 4.16 percent interest. Obviously, ${}^2_tS^E = {}^1_tS^E + {}^0_tS^E$, and ${}^2_tAP^E = {}^1_tAP^E + {}^0_tAP^E$.

If we look at free surplus at the end of year 6, we see the result of having invested \$15 in a product five years ago. We have \$29.79 of free surplus instead of \$18.39.

The internal rate of return calculated on column 1 book profits is approximately 28 percent. The rate-of-return calculation mentioned in Section I, D, of the paper utilizes ${}^2_tAP^E$ or ${}^2_tS^E$, that is, $15(1+i)^5 = 29.79$. It produces a rate of return just under 15 percent, which is, of course, the weighted average of the 28 percent internal rate of return on book profits and the 4.16 percent rate on the reinvestment of book profits. I do not think it is particularly useful. If we let a portion of renewal book profits be disassociated from the product (see Mr. Lee's formula [8]), the rate of return will be between 15 and 28 percent. The 28 percent rate of return is reached if all renewal book profits are disassociated from the product (e.g., paid out, instead of being reinvested at 4.16 percent).

Free surplus should not be allowed to accumulate at 4.16 percent. It should be "leveraged" by reinvesting it in new business. Statutory benchmark surplus, or the required surplus that I referred to earlier, is taken into account in my modification of Anderson's book profit formula in determining a rate of return. Generally, it is desirable to operate with free surplus as close to zero as possible.

HEMANT TILAK:

I first want to thank Mr. Lee for a paper that is both informative and interesting. It is sure to generate a great deal of thought in the future.

I developed the same formula for ${}^1_tAP_x$ in the course of some asset share work at the Canada Life Assurance Company late last year. The approach to the formula was somewhat different, resulting primarily from an investigation into the allocation of investment income and taxes between the traditional book profits and the accumulation of surplus. To explain the development more clearly, it is necessary to write the profit formulas differently.

It is possible to write the formulas for ${}^1_tAP_x$ and ${}^2_tAP_x$ in a manner that establishes more clearly the concepts underlying their formulation. Defining

$${}_tBP'_x = {}_tBP_x(1 + i_t^A) ,$$

$${}^1_tAP'_x = {}^1_tAP_x(1 + i_t^A) ,$$

and

$${}^2_tAP'_x = {}^2_tAP_x(1 + i_t^A) ,$$

it follows from formulas (6) and (7) that

$${}^1_tAP'_x = {}_tBP'_x + i_t^A \sum_{s=1}^{t-1} {}^1_sAP'_x \tag{6.1}$$

and

$${}^2_tAP'_x = {}_tBP'_x + i_t^A \sum_{s=2}^{t-1} {}^2_sAP'_x . \tag{7.1}$$

We now note that formulas (6.1) and (7.1) can also be written in terms of the traditional book profits. First, we note that

$$\sum_{r=1}^{t} {}^{1}_{r}AP'_x = \sum_{r=1}^{t} {}_{r}BP'_x \prod_{s=r+1}^{t} (1 + i^A_s) \tag{6.2}$$

and

$$\sum_{r=2}^{t} {}^{2}_{r}AP'_x = \sum_{r=2}^{t} {}_{r}BP'_x \prod_{s=r+1}^{t} (1 + i^A_s) \,. \tag{7.2}$$

Furthermore, assuming ${}_0S_x = 0$, it is true that

$${}_tp^T_{[x]} \; {}_tS_x = \sum_{r=1}^{t} {}_{r}BP'_x \prod_{s=r+1}^{t} (1 + i^A_s) \,,$$

and hence that

$${}_tp^T_{[x]} \; {}_tS_x = \sum_{r=1}^{t} {}^{1}_{r}AP'_x \tag{6.3}$$

$$= \sum_{r=2}^{t} {}^{2}_{r}AP'_x + {}_1p^T_{[x]} \; {}_1S_x \prod_{s=2}^{t} (1 + i^A_s) \,. \tag{7.3}$$

Finally, substituting from equation (6.3) in equation (6.1), and from (7.3) in (7.1), we have

$${}^{1}_{t}AP'_x = {}_tBP'_x + i^A_t ({}_{t-1}p^T_{[x]} \; {}_{t-1}S_x) \tag{6.4}$$

and

$${}^{2}_{t}AP'_x = {}_tBP'_x + i^A_t \Big[{}_{t-1}p^T_{[x]} \; {}_{t-1}S_x - {}_1p_{[x]} \; {}_1S_x \prod_{s=2}^{t-1} (1 + i^A_s) \Big] \,. \tag{7.4}$$

Equations defining the internal rate of return for the three methods may then be defined as follows:

$$\sum_{r=2}^{t} {}_{r}BP'_x \prod_{s=r+1}^{t} (1 + i_{\text{IRR}}) = -{}_1p^T_{[x]} \; {}_1S_x (1 + i_{\text{IRR}})^{t-1} \,,$$

$$\sum_{r=2}^{t} {}^{1}_{r}AP'_x \prod_{s=r+1}^{t} (1 + i_{\text{IRR}}) = -{}_1p^T_{[x]} \; {}_1S_x (1 + i_{\text{IRR}})^{t-1} \,, \tag{6.5}$$

$$\sum_{r=2}^{t} {}^{2}_{r}AP'_x = -{}_1p^T_{[x]} \; {}_1S_x (1 + i_{\text{IRR}})^{t-1} \,. \tag{7.5}$$

The rates of return determined by these formulas will differ. Referring to them as ${}^0i_{\text{IRR}}$, ${}^1i_{\text{IRR}}$, and ${}^2i_{\text{IRR}}$, respectively, we have the following relationships for the special case when $i^A_s = i^A$ for all s:

$${}^0i_{\text{IRR}} \lesseqgtr i^A \,, \quad {}^1i_{\text{IRR}} \lesseqgtr 0 \,, \quad \text{and} \quad {}^2i_{\text{IRR}} \lesseqgtr i^A \,,$$

all depending on ${}_tS_x \lesseqgtr 0$.

Over a given period, the rates ${}^{0}i_{\text{IRR}}$ and ${}^{1}i_{\text{IRR}}$ are unique provided that there is only one reversal in sign for the corresponding streams of profits. In the third case, ${}^{2}i_{\text{IRR}}$ is always unique.

Where the rate of return is not unique, the following approach is worth considering.[1] Treating the negative profits as investments in the policy and positive profits as returns from the policy, we define the rate of return as that rate that equates the values of these two streams. The value of the investment stream is calculated at the portfolio rate, and the value of the returns from the policy is calculated at i_{IRR}. We thus have the following relationships:

$$\sum_{r=1}^{t} {}_{r}BP_x^{+}(1 + {}^{0}i_{\text{IRR}})^{-r+1} = -\sum_{r=1}^{t} {}_{r}BP_x^{-} \prod_{s=2}^{r} v_s^A ,$$

and

$$\sum_{r=1}^{t} {}^{1}_{r}AP_x^{+}(1 + {}^{1}i_{\text{IRR}})^{-r+1} = -\sum_{r=1}^{t} {}^{1}_{r}AP_x^{-} \prod_{s=2}^{r} v_s^A .$$

An approach consistent with the basis of ${}^{2}_{t}AP_x'$ would require that

$${}^{2}_{t}AP_x' = {}_{t}BP_x' + i_t^A \sum_{s=1}^{t-1} {}^{2}_{s}AP_x^{+} ,$$

and

$$(1 + {}^{2}i_{\text{IRR}})^{-t+1} \sum_{r=1}^{t} {}^{2}_{r}AP_x^{+} = -\sum_{r=1}^{t} {}^{2}_{r}AP_x^{-}(1 + {}^{2}i_{\text{IRR}})^{-r+1} .$$

It is interesting to interpret the three approaches in terms of the Canadian annual statement. Ignoring unusual items (charged through the surplus statement) and taxes, the three profit formulas and the corresponding surplus accumulation formulas represent the "income statement" and the "reconciliation of surplus statement."

For Anderson's method, the formula representations

$${}_{t}BP_x' = {}_{t-1}p_{[x]}^{T}\ {}_{t-1}V_x(1 + i_t^A) + CF' - {}_{t}p_{[x]}^{T}\ {}_{t}V_x$$

and

$${}_{t}p_{[x]}^{T}\ {}_{t}S_x = {}_{t-1}p_{[x]}^{T}\ {}_{t-1}S_x(1 + i_t^A) + {}_{t}BP_x'$$

imply an allocation of total investment income between the income statement and the surplus statement.

For the second method, the formula representations

$${}^{1}_{t}AP_x' = {}_{t-1}p_{[x]}^{T}\ {}_{t-1}V_x(1 + i_t^A) + i_t^A\ {}_{t-1}p_{[x]}^{T}\ {}_{t-1}S_x + CF' - {}_{t}p_{[x]}^{T}\ {}_{t}V_x$$

[1] James C. Van Horne, *Financial Management and Policy* (Englewood Cliffs, N.J.: Prentice-Hall, 1968).

and

$$ {}_{t}p^{T}_{[x]}\ {}_{t}S_x = {}_{t-1}p^{T}_{[x]}\ {}_{t-1}S_x + {}^{1}_{t}AP'_x $$

imply an allocation of all investment income to the income statement. This is the method currently used in the Canadian annual statement, and was the basis for the development of this profit formula at Canada Life earlier this year.

For the third method, we view the initial loss as a "contribution to surplus by shareholders." The formula representations

$$ {}^{2}_{t}AP'_x = {}_{t-1}p^{T}_{[x]}\ {}_{t-1}V_x(1+i^{A}_{t}) + i^{A}_{t}\left[{}_{t-1}p^{T}_{[x]}\ {}_{t-1}S_x - {}_{1}p^{T}_{[x]}\ {}_{1}S_x \prod_{s=2}^{t-1}(1+i^{A}_{s})\right] + CF' - {}_{t}p^{T}_{[x]}\ {}_{t}V_x $$

and

$$ {}_{t}p^{T}_{[x]}\ {}_{t}S_x = {}_{t-1}p^{T}_{[x]}\ {}_{t-1}S_x + {}^{2}_{t}AP'_x $$

imply that all investment income, including that earned on contributions by shareholders, is allocated to the income statement. The surplus statement should of course reflect the shareholders' contribution in the first policy year.

Looked at in this manner, the three rates of return may be interpreted as follows: (1) ${}^{0}i_{\text{IRR}}$ represents the effective rate of interest paid by the policy on the surplus invested in the policy; (2) ${}^{1}i_{\text{IRR}}$ represents the additional growth rate experienced by the amounts invested in the policy, over that which these amounts would otherwise have experienced; and (3) ${}^{2}i_{\text{IRR}}$ represents the effective rate of interest earned by shareholders on their investment in the policy.

In closing, I would like to congratulate Mr. Lee for his fine contribution to the actuarial literature.

JAMES W. LAMSON:

Mr. Lee's paper provides a convenient way to account for interest earnings on surplus when projecting a life company's earnings. If it is assumed that the company retains all the ${}_{t}BP_x$'s as free surplus, then using the ${}^{1}_{t}AP_x$'s in place of the ${}_{t}BP_x$'s to project the future earnings of newly issued business includes automatically most of the earnings on surplus in the projection—a piece of the puzzle easily overlooked.

At several points in the paper Mr. Lee suggests that the "sum of the earnings discounted to issue generated by formulas (3) and (4)" be employed for various purposes. Since it is unclear from the wording in the paper precisely what is meant by these "earnings discounted to issue," this discussion will focus on two possible meanings and their ramifications.

If the first n policy years are considered, discounting ${}^1_1AP_x$, ${}^1_2AP_x$, ${}^1_3AP_x, \ldots, {}^1_nAP_x$ to issue could mean either

$$\sum_{t=1}^{n}\prod_{s=1}^{t-1}v_s^D\ {}^1_tAP_x \quad \text{or} \quad \left(\prod_{s=1}^{n-1}v_s^D\right)\sum_{t=1}^{n}{}^1_tAP_x\,.$$

If the first formula above is used in place of the discounted value of book profits in a profit study, a serious overstatement of policy profits will be made. If the second formula is to be employed as a profit measure, one should be aware of a simpler and more complete measurement of accumulated profit, which is already available.

In considering the first formula above, one should recognize that the ${}_{t-1}p^T_{[x]}\ d^A_t\ {}_{t-1}S_x$ component of ${}^1_tAP_x$ (which is the difference between ${}^1_tAP_x$ and ${}_tBP_x$) is simply the company's reward for deferring receipt of the ${}_tBP_x$'s as they arise. Such deferral of receipt is assumed by Mr. Lee to be accomplished by investing the arising surplus in assets earning interest at the asset rate.

If, indeed, the ${}_tBP_x$ portions of the ${}^1_tAP_x$'s are invested as they arise in interest-bearing assets, they are not available as a positive cash flow for discounting purposes. Discounting both the stream of ${}_tBP_x$'s and the interest on surplus would be like having your cake and eating it. This procedure would simply result in a false overstatement of policy profits, presumably leading an actuary to set gross premiums at inadequate levels. The error in discounting both the stream of ${}_tBP_x$'s and the interest on surplus can be seen easily by reference to a simple bond analogy.

Consider a \$1,000 par value, two-year annual coupon bond with coupons of \$100 each. If this bond is purchased at par, it will yield 10 percent to the purchaser. This fact is demonstrated by noting that the present value of the cash outflow (\$1,000) equals the present value of the cash inflows, each present value taken at 10 percent.

$$1{,}000 = \frac{100}{1.10} + \frac{100}{(1.10)^2} + \frac{1{,}000}{(1.10)^2}\,.$$

Now, if the purchaser chooses to reinvest the first \$100 coupon (analogous to book profits) at a 10 percent yield, a calculation similar to discounting the ${}^1_tAP_x$ would yield the following formula for the determination of a yield rate (y):

$$1{,}000 = \frac{100}{1+y} + \frac{100}{(1+y)^2} + \frac{1{,}000}{(1+y)^2} + \frac{10(\text{interest on first coupon})}{(1+y)^2}\,.$$

Solving for y, we obtain a yield of 10.475 percent, which is clearly erroneous.

However, close examination of what constitutes a cash inflow results in an equation that correctly accounts for the subsequent reinvestment of coupon interest by discounting the first coupon for two years:

$$1{,}000 = \frac{100}{(1+y)^2} + \frac{1{,}000}{(1+y)^2} + \frac{100(\text{first coupon})}{(1+y)^2} + \frac{10}{(1+y)^2}\,,$$

which gives $y = 10$ percent.

Clearly, the first formula for discounting the ${}^1_tAP_x$'s is an incorrect interpretation of the words "sum of the earnings discounted to issue generated by formulas (3) and (4)."

Turning to the second formula,

$$\left(\prod_{s=1}^{n-1} v_s^D\right) \sum_{t=1}^{n} {}^1_tAP_x\,,$$

I submit

$$\left(\prod_{s=1}^{n} v_s^D\right) {}_np_{[x]}^T \; {}_nS_x$$

as a much simpler and more reasonable measure of a product's profitability. It does, of course, extract that measure at the end of the nth policy year rather than at the beginning, as the second formula (presumably) does.

The profit measure suggested above reflects an accurate crediting of interest to accumulated surplus. An adjustment to $\Sigma_{t=1}^n \; {}^1_tAP_x$ is necessary to make it an accurate representation of accumulated profit. In its place, $\Sigma_{t=1}^n \; {}^1_tAP_x(1+i_t^A)$ should be substituted, resulting in

$$\left(\prod_{s=1}^{n} v_s^D\right) \sum_{t=1}^{n} {}^1_tAP_x(1+i_t^A)$$

as the discounted value. It can be shown that the adjusted profit measure and the one suggested by this discussion are indeed equivalent, a fact that stems from the equivalence of ${}_np_{[x]}^T \; {}_nS_x$ and $\Sigma_{t=1}^n \; {}^1_tAP_x(1+i_t^A)$.

One can easily show (by accumulating both sides of eq. [5] at the asset accumulation interest rates) that ${}_np_{[x]}^T \; {}_nS_x$ is simply the accumulated value of each year's book profit, or, expressed as a formula, that

$${}_np_{[x]}^T \; {}_nS_x = \sum_{t=1}^{n} \prod_{s=t}^{n} (1+i_s^A) \; {}_tBP_x\,.$$

With this in mind, the profit measure $(\Pi_{s=1}^n v_s^D) \; {}_np_{[x]}^T \; {}_nS_x$ is more appropriate than the second formula, especially if one looks upon ${}_np_{[x]}^T$

${}_nS_x$ as equal to the iterative accumulation of book profits, as in

$${}_np^T_{[x]}\ {}_nS_x = (1 + i^A_n)({}_{n-1}p^T_{[x]}\ {}_{n-1}S_x + {}_nBP_x)\ ,$$

rather than as equal to

$$\sum_{t=1}^{n} \prod_{s=t}^{n} (1 + i^A_s)\ {}_tBP_x\ .$$

Clearly, all interest due on surplus *is* credited with this formula.

Since this measure of accumulated profit is available, how might it be used? When the sum of the discounted book profits is employed as a profit measure, it is implicitly assumed that the book profits can be either removed as they arise or reinvested at a rate at least as high as the discounting rate of interest. If, for some reason (such as attainment of a certain ratio of assets to liabilities), the profits cannot be removed from the company, and the discounting interest rate cannot be achieved within the investment guidelines of the company, the sum of the discounted book profits is not an appropriate profit measure. In these circumstances, $(\Pi^n_{t=1}\ v^D_t)\ {}_np^T_{[x]}\ {}_nS_x$ is a valuable measure of true profitability. These points are very ably discussed by Samuel Turner in his paper on actuarial appraisal valuations.[1]

In this connection, it is interesting to note that, if $i^D_t > i^A_t$ for all t, then

$$\left(\prod_{t=1}^{n} v^D_t\right)\ {}_np^T_{[x]}\ {}_nS_x < \sum_{t=1}^{n} \prod_{s=1}^{t-1} v^D_s\ {}_tBP_x\ ,$$

and equality exists if $i^D_t = i^A_t$ for all t.

(AUTHOR'S REVIEW OF DISCUSSION)

DAVID S. LEE:

I am grateful to each of the reviewers for their fine discussions. The additional ideas presented in the discussions contribute significantly to the quality of the paper.

I agree with Mr. Tilley's statement that projecting marginal tax rates into the future can be a tricky and difficult process. This problem can be circumvented by not projecting them at all, using instead the most recent year's marginal tax rates for all future policy years. For many life insurance companies this should produce only slight errors in the policy years shortly after issue, provided that the marginal tax rates vary only slightly from year to year. The potential error in projecting

[1] Samuel H. Turner, "Actuarial Appraisal Valuations of Life Insurance Companies," *TSA*, XXX, 139.

taxes into the distant future is likely to be considerable regardless of the method used, but the discounting effect minimizes this error. The main danger in using marginal tax rates is that a change in a company's tax situation will alter the marginal rates considerably.

Mr. Tilley suggests reading his paper "The Pricing of Nonparticipating Single Premium Immediate Annuities," which also appears in this volume of the *Transactions*. I found his paper, for which he was awarded the Triennial Prize, to be interesting and well written.

Mr. Metz presents a formula utilizing a generalized fund approach such that, depending on the definition of the generalized fund, any of the before-tax formulas developed in the paper can be represented by a single formula.

Messrs. Sommer and Collett derive the following expression as an alternative method of expressing formula (3):

$$ {}^{1}_{t}AP_x = {}_{t}BP_x + d_t^A \sum_{s=1}^{t-1} {}^{1}_{s}AP_x(1 + i_s^A) \, . $$

When the formula is expressed in this form, it is clear that the terms inside the summation represent profits as of the end of each policy year. It is then suggested that formula (7) be modified as follows, so that the terms inside the summation also represent profits as of the end of each policy year:

$$ {}^{2}_{t}AP_x = {}_{t}BP_x + d_t^A \sum_{s=2}^{t-1} {}^{2}_{s}AP_x(1 + i_s^A) \, . $$

I agree that this version of formula (7) is conceptually correct. The modification is needed only in formula (7) and not in formula (11), which is the after-tax counterpart of formula (7), discussed in Section II of the paper. The reason no adjustment is needed in the after-tax formulas is that the stream of ${}^{2}_{t}AP_x^{AT}$'s is already expressed as of the end of the year.

Messrs. Sommer and Collett suggest as an additional generalization of formula (8) beginning the summation in the first year rather than the second. This yields the following expression for ${}^{2}_{t}AP_x$:

$$ {}^{2}_{t}AP_x = {}_{t}BP_x + v_t^A i_t^S \sum_{s=1}^{t-1} {}^{2}_{s}AP_x(0.01p_s)(1 + i_s^A) \, . $$

With suitable choice of the p_t's, this generalized formula encompasses all the other formulas developed in the paper. Messrs. Sommer and Collett argue that this change removes the restriction that the initial investment must equal the first-year book loss. However, closer investigation of this generalized formula reveals that this is not so. In policy year 1, the summation goes from one to zero, the second term vanishes, and ${}^2_1AP_x = {}_1BP_x$. In order to allow the initial investment to be something other than the first-year book loss, the following modification may be considered:

$t = 1$:

$$ {}^2_1AP_x = {}_1BP_x - R \, , $$

where R represents the amount of first-year book profits retained as contingency funds backing the product;

$t > 1$:

$$ {}^2_tAP_x = {}_tBP_x + v_t^A i_t^S \Big[R(1 + i_1^A) + \sum_{s=1}^{t-1} {}^2_sAP_x(0.01p_s)(1 + i_s^A) \Big] \, . $$

Mr. Sondergeld suggests that the paper would have been easier to follow if all formulas had been defined consistently either at the beginning or at the end of the policy year. In order to maintain consistency with the literature that has been published to date, Section I of the paper defines policy-year profits as of the beginning of the policy year. However, when federal income taxes are introduced into the calculations, the formulas become more complicated if cash flows are discounted to the beginning of the policy year. One method of handling the situation is to accumulate cash flows to the end of the policy year at the before-tax asset earnings rate, pay taxes at the end of the policy year, then discount the remaining cash flow back to the beginning of the policy year at an after-tax asset earnings rate. A difficult question to be addressed is the interest rate at which earnings will be discounted to the beginning of the policy year. It could be the discount rate or some other appropriate after-tax rate. I prefer viewing all formulas as defined in Section II of the paper at the end of the policy year, since this avoids addressing the question.

Mr. Sondergeld's expansion of the table in Section II, G, helps clarify some of the concepts discussed in earlier sections of the paper. Column 4 exhibits the free surplus generated by the product as of the end of each policy year. However, if policy-year earnings are defined by ${}^2_tAP_x$, and if free surplus is defined so that it does not include any contingency funds associated with the product, then the free surplus associated with the product becomes the negative of column 2 for policy years 1–5. At the end of policy year 6, \$29.79 is released to free surplus, and the free surplus is \$29.79 − \$18.39 = \$11.40.

Suppose that the generalized version of ${}^2_tAP_x$, defined by formula (8), is used to measure policy-year earnings. Assume that statutory benchmark surplus is defined so that $p_t = 0.4$ for policy years 2–6, and that all funds associated with the product are returned to free surplus at the end of policy year 6. Under this scenario, the relationship between free surplus and policy-year earnings is described by Table 1 of this review.

An internal rate of return calculated using the stream of earnings returned to surplus each year is equal to 20.5 percent. As Mr. Sondergeld suggests, this is lower than the internal rate of return calculated using the stream of book profits.

Mr. Tilak demonstrates that each of the three basic formulas developed in the paper implies an allocation of investment income for a line of business between the income statement and the reconciliation of surplus. Anderson's method implies that $i_x^a \; {}_tV_x \; {}_{t-1}p_{[x]}^T$ is allocated to the income statement, and $i_x^a \; {}_tV_x \; {}_{t-1}p_{[x]}^T$ is allocated to the reconciliation of surplus.

TABLE 1

Policy Year t	${}_t^2AP_x$ (1)	Earnings Returned by Product (2)	Earnings Returned to Free Surplus (3)	Free Surplus (4)
1	−$15.00	....	−$15.00	−$15.00
2	8.00	$3.20	4.80	− 10.82
3	6.13	2.45	3.68	− 7.59
4	3.23	2.09	3.14	− 4.77
5	4.32	1.73	2.59	− 2.37
6	4.39	− 9.47	13.86	11.40

NOTE.—$(2)_t = (1)_t \times 0.4$; $(3)_t = (1)_t - (2)_t$; $(4)_t = (4)_{t-1}(1.0416) + (3)_t$.

The use of either ${}_t^1AP_x$ or ${}_t^2AP_x$ implies that all investment income is allocated to the income statement. The use of ${}_t^2AP_x$ implies that, in addition to the asset share, interest earnings on the shareholder contribution, which subsidizes the first-year book loss, are credited to the income statement. If federal income taxes are included in the analysis, Anderson's method allocates taxes between the income statement and the reconciliation of surplus in a manner consistent with the allocation of investment income. Both ${}_t^1AP_x$ and ${}_t^2AP_x$ imply an allocation of all federal income tax to the income statement. Therefore, if earnings are considered by line of business, Anderson's method implies that surplus may be considered to be a separate line supported by investment income and paying federal income tax on that income. The other formulas discussed in the paper are not consistent with the consideration of surplus as a separate line.

Mr. Tilak also develops and interprets a rate of return for each of the formulas discussed in the paper.

Mr. Lamson carefully distinguishes between the "sum of the discounted profits" and the "discounted sum of the profits." This paper employs as a profit measure the discounted sum of the profits in formulas (3) and (4). Anderson's "present value of profit" index is based on the sum of the discounted book profits.

Mr. Lamson points out that ${}^{1}_{t}AP_x$ can be an extremely valuable profit index if certain constraints are placed on the surplus of the company.

Formula (1) is referred to in the paper as Anderson's method. Most actuaries who use this approach for pricing and profit analysis refer to the associated formulas by this designation. However, an approach with certain similarities to Anderson's method was presented in the 1951 *Transactions* in a paper written by Harwood Rosser entitled "A Present Value Approach to Profit Margins and Dividends."

Once again, I would like to express my gratitude to all the reviewers for their contributions.