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CONSTANT REPLACEMENT RATIOS IN RETIREMENT: A THEORETICAL APPROACH

BARNET N. BERIN AND ANTHONY B. RICHTER

ABSTRACT

Recognizing the increase in the rate of inflation over recent years in the United States, pension plans have been improving benefits for retired employees. A planning model is developed for such increases, based on periodic examination of the increase in pension necessary to keep a replacement ratio constant. This ratio, R, is initially related to the private pension plus social security benefit, divided by final average salary.

The replacement ratio is calculated periodically in retirement years and the relationships among the various elements involved in maintaining it are explored.

The meaning and significance of the model are discussed. Both philosophical and practical issues are explored, and examples of the approach are included.

I. INTRODUCTION

For a substantial part of the history of private pension plans in the United States, inflation was not a serious problem and attention was focused on introducing, and improving, benefits for active, working employees. Over the last ten years, however, as inflation has increased, private pension plans have begun to improve benefits for those already retired. These benefit increases may reflect a belief that there is an obligation to maintain, at least partially, the living standards of workers who retire from service. They also may reflect a generational compact whereby each working-life generation contributes to the prior generation's welfare. The willingness to contribute stems from recognition that the continuous income-producing capacity of a company, which assures its viability and success, is created by all generations of workers, and that retirement is inevitable for each working generation. As a practical matter, increases in retiree pensions have become part of the collective bargaining process, so that some portion of the settlement, formerly reserved for active employees, now is allocated to pensioners.

In this paper, we investigate an approach to retirement planning based on the following assumptions:

1. At retirement, an employee receives a private pension and a social security pension. The ratio of the sum of these two pensions to the employee's final average salary is the replacement ratio. For most employees, this ratio is an important part of retirement planning.
2. As wage levels increase during the employee's retirement, the company periodically reviews the employee's total retirement income, based on a planning model designed to maintain a constant replacement ratio. This is accomplished by basing the need to increase the private pension plan benefit on the year-by-year relationships between changes that occur after retirement in the social security benefit and in the average salary of active employees.
3. The effect of taxes before retirement is recognized by plan design, and after retirement by considering after-tax payments in the review process leading to postretirement increases.

The premise of a constant replacement ratio is subject to debate, and there are certainly practical and financial limits to its application. However, as an initial step in the planning process, construction of an analytic model based on that premise can facilitate the study of a problem that has been receiving increasing attention. Section II presents such a model. The simple approach taken here is a bit deceptive. Attention to the conditions that make the replacement ratio constant leads to insight into the retirement planning process and reveals some interesting relationships that could prove useful as part of periodic studies leading to pension improvements for retirees. Section III develops these relationships. The theoretical results are tested in Section IV, using data for the period 1970–79 as examples of the approach.

II. THE PLANNING MODEL

Assume a private pension benefit at age 65 of P, a primary social security benefit at age 65 of SSB, a final average salary at age 65 of S, and a replacement ratio of R. By definition,

$$R = \frac{P + SSB}{S} . \tag{1}$$

S is an average over whatever period of time, one year or greater, is appropriate for the company or industry being considered.

To investigate the implications of a constant replacement ratio in retirement, assume annual percentage increases (a_i, b_i, and c_i) in each of the three items of expression (1). Each of these increases can be expressed as an accumulation so that, after n years of retirement, the replacement ratio can be expressed as

$$R_n = \frac{P\prod_{i=1}^{n}(1 + a_i) + SSB\prod_{i=1}^{n}(1 + b_i)}{S\prod_{i=1}^{n}(1 + c_i)} . \qquad (2)$$

It is important to note that $R = R_n$ is not based on a simple indexation of salary, but on the relationships in equation (2), where a must be determined to satisfy the equation. For example, suppose an employee retires at age 65 in 1970 with $R = 79$ percent. The planning model periodically would determine what values of a_i are required to keep $R_n = 79$ percent, by projecting all the other elements in (2) and solving for a_i.

Consider the meaning of a, b, and c. The simplest to understand is b. This is, of course, the increase in social security benefits, as specified by the Social Security Administration over the years, presently based on the Consumer Price Index. For our purposes, we will assume that this is an appropriate measurement of inflation for retirees, even though it has come under criticism in this respect. Some modification of this index might be equally valid. Under social security rules, $b \geq 0$.

There are several possible interpretations of c. It could be based on the retiree's former salary, as defined by a job or grade category, traced forward after the employee's retirement. Traditionally, this grid is updated periodically. While there are companies that can follow this progression, there are practical problems with job changes, both subtle and technological, and even job replacement programs. Alternatively, c could be the average increase in salary for the company, or even for the country. Except under unusual circumstances, $c \geq 0$. The relationship between b and c is the key to later developments.

If a salary increase can be divided into components for merit, seniority, productivity, inflation, and residual elements (presumably small in effect), then the planning model can test the effects of including or excluding various elements. While it is at least arguable that this compartmentalization can be accomplished, the planning model should be seen as philosophically neutral, providing a basis for examining the meaning and extent of certain retired life pension increases based on various choices of c in the denominator.

The constant c can be adjusted to reflect inflation only, or to reflect inflation and one or more of merit, seniority, and productivity. How this is approached at the planning stage will reflect company philosophy. While allowing for different possibilities, we will assume that c is chosen to be the increase in average wage for employees covered under social security.

Keeping S unchanged, or $c = 0$, would make R_n increase as long as $b > 0$. A special case of constant R_n results from $a = b = c = 0$. An interesting case, which is not trivial, is $a = b = c$. If, in choosing c, salary increases are stripped of merit, seniority, and productivity components, the remaining elements are inflation and residual (one hopes small) amounts. Ignoring the latter, if the inflation rate recognized in salary increases equals the inflation rate recognized in the social security benefit, then $a = b = c$.

The value of a is, of course, to be derived. It represents the percentage increase needed in the private pension benefit to preserve the replacement ratio (i.e., so that $R = R_n$ above). In practice, $a \geqslant 0$, since a negative adjustment to the private pension is not permitted: zero would be used instead, but the negative percentage would be included in the accumulation test in the following years. Where the subscript i is not used in the expressions that follow, a, b, and c refer to percentage increases over some specified period of time.

III. SOME RELATIONSHIPS

Equations (1) and (2) in the preceding section imply that $R_n = R$ if and only if

$$P(a - c) + SSB(b - c) = 0 . \tag{3}$$

For equation (3) to hold, either one expression in parentheses must be negative and one positive, or both must be equal to zero. For our purposes, we can assume that all the elements in (3) represent simple functions and that since P and SSB are positive, one of the items in parentheses must be negative. Having made this assumption, it follows that

$$\text{If } b > c, \quad \text{then } a < c \quad \text{and} \quad a < c < b , \tag{4}$$

and

$$\text{If } b < c, \quad \text{then } a > c \quad \text{and} \quad a > c > b . \tag{5}$$

Statement (4) says that the replacement ratio can be constant if the increase in private pension is less than the increase in average wage, which, in turn, is less than the increase in social security benefits. This might be considered an unfavorable economic scenario. Statement (5) says that the replacement ratio can be constant if the increase in private pension is greater than the increase in average wage, which, in turn, is greater than the increase in the social security benefits. This might be considered a favorable economic scenario.

Notice the reversal from (4) to (5), corresponding to what may be considered a reversal from an unfavorable to a favorable economic scenario. There are six possible inequalities among a, b, and c, but only two will keep $R = R_n$ constant. In (3), if $b = c$, a must equal c, which is, of course, apparent from (2), where $R = R_n$ if $a = b = c$.

If b were close to zero in (5), c likely would be a much lower percentage than the increases we have been accustomed to in recent years. If the rate of inflation (and, by necessity, b) were close to zero, the constant replacement ratio problem would be nearly solved.

From equation (3), we find that

$$a = c - \frac{SSB}{P}(b - c) \tag{6}$$

$$= \left(1 + \frac{SSB}{P}\right)c + \left(-\frac{SSB}{P}\right)b\,. \tag{7}$$

Equation (6) indicates the adjustment in private pension necessary to preserve replacement ratios. Since SSB/P is positive, under unfavorable economic conditions $b > c$, so that the percentage adjustment in private pensions is less than c, the increase in average wage. Under favorable economic conditions, $b < c$, so that the percentage adjustment in private pensions is in excess of c, the increase in average wage. If $b = c$, the percentage adjustment in private pensions is equal to c, the increase in average wage, and $a = b = c$. While this is simply another way of showing that statements (4) and (5) hold, it identifies the calculation technique and permits an interesting comparison.

Notice the relationship between these results and the various proposals to index social security benefits by the lesser of the increase in the Consumer Price Index and the increase in the average wage. In our terms, the proposed social security pensioner increase equals the lesser of b and c, being equal to c under unfavorable economic conditions and equal to b under favorable economic conditions. This approach was suggested to reduce the cost of the social security system where the pensioner increase is currently equal to b.

Equation (7) shows that a is a weighted average of b and c, where the weight attached to c is greater than one and the weight attached to b is negative.

If SSB/P is equal to N, then N, for practical purposes, would be a positive number for a low-paid employee and near zero for a high-paid employee. On this basis,

$$a = c - N(b - c) \quad \text{for a low-paid employee ,} \tag{8}$$

and

$$a \doteq c \quad \text{for a high-paid employee .} \tag{9}$$

From formulas (8) and (9), we can show that the effect of a varies with salary. If $b > c$, an unfavorable economic scenario, then $a < c$ and a increases with increasing salary. If $b < c$, a favorable economic scenario, then $a > c$ and a decreases with increasing salary. If $b = c$, then a is the same for all salaries and $a = b = c$.

IV. APPLICATION OF THE MODEL

To investigate equation (6) further, it is necessary to make some assumptions in order to be more precise about the effect of the ratio of SSB to P. In the spirit of a planning model, assume the following:

1. Social security replacement ratios are as shown in Table 1.

TABLE 1

SOCIAL SECURITY REPLACEMENT RATIOS

EMPLOYEE'S PAY IN FINAL YEAR S	ANNUAL SOCIAL SECURITY BENEFIT	
	Amount SSB	Percentage of Final Year's Pay SSB/S
\$ 5,000	\$3,394	68%
10,000	5,216	52
15,000	6,664	44
20,000	7,022	35
25,000	7,141	29
30,000	7,200	24
40,000	7,200	18
50,000	7,200	14
60,000	7,200	12

NOTE.—Amounts based on social security law in effect for retirements in 1982, assuming 5 percent pay increases over the working career. The social security replacement ratio is SSB/S. Benefits reflect the law in effect January 1, 1981, ignoring the transitional rule.

2. The plan objective is to provide a total replacement ratio of 80 percent where final pay is \$5,000, and 60 percent where final pay is \$50,000. Assuming that the ratios in between are linear, this straight line can be defined by a replacement ratio formula $R = (\$37{,}000 - 0.2S)/\$45{,}000$ (see Fig. 1).

Using these assumptions, it is possible to develop values of *SSB/P* so that we can explore the implications of formula (6) by an example (see Table 2 and Fig. 2).

As a result of the social security amendments of 1977, benefits were reduced to correct for the double indexing under the 1972 act. Benefit reductions were phased in over a period of years, the last of which is 1981. Therefore, 1982 ratios were selected for Table 1 because it is the first year in which social security retirement benefits fully reflect the 1977 amendments. Thus, the ratios shown in Table 1 can be thought of as representing an appropriate level of social security replacement ratios. The 1977 amendments, in effect, correct the error which was part of the 1972 law.

In Table 2, line 2 reflects assumption 2 above, as depicted in Figure 1. Line 3 is taken from Table 1. Line 4 is line 2 minus line 3, and line 5 is equal to line 3 divided by line 4. Using this approach, we extract values to construct Figure 2.

We are now able to quantify the earlier results. The examples chosen are based on retirement in 1970. A replacement ratio is determined, and the improvement in private pensions is calculated on a one-year, two-year, and three-year level equivalent basis. Keep in mind that the period was turbulent (b_i varied from 0 to 20 percent), that this is a planning model, that the Consumer Price Index is only a proxy for the rate of inflation (let alone the rate of retiree inflation), and that many companies provided increases on the order of 2–4 percent a year during this period. Additional considerations involved in judging the increases in retiree pensions include the change in tax status after retirement, the nontaxability of the social security benefits, the availability of other forms of savings,[1] the availability of medicare to reduce medical expenses, the reduction in work-related expenses after retirement, and possibly some family support in retirement. While the replacement ratio is an important part of retirement planning for most retirees, it may not represent the sole source of retirement income or support.

[1] Possibly including qualified voluntary employee contributions, now available as a result of the 1981 Economic Recovery Tax Act.

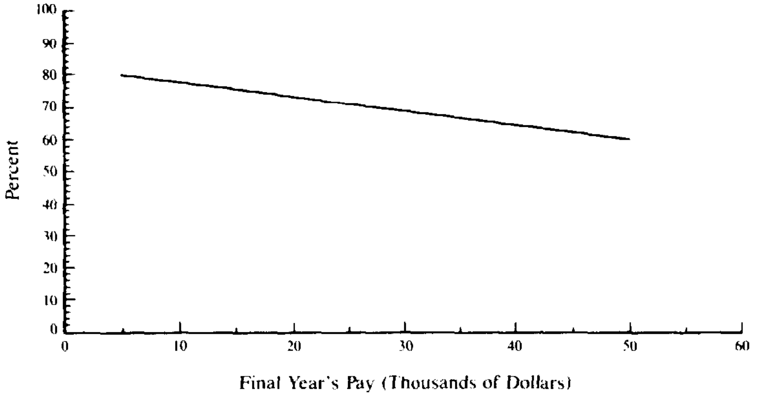

FIG. 1.—Private pension plus social security benefits as a percentage of final year's pay

TABLE 2

REPLACEMENT RATIOS AT RETIREMENT FOR EMPLOYEES IN SAMPLE SALARY RANGES

	SALARY RANGE		
	Low	Medium	High
1. Representative salary (*S*)	\$5,000	\$20,000–\$35,000	\$50,000
2. Target overall replacement ratio (*P* + *SSB*)/*S*	80%	73⅓%–66⅔%	60%
3. *SSB*/*S*	68%	35%–21%	14%
4. *P*/*S*	12%	38⅓%–45⅔%	46%
5. *SSB*/*P*	5.67	0.91–0.46	0.30

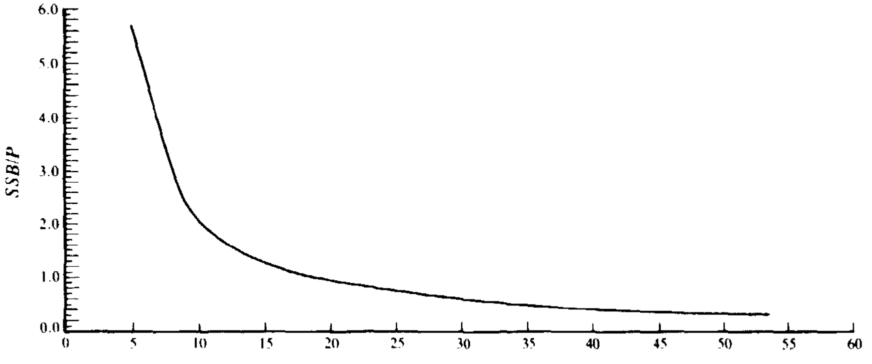

FIG. 2.—Ratio of *SSB* to *P* by salary level

Example 1

Assume that an employee retires in 1970 at age 65, with a final salary equal to the social security wage base at that time of $7,800, a history of earnings equal to the social security wage base, and a replacement ratio, $(P + SSB)/S$, at retirement of 78.8 percent. This is based on the straight line of Figure 1. (Almost 75 percent of the working population earned less than $7,800 in 1970. The corresponding percentage in 1981, at the social security wage base of $29,700, is about 92 percent.)

For those retiring in 1970 at the maximum, *SSB* was $2,277.60. Solving for the private pension, P = $3,868.80. The ratio SSB/P then becomes 58.9 percent. This is not consistent with Figure 2, which is based on the target overall replacement ratio in Table 2. This discrepancy stems from assuming current social security replacement ratios applied in 1970.

The formula for a, the percentage increase in P needed to maintain the 78.8 percent replacement ratio, is as follows: $a = 1.59c - 0.59b$. The basis for b is assumed to be the actual increases in social security benefits granted since 1970. The basis for c is assumed to be the historical average annual nominal earnings increase percentages as shown in the 1980 social security trustees' reports. In effect, c tracks the total earnings of those covered under social security. These percentages are shown below.

Year i	Basis for b_i	Basis for c_i	Year i	Basis for b_i	Basis for c_i
1970	0.0%*	4.9%	1975	8.0%	6.6%
1971	10.0	4.9	1976	6.4	8.4
1972	20.0	7.3	1977	5.9	7.1
1973	0.0	6.9	1978	6.5	8.1
1974	11.0	7.4	1979	9.9	8.4

* The 15 percent social security benefit increase granted in January 1970 is already reflected in the *SSB* figure of $2,277.60 above. If 15.0 percent, instead of 0.0 percent, is used as b_i in 1970, and the calculation is repeated using the smaller social security benefit of $1,980.52 (i.e., $2,277.60 divided by 1.15), there is no substantial difference in the result.

On the basis of the above, yearly figures for the value of a_i are as follows:

Year i	a_i	Year i	a_i
1970	7.8%	1975	5.8%
1971	1.9	1976	9.6
1972	(0.2)	1977	7.8
1973	11.0	1978	9.0
1974	5.3	1979	7.5

The values of a_i shown represent the yearly increases in the amount of this employee's private pension needed to ensure that the ratio of total retirement income (exclusive of any other source of income) to the average wage level for active employees remains approximately the same throughout the employee's retirement years.

Because the yearly values of a_i vary widely (especially in the early 1970s), less frequent increases are calculated on a level annual compound interest equivalent basis. The two- and three-year level annual equivalents of the above values of a_i are shown below.

2-Year Level Annual Equivalents		3-Year Level Annual Equivalents	
Years	a	Years	a
1970–71	4.8%	1970–72	3.1%
1972–73	5.3	1973–75	7.3
1974–75	5.6	1976–78	8.8
1976–77	8.7		
1978–79	8.2		

To complete the numerical examples, compare a low-paid retiree with a high-paid retiree.

Example 2

The value of a_i will vary by final average salary at retirement. This can be illustrated by choosing employees with final average salaries of $5,000 and $50,000.

$5,000 employee:

$$a = 1.77c - 0.77b \quad \text{and} \quad SSB = \$1{,}736.40 \,.$$

$50,000 employee:

$$a = 1.08c - 0.08b \quad \text{and} \quad SSB = \$2{,}277.60 \,.$$

Yearly values of a and the level annual equivalents over two and three years for these two employees are shown on the facing page.

Of the ten years considered, 1970–79, inequality (4) holds in the five years 1971, 1972, 1974, 1975, and 1979, while inequality (5) holds in the remaining five years. The value of a increases with increasing salaries in the years in which inequality (4) holds, and decreases with increasing salaries in the years in which inequality (5) holds. This is consistent with the discussion of (8) and (9) in Section III.

Year(s)	Final Average Salary	
	$5,000	$50,000
1970	8.7%	5.3%
1971	1.0	4.5
1972	(2.5)	6.3
1973	12.2	7.5
1974	4.6	7.1
1975	5.5	6.5
1976	9.9	8.6
1977	8.0	7.2
1978	9.3	8.2
1979	7.3	8.3
1970–71	4.8	4.9
1972–73	4.6	6.9
1974–75	5.0	6.8
1976–77	8.9	7.9
1978–79	8.3	8.3
1970–72	2.3	5.4
1973–75	7.4	7.0
1976–78	9.1	8.0

V. CONCLUSION

For planning purposes, P is known, and it is possible to define S and to estimate SSB for a group of employees so that $R = (P + SSB)/S$ can be determined. Each of these terms (P, SSB, and S) increases yearly by the rates a_i, b_i, and c_i, respectively. Once the choice of c is made, which involves company philosophy and practical considerations, a can be found. The relationships between a, b, and c, and the conditions under which $R = R_n$, provide insight into the process of determining increases in retiree benefits, subject, of course, to the ability to meet the costs of such increases.

DISCUSSION OF PRECEDING PAPER

ROBERT J. MYERS:

Messrs. Berin and Richter are to be congratulated on their paper dealing with the interrelationship of social security benefits and private pensions from a theoretical standpoint—but with significant practical applications.

My discussion will deal with only one aspect of the paper—namely, Table 1, which presents social security benefit replacement rates. The authors indicate that the figures, which relate to a person working through 1981 and retiring in January 1982, are based on the law in effect on January 1, 1981 (not taking into account the transitional-guarantee provision). In my opinion, the figures are incorrect, because no account is taken of the benefit increase of 11.2 percent for June 1981, even though the wages in 1981 *are* used both in the computation of the benefit and in the denominator of the replacement rate. The caveat of the authors that the figures are based on the law as of January 1, 1981, is not controlling, because that law provides for ongoing future automatic benefit increases. As a result, the benefit amounts and replacement ratios presented are too low and should be increased by 11.2 percent in each case.

Another problem in connection with these figures is that they are based on the simplistic assumption that wages increased by 5 percent in all past years back to 1951. It would have been better to use the actual wage increases, which varied significantly over the period. One way to do this is to use the wage-indexing series that is applied to social security earnings records in order to calculate benefit amounts under the wage-indexed computation method that will be used exclusively in the long-run future (as used by the authors in developing the figures in Table 1).

The wage histories developed by the authors are somewhat higher in every past year than those based on past wage trends. For example, under the approach used by the authors, the earnings for 1951 comparable to $10,000 in 1981 are $2,314, whereas using past wage trends they are only $2,059. As a consequence of using this "simplified" method, the hypothetical workers have steadily decreasing relative wage levels as they become older, which is not realistic.

The accompanying table presents data on the annual benefits and replacement rates as presented by the authors and as adjusted for the June 1981 benefit increase. The table also presents comparable figures using

COMPARISON OF SOCIAL SECURITY BENEFIT REPLACEMENT RATES

Pay in Final Year	Annual Benefit*		Replacement Rate†		
	Berin-Richter Method	Myers Method	Berin-Richter Method		Myers Method
			Original	Revised	
$ 5,000	$3,394	$3,421	68%	75%	68%
10,000	5,216	5,094	52	58	51
15,000	6,664	6,767	44	49	45
20,000	7,022	7,711	35	39	39
25,000	7,141	7,921	29	32	32
30,000	7,200	8,006	24	27	27
40,000	7,200	8,006	18	20	20
50,000	7,200	8,006	14	16	16
60,000	7,200	8,006	12	13	13

* Benefit rate for January multiplied by 12.

† Defined as annual benefit divided by pay in final year of employment.

what I believe to be the more appropriate method of projecting earnings back into the past. As it happens, the replacement rates based on my procedure are very close to those originally developed by the authors for earnings of $15,000 and under but are somewhat larger for higher earnings.

DONALD P. HARRINGTON:

The planning model developed by Messrs. Berin and Richter is very useful in analyzing the implications of postretirement benefit improvements. The formula is concise and does contain the significant elements that should be considered when a postretirement adjustment is proposed. A straightforward application of the formula, however, does raise some philosophical issues. The primary issue relates to the increase in pay and its relation to inflation. An automatic application of the formula (the authors are not recommending this) implies that the increases in pay should be passed along to the already retired, irrespective of the relation to inflation. To the extent that such an increase in pay exceeds inflation, the pension adjustment will exceed the increase in pay received by active employees. In this situation, I believe it would be difficult to justify a constant replacement ratio. Perhaps in the unfavorable economic scenario when inflation exceeds increases in pay a constant replacement ratio may be acceptable, but that would be only if the plan sponsor can afford to pass along improvements to the retired. Considering these issues, and the fact that the economic environment tends to be cyclical, my inclination would be to avoid focusing on a constant replacement ratio in retirement.

Alternatively, the numerator in the algorithm should be examined, since this expression in the formula can be used to measure the change in purchasing power from the date of retirement to the date of the pension adjustment. Accordingly, by setting

$$P\prod_{i=1}^{N}(1 + a_i) + SSB\prod_{i=1}^{N}(1 + b_i) = R_0S\prod_{i=1}^{N}(1 + k_i) ,$$

we can measure this change. While the form of the algorithm appears to be unchanged, the implications are significant. In this situation R_0S is the initial combined benefit, and it is treated as a unit amount. The variable k reflects the appropriateness of the CPI for retirees in general and the plan sponsor in particular. The plan sponsor may pay for other benefits, or past formula adjustments may have been extended to the retired, and these considerations should be taken into account. Finally, the improvement in pay should be considered, but only to balance equity between actives and retirees. Under this scenario, ad hoc adjustments are substantially more *subjective* than anyone might expect. No doubt the authors recognize these distinctions, but I emphasize the issue of subjectivity particularly when a plan sponsor is concerned that an automatic pattern of ad hoc increases could create a contractual commitment.

Further analysis of the equation

$$P\prod_{i=1}^{N}(1 + a_i) = \overline{R_0S}\prod_{i=1}^{N}(1 + k_i) - SSB\prod_{i=1}^{N}(1 + b_i)$$

is helpful in drawing additional conclusions.

Given

$$P(a) = \overline{R_0S}(k) - SSB(b) \quad \text{and} \quad k < b ,$$

then

$$P(a) = \overline{R_0S}(k) - SSB(k) - SSB(\Delta) , \quad \text{where } 0 < \Delta < 1 ,$$

or

$$P(a) = P(k) - SSB(\Delta) ,$$

and it is evident that $a < k$. The conclusion, therefore, is that if $k < b$, the plan sponsor may take credit for the excess indexing provided by social security. Also, under the same circumstances, the longer the duration of retirement, the smaller the adjustment factor a, since the social security benefit is converging on the total retirement benefit. The plan sponsor may conclude from this relationship that should this condition, $k < b$, continue for a sufficient period of time, it would *not* be necessary to continue making periodic ad hoc increases.

Finally, one more issue should be addressed. It is apparent that $\lim_{S_0 \to \infty} (SSB/S_0) = 0$. Therefore, as S_0 becomes infinite, the value of a approaches k, and b is irrelevant. To the plan sponsor, this means that any single percent adjustment will benefit various classes more than others. To the extent that the class selected does not have the highest ratio SSB/S_0, and given that $k < b$, the adjustment will be larger than the lowest-paid retiree needs.

My particular experience has been that k is more subjective than pensioners might imagine, and that, to date, $k < b$.

CECIL J. NESBITT:

My thinking in regard to this paper has been influenced by two recent reports [1, 2]. The graded benefit annuities utilize the concept of assuming a low interest rate and applying investment gains to purchase incremental annuities. Such annuities are becoming more common for retiree incomes and may well become a significant form in the future. One of the reports [2] explores the possibility that the CPI overstates the cost-of-living increases for retirees and thereby greatly affects the need for retirement income adjustments, inversely according to the proportion of total retirement income that is provided through social security. Instead of using the criterion of constant replacement ratio, as in the present paper, the report considers a purchasing-power index. We shall see that the two concepts are closely related, as one might expect.

Neither the paper nor the report [2] takes into account retirement income provided by the retiree's own savings and investments. This is an important factor for higher-income employees and, of course, is one leg of the oft-mentioned three-legged stool for retirement income. I shall take it into account but thereby shift emphasis from the employer's responsibility to that of the individual. The means of doing so will be an additional component, A, representing retirement income derived from the individual's own accumulation. I shall assume that A is in the form of an increasing annuity, possibly arranged by successive annuity purchases from time to time.

While one could use a set of time-indexed components, P_n, SSB_n, A_n, $n = 0, 1, 2, \ldots$ for the retirement income, I shall follow the authors' terminology and assume that P, SSB, and A are the components at the beginning of an interval, and that over the interval these components increase by the rates a, b, and d. Further, salary S at the beginning of the interval increases by rate c, and cost of living for the retiree by rate $\bar{c}$.

For the authors' purpose of maintaining a constant replacement ratio, we would have

$$\frac{P(1 + a) + SSB(1 + b) + A(1 + d)}{S(1 + c)} = \frac{P + SSB + A}{S}, \tag{1}$$

which does not depend on S. If, instead of constant replacement ratio, we aim at constant purchasing power, the rate c would be replaced by $\bar{c}$, and equation (1) would become

$$P(1 + a) + SSB(1 + b) + A(1 + d) = (1 + \bar{c})(P + SSB + A). \tag{2}$$

Also, if instead of the employer taking responsibility through choice of a to maintain equation (1) or (2), the retiree assumes such responsibility through management of A, then from equation (2) it follows that d should be not less than the level indicated by equation (3), namely,

$$d = \bar{c} + \frac{P}{A}(\bar{c} - a) - \frac{SSB}{A}(b - \bar{c}). \tag{3}$$

In other words, the rate of increase in the retirement income from the retiree's own funds to maintain constant purchasing power equals $\bar{c}$ plus an amount to compensate for less than full cost-of-living adjustment of P, and minus an amount for overcompensation of SSB.

In particular, if $a = \bar{c}$,

$$d = \bar{c} - \frac{SSB}{A}(b - \bar{c}) \tag{4}$$

and may be less than $\bar{c}$. If $a = 0$,

$$d = \left(1 + \frac{P}{A}\right)\bar{c} - \frac{SSB}{A}(b - \bar{c}) \tag{5}$$

and may be greater than $\bar{c}$.

There are many ramifications that could be considered for higher-income employees, including the effect of earnings after retirement. The authors have supplied us with a framework in which to explore these ramifications, and thereby have contributed to thought on pensions.

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DONALD E. BODEN*:

My personal experience in discussing with companies whether or not to increase pensions in payment has been that their primary concern has been to maintain the credibility of the pension plan itself. The opinion is often expressed that without increases in pensions, the plan will fail the retirees, and this failure will become known to present employees and will seriously devalue the pension plan in the compensation package.

Defining c is an obvious problem. I was puzzled for a while by the phrase "c . . . reflecting inflation only" until I concluded that "reflecting inflation" did not mean "equal to inflation." But c does pose problems, and there is no good answer. There are two situations in Germany where similar problems have arisen.

A 1974 German law requires all private pension plans to review all pensions in payment every three years and to increase them for changes in cost of living. The law provided no guidance on how to deal with changes in social security and wage levels. One court case said that the amount of increase in pension never had to exceed the increase in earnings of an active employee during the same period. This raised so many questions on how to make the comparison that a later court ruling threw out this approach. The second situation is the practice of providing individual pension arrangements for executives where the pension in payment is linked to the movement of the midpoint of the civil service salary range. Here you have the obvious problems of civil service grades being affected by content and structural changes.

Certainly even linking c to the movement of a company-wide average is fraught with problems such as the examples we have seen of companies

* Mr. Boden, not a member of the Society, is a member of the American Academy of Actuaries.

in Europe changing product mix from low to high technology, causing large increases in average earnings.

(AUTHORS' REVIEW OF DISCUSSION)

BARNET N. BERIN AND ANTHONY B. RICHTER:

We agree with Mr. Myers that his approach to Table 1 is superior to that taken in the paper. The replacement rates based on his procedure are very close to those originally developed: about the same for low-paid and high-paid, with a maximum positive excess of 3–4 percent, from $20,000 to $30,000 of final pay. These differences do not damage the formulas or the concepts, but do influence the numerical results slightly. The examples were meant to be illustrative of the technique, which is, of itself, a first step in the process.

Mr. Harrington sees the paper as very useful in analyzing the implications of postretirement benefit improvements and introduces several practical observations, including some cautions about applying the technique. The relationship between $a < c < b$ and $a > c > b$ is emphasized and the effect of increasing salary discussed (where b is not significant). Mr. Harrington introduces variables k and Δ in an interesting extension. Note that $\Delta = b - k$, and the relationship is for $a < k < b$.

Regarding Professor Nesbitt's comments, the authors did make the point that there are many considerations involved in selecting and quantifying results. Early drafts of the paper made explicit reference to section 401(k) plans. Professor Nesbitt's formula (1) may still involve S, if only indirectly, since c could depend upon S in the particular situation that he describes. Formulas (3), (4), and (5) are developed from the point of view of the retiree and are of general interest. Introducing private savings into the equation may cause a_i to increase less sharply with increases in final pay.

Klaus Hubeck examines the problem discussed by Mr. Boden (and its seven-year history) in a paper on pensions under inflationary conditions recently published (May 1982) for the Eighth Conference of the International Association of Actuaries (II, 27), "The Adjustment of Company Pensions: A German Model—after Years of Discussion."

In response to a question from other discussants, the results of the paper hold true for employees who work for several employers, vesting in each, if the social security benefit is assumed to be earned proportionately over a working lifetime.

We thank Messrs. Myers, Harrington, Nesbitt, Boden, and others for their discussions. Their comments are useful and are appreciated.