# THE GENERALIZED ORDINARY DIVIDEND FORMULA UNDER TEFRA 

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#### Abstract

This paper extends the author's previous study, "An Expanded Financial Structure for Ordinary Dividends,'" to reflect amendments to the life insurance company federal income tax statutes introduced by the Tax Equity and Fiscal Responsibility Act of 1982 (TEFRA). The generalized dividend formula is based on the contribution principle and incorporates into a single structure three methods described in actuarial literature: the source-of-earnings method, the asset-shares method, and the fund method. Companies using the generalized structure of the previous study or any of these methods will find the precise new federal income tax (FIT) factors of the generalized formula helpful in allocating FIT among ordinary dividend classes. A permanent replacement for TEFRA is being sought, and if the new law is based on gain from operations with some limit on dividend deductibility, then this approach should be readily adaptable to it.

In the previous study, generalized formulas were developed in detail for a typical Phase 1 mutual company in Fraser situation B. Formulas in this paper are developed in detail for a Phase 2 Negative mutual company in Fraser situation A, typical under TEFRA for a mutual company.


## 1. BACKGROUND REQUIRED

Familiarity is assumed with my previous study [1] and with John C. Fraser's classic FIT paper [2]. The terminology of those papers is used in this one, extended in the text as necessary. ${ }^{1}$ This paper does not attempt to recreate the derivations, logic, or discussions of my previous study and, in fact, should be regarded strictly as an addendum to that work, updating it for the TEFRA changes.

The Fraser situations, tax formula, and marginal factors that result from TEFRA are presented in Appendix I in the detail needed for this paper. The formulas are developed along the lines used by Fraser [2]. Appendix II sketches the derivation under TEFRA of the basic generalized dividend formula (4) from my previous paper [1].

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## 11. THE DIVIDEND TAX TERM IN SITUATION A

The dividend tax term $(F I T)_{n}$ in formula (1) of [1], page 318, repeated in Appendix II, is on a marginal basis applying to a policy year. The calendar-year format for the tax shown in Appendix I reduces to the following on a policy-year basis, neglecting $X$, which is zero for a large company, and other constants as immaterial at the policy level:

$$
\begin{aligned}
(F I T)_{n}= & r\left\{\left(\pi_{n}-E_{n}^{\prime}-E_{n}^{\prime \prime}\right)\right. \\
& +h i^{\prime}\left(\pi_{n}+f_{n-1} V_{n-1}-E_{n}^{\prime}-1 / 2 E_{n}^{\prime \prime}-1 / 2 q_{n-1}^{\prime} F_{n}\right) \\
& -\left[V_{n}\left(1-q_{n-1}^{\prime}-w_{n-1}^{\prime}\right)-V_{n-1}\right] \\
& -q_{n-1}^{\prime}\left[F_{n}-(T D)_{n}\right] \\
& -w_{n-1}^{\prime}\left[C_{n}-(T D)_{n}\right] \\
& -a\left(1-m q_{n-1}^{\prime}\right) D_{n} \\
& -a\left(q_{n-1}^{\prime}+w_{n-1}^{\prime}\right)(T D)_{n} \\
& +1 / 2(1-h) i\left(V_{n-1}+P_{n}+V_{n}\right) \\
& -(\text { Sec. } 818(\mathrm{c}) \text { factor on CRVM policies })\},
\end{aligned}
$$

where

$$
\begin{aligned}
a=a_{1} & =1.00 \text { on pension policies; } \\
a=a_{2} & =0.775 \text { on nonpension policies in mutual companies } \\
& =0.85 \text { on nonpension policies in stock companies } .
\end{aligned}
$$

Also, there is an FIT deduction of $-r E_{\mathrm{o}}$ at issue, if TEFRA was applicable at issue. On CRVM policies, this credit appears as $-r\left(E_{0}-\left(P_{2}-P_{1}\right)\right]$ at issue and as $-r\left[E_{1}+\left(P_{2}-P_{1}\right)\right]$ in the first policy year. If TEFRA was not applicable at issue, there is no credit for issue expenses.

The 818(c) factor will be discussed in the following section.

## 1II. GENERALIZED DIVIDEND FORMULA IN SITUATION A

If the expression for $(F I T)_{n}$ in the preceding section is introduced into formula (1) of [1], page 318, and formula (1) is transformed into formula (2) and then into formula (4), as outlined in Appendix II, one obtains the following for TEFRA situation $A$.

New Formula (3): The Surplus Function $S_{n}$

$$
\begin{gather*}
S_{n}=\frac{B_{\mathrm{n}}(1-x)-\left(1-m q_{n-1}^{\prime}\right)(1-a r)\left(\Delta D_{n}\right)+S_{n-1}\left(1+i^{\prime}\right)}{1-q_{n-1}^{\prime}-w_{n-1}}  \tag{3}\\
S_{0}=-E_{0}(1-x),
\end{gather*}
$$

where

$$
\left.\begin{array}{rl}
x & =\mathrm{r} \\
& =0 \\
& \text { if TEFRA is in effect at issue and } n \leqslant k \\
& \text { if TEFRA is no in effect at issue and } n \leqslant k
\end{array}\right) \text { if } n>k .
$$

Note that on CRVM policies,

$$
S_{\mathrm{O}}=-\left[E_{O}-\left(P_{2}-P_{1}\right)\right](1-x)
$$

and

$$
B_{m(\not \times k)}=\frac{E_{0}-\left(P_{2}-P_{1}\right)}{a_{\text {衣 }}},
$$

with $E_{1}+\left(P_{2}-P_{1}\right)$ substituted for $E_{1}$ in the first-year dividend.
Also, note that the 818(c) credit in the first year on CRVM policies also can be applied against $E_{0}$, reducing the amortizable issue expense to zero in many cases; then, $k$ would become zero.

In Section V, a dynamic or floating definition of $B_{n(\nmid * k)}$ is given, which ensures that the values of $\left(-S_{n}\right)$ for $n \leqslant k$, as established at introduction of the policy class, are sustained as $i^{\prime}, q_{n-1}^{\prime}$, and $w_{n-1}^{\prime}$ change in later years, thereby ensuring recoverability of issue expenses automatically.

New Formula (4): The Generalized Dividend $D_{n}$

$$
\begin{aligned}
& \left.\left(1-m q_{n}^{\prime}\right)\right) D_{n} \\
& \quad=\frac{1-r}{1-a r}\left\{\left(P_{n}+V_{n-1}\right)\left(1+i^{\prime}\right)-V_{n}\right.
\end{aligned}
$$

$$
\begin{align*}
& +\left(\pi_{n}-P_{n}\right)\left(1+i^{\prime}\right)-E_{n}^{\prime}\left(1+i^{\prime}\right)-E_{n}^{\prime \prime}\left(1+1 / 2 i^{\prime}\right) \\
& -q_{n-1}^{\prime}\left[F_{n}\left(1+1 / 2 i^{\prime}\right)-V_{n}\right] \\
& \left.-w_{n-1}^{\prime}\left(C_{n}-V_{n}\right)\right\} \\
& -\frac{(1-a) r}{1-a r}\left[\left(q_{n-1}^{\prime}+w_{n-1}^{\prime}\right)(T D)_{n}\right] \\
& +\frac{(1-h) r}{1-a r}\left\{i^{\prime}\left[\left(\pi_{n}+V_{n-1}\right)-E_{n}^{\prime}-1 / 2 E_{n}^{\prime \prime}-1 / 2 q_{n-1}^{\prime} F_{n}\right]\right.  \tag{4}\\
& \left.-1 / 2 i\left(V_{n-1}+P_{n}+V_{n}\right)\right\} \\
& - \begin{cases}\frac{1-x}{1-a r} B_{n(>k)}+\left(1-m q_{n-1}^{\prime}\right)\left(\Delta D_{n}\right) & \text { for } n \leqslant k \\
\frac{1}{1-a r} B_{n(>k)} & \text { for } n>k\end{cases} \\
& +\frac{1}{1-a r}\left(G_{n}+R_{n}\right) \\
& +\frac{r}{1-a r} \text { (818(c) factor on CRVM policies). }
\end{align*}
$$

The $818(\mathrm{c})$ credit at $n=1$ would be applied against amortizable issue expenses first, and any balance would be spread forward on an actuarial equivalence basis; this factor is zero if TEFRA is not in effect at issue, that is, for policies issued before January 1, 1982.

It is convenient that on in-force policies formulas (3) and (4) accept $S_{n-1}$, accumulated on the historical Phase 1 FIT basis prior to TEFRA, as input into the new $S_{n}$, and $D_{n}$, now reflecting the Phase 2 Negative FIT under TEFRA.

The 818(c) Factor on CRVM Policies
The $818(\mathrm{c})$ term is derived by differencing the tax term $(F I T)_{n}$ in Section II with respect to reserves and inserting the approximate expression for the excess of net level premium (NLP) over CRVM reserves allowed in the statute. The difference in the tax is as follows:

$$
\begin{align*}
& -Y r\left\{\left[F_{n}^{\prime}(1+1 / 2 i v)-V_{n}\right]\left(1-q_{n-1}^{\prime}-w_{n-1}^{\prime}\right)-\left[F_{n}^{\prime}(1+1 / 2 i v)-V_{n-1}\right]\right.  \tag{5}\\
& \left.\quad-1 / 2(1-h) i\left(2 F_{n}^{\prime}-V_{n-1}-P_{n}-V_{n}\right)\right\}
\end{align*}
$$

where

$$
\begin{aligned}
F_{n}^{\prime} & =F_{n}-(T D)_{n} \\
Y & =0.021 \text { for policies issued prior to April 1, } 1982 \\
& =0.019 \text { for policies issued after March 31, } 1982 .
\end{aligned}
$$

The 818(c) dividend term is the change in tax (5) multiplied by the factor $-1 /(1-a r)$. Hence, in formula (4), the 818 (c) factor in CRVM policies is as follows:

$$
\begin{aligned}
& Y\left\{\left(F_{n}^{\prime}-F_{n-1}^{\prime}\right)(1+1 / 2 i v)-\left(V_{n}-V_{n-1}\right)-\left(q_{n-1}^{\prime}+w_{n-1}^{\prime}\right)\left[F_{n}^{\prime}(1+1 / 2 i v)-V_{n}\right]\right. \\
& \left.\quad-1 / 2(1-h) i\left(2 F_{n}^{\prime}-V_{n-1}-P_{n}-V_{n}\right)\right\} .
\end{aligned}
$$

In the common case of $F_{n-1}^{\prime}=F_{n}^{\prime}=F^{\prime}$ and $P_{2} \ngtr{ }_{19} P_{1+1}$ for a policy issued after March 31, 1982, the 818(c) dividend term is as follows:

$$
\begin{aligned}
& \text { For } n=1 \text { : } \\
& \qquad \frac{0.019 r}{1-a r}\left[F^{\prime}(1+1 / 2 i v)\left(1-q_{0}^{\prime}-w_{0}^{\prime}\right)-(1-h) i\left(F^{\prime}-1 / 2 P_{1}\right)\right] .
\end{aligned}
$$

For $n>1$ :

$$
\begin{gathered}
\frac{-0.019 r}{1-a r}\left\{\left(V_{n}-V_{n-1}\right)+\left(q_{n-1}^{\prime}+w_{n-1}^{\prime}\right)\left[F^{\prime}(1+1 / 2 i v)-V_{n}\right]\right. \\
\left.+(1-h) i\left[F^{\prime}-1 / 2\left(V_{n-1}+P_{n}+V_{n}\right)\right]\right\}
\end{gathered}
$$

As noted earlier, the tax savings, formula (5), usually would be used to reduce the amortizable issue expense, $\left[E_{0}-\left(P_{2}-P_{1}\right)\right](1-r)$, the balance, if any, being spread forward on an actuarial equivalence basis against the negative 818(c) dividend term for $n>1$.

## Discussion of Formulas (3) and (4)

The first term in formula (4) equals $(1-r) /(1-a r)$ times the statutory gain prior to FIT, amortization of issue expenses, profit charges, and dividend, where the multiplier is 83.9 percent for nonpension policies in mutual companies, 88.7 percent for such policies in stock companies, and 100 percent for pension policies in all companies. This is the major part of the dividend.

For policies issued after December 31, 1981, issue expenses are de-
ductible from taxable income, and the charge for amortization of issue expenses has the same multiplier; for policies issued prior to January 1 , 1982, there is no such credit for issue expenses and the multiplier of the amortization charge is larger, $1 /(1-a r)$. Further understanding develops by study of the complete ( $(F I T)_{n}$ expression in the next section.

iv. THE COMPLETE DIVIDEND TAX FUNCTION (FIT) $n$

## Derivation

If the generalized dividend $D_{n}$ of formulas (3) and (4) is introduced into the tax term given by the formula in Section II, one derives the complete tax function expressed in policy and experience factors only. The easiest derivation of this function is to note that ( $F I T)_{\text {" }}$ is the excess of the hypothetical dividend, assuming $r=0$. over the actual dividend in Situation A under TEFRA. The concept of the floating $B_{m(\nmid \alpha)}$, set forth in Section V, is used in this derivation, so that $B_{n+5}$ on the $r=0$ assumption reproduces the values of $\left(-S_{"}\right)$ established for the actual tax basis. The result is as given below.

Formula for the Complete (FIT) Function

$$
\begin{aligned}
& (F I T)_{n}\left(1-m q_{n-1}^{\prime}\right) \\
& =\frac{(1-a) r}{1-a r}\left\{\left(P_{n}+V_{n-1}\right)\left(1+i^{\prime}\right)-V_{n}\right. \\
& +\left(\pi_{n}-P_{n}\right)\left(1+i^{\prime}\right)-E_{n}^{\prime}\left(1+i^{\prime}\right)-E_{n}^{\prime \prime}\left(1+1 / i^{\prime}\right) \\
& -q_{n-1}^{\prime}\left[F_{n}\left(1+1 / 2 i^{\prime}\right)-V_{n}\right] \\
& \left.-w_{n, 1}^{\prime}\left(C_{n}-V_{n}\right)\right\} \\
& +\frac{(1-a) r}{1-a r}\left[\left(q_{n-1}^{\prime}+w_{n-1}^{\prime}\right)(T D)_{n}\right] \\
& -\frac{(1-h) r}{1-a r}\left\{i^{\prime}\left[\left(\pi_{n}+V_{n-1}\right)-E_{n}^{\prime}-1 / 2 E_{n}^{\prime \prime}-1 / 2 q_{n-1}^{\prime} F_{n}\right]\right. \\
& \left.-1 / 2 i\left(V_{n-1}+P_{n}+V_{n}\right)\right\} \\
& + \begin{cases}\frac{a r}{1-a r} B_{n(\nmid k)}-h r i^{\prime}\left(-S_{n-1}\right)-\operatorname{ar}\left(1-m q_{n-1}^{\prime}\right)\left(\Delta D_{n}\right) n \leqslant k \\
\frac{a r}{1-a r} B_{n(\leqslant-<1}, & n>k\end{cases}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{a r}{1-a r}\left(G_{n}+R_{n}\right) \\
& -\frac{r}{1-a r}(818(\mathrm{c}) \text { factor on CRVM policies }) .
\end{aligned}
$$

## Discussion of the Complete (FIT) ${ }_{n}$ Function

This formula indicates the manner in which dividends before FIT should be adjusted for allocation of FIT under TEFRA, assuming that FIT is to be reflected fully in dividends. Whether or not a company uses the full generalized dividend scale, this formula can be a guide to allocation of FIT among dividend classes.

The dominant first term is ( $1-a) r /(1-a r$ ) times the statutory operating gain prior to amortization of issue expenses, profit charges, FIT, and dividends. The multiplier is 16.1 percent for nonpension policies in mutual companies, 11.3 percent for such policies in stock companies, and 0 percent for pension policies in all companies. Where the policy is issued on or after January 1, 1982, so that $x=r$, the same multiplier applies to $B_{m(x A)}$. the charge for amortization of issue expenses.

The remaining terms show that the tax function differs from a percentage of the statutory operating gain prior to profit charges, FIT, and dividends, being affected in addition by the value of $a$, percentage of taxexempt investment income ( $1-h$ ), 818(c) adjustment, profit charges, and the schedule of amortizing issue expenses. The enhancement of the effect of tax-exempt investment income after pass-through to dividends is especially notable.

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V. "FLOATING" ISSUE-EXPENSE AMORTIZATION CHARGE B}\mp@subsup{B}{n(>G)}{
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In his discussion of [1], Dr. Thomas Kabele noted that the schedule of ( $-S_{n(\nmid k)}$ ) determined at introduction of a policy class could be preserved by allowing $B_{n(\ngtr k)}$ to vary, or float, thereby ensuring the recoverability of the amortizable issue expenses. This is accomplished in the following way.

Let $S_{m \neq k}$, be defined as in formula (3):

$$
\left(-S_{n}\right)=\frac{\left(1+i^{\prime \prime}\right)\left(-S_{n-1}\right)-B_{n}^{\prime}(1-x)+\left(1-m q_{n-1}^{\prime}\right)(1-a r)\left(\Delta D_{n}\right)}{1-q_{n-1}^{\prime}-w_{n-1}^{\prime}}
$$

where $i^{\prime \prime}=(1-h r) i^{\prime}, q_{n-1}^{\prime}, w_{n-1}^{\prime}, B_{n}^{\prime}$, and $\left(\Delta D_{n}\right)$ are all based on the assumptions at introduction of the policy class.

The floating $\mathrm{B}_{n(>k)}$ then is determined as follows in later calendar years:

$$
\begin{aligned}
& B_{n(>k)}=\left(1-m q_{n,-}^{\prime}\right)(1-a r)\left(\Delta D_{n}\right)+\left(1+i^{\prime}\right)\left(-S_{n-1}\right)- \\
& \quad\left(1-q_{n-1}^{\prime}-w_{n-1}^{\prime}\right)\left(-S_{n}\right),
\end{aligned}
$$

where $i^{\prime \prime}, q_{n}^{\prime}$, and $w_{n-1}^{\prime}$ are the new values in the current year's dividend scale, and $S_{n}, S_{n-1}$, and ( $\Delta D_{n}$ ) remain unchanged.

This floating $B_{n(\neq k)}$ should be a standard feature of the generalized dividend formula. As noted previously, an adaptation of this process was used in Section IV.

## APPENDIXI

## TEFRA MODIFICATIONS OF CERTAIN FRASER FORMULAS

This analysis is confined largely to modifications of the formulas from [2] needed for the new TEFRA Phase 2 Negative situation, where most mutual life insurance companies will be located.

## DEFINITIONS

Mr. Fraser's notation is used with several extensions:
$D^{P}=$ Policyholder dividends on pension policies;
$D^{N^{P}}=$ Policyholder dividends on nonpension policies;
$a_{1}=a$ factor applying to pension policies ( $=1.00$ );
$a_{2}=a$ factor applying to nonpension policies ( $=0.775$ for mutual companies; $=0.85$ for stock companies);
$N=$ Nonparticipating premiums defined in section 809(d)(5); and
$H=$ Accident and health and group life premiums defined in section 809(d)(6).

Hence, assuming limitations on 0.03 N and 0.02 H do not apply, Fraser's $D$ becomes

$$
D=D^{P}+D^{N P}+0.03 N+0.02 H .
$$

Also,

$$
\begin{aligned}
X & =\$ 2,000,000-D / 4, \text { where } 0 \leqslant X \leqslant \$ 1,000,000 ; \\
r & =\text { Tax rate }(0.46) ; \\
\text { Tax } & =r(T n)+0.28(\text { Long-term capital gains) }- \text { Tax credits }- \text { Small } \\
& \text { constant; and } \\
T & =r(T I), \text { where } T I \text { (taxable income) is as defined below. }
\end{aligned}
$$

SITUATIONS AND TAXABLE INCOME
The new TEFRA limitation on $D$ is as follows:
$D \ngtr$ larger of $(A)$ and an elective alternative limitation (B):

$$
\begin{aligned}
& (A)=X+[(G-I \not) \nless 0] \\
& (B)=X+a_{1} D^{P}+a_{2}\left(D^{v r}+0.03 N\right),
\end{aligned}
$$

except that

$$
(B)=a_{1} D^{P}+\left(1+a_{2}\right)\left(D^{N P}+0.03 N\right) \text { if }\left(D^{N P}+0.03 N\right)<X .
$$

Corresponding to [2], page 57, TI becomes the following:
$T I=$ Sum of $(a)$ and $(b)$ :
$(a)=$ Smaller of (1) and (2):
(1) $=I$
(2) $=G-$ Smaller of (i) and (ii):
(i) $=D$
(ii) $=$ Larger of $(x)$ and an elective ( $y$ ):

$$
\begin{aligned}
& (x)=X+[(G-I)<0] \\
& (y)=X+a_{1} D^{P}+a_{2}\left(D^{v P}+0.03 N\right)
\end{aligned}
$$

$$
(b)=1 / 2(G-D-I) \nless 0
$$

The new Fraser situations and taxable income then may be displayed as shown on page 131.

As previously, situations A and C are Phase 2 Negative, situation B is Phase 1, and situation D is Phase 2 Positive. In large companies, $X=0$ and situation C is null.

## MARGINAL FACTORS IN SITUATION A FOR $X=0$

Some of Fraser's marginals are changed by TEFRA in all situations except situation C. In particular, the "Geometric Menge" adjustment in situation $B$ changes the marginals for assets, nontaxable investment income, taxable investment income, and nonpension reserves; this is true also in situation $D$, where the marginals are the average of those in situations C and B . Only situation A for $X=0$, applying to large mutuals, previously in situation B, will be treated here. Corresponding to Fraser's formulas for situation A ([2], pp. 63-66 and 96-98), and with the application of his derivations, the following emerges for situation A with $X=$ 0 , shown in the first line of the display on page 681:

$$
T=r\left[G^{\prime}+I^{r}-B^{\prime}-B^{\prime \prime}+\frac{I^{N T}}{I^{T}+I^{N T}}\left(i^{N P} V^{N P}+i^{P} V^{P}+B^{\prime}\right)-X\right.
$$

$$
\left.-a_{1} D^{P}-a_{2}\left(D^{v P}+0.03 N\right)-(818(\mathrm{c}) \text { factor on CRVM policies })\right] .
$$

Here, $G^{\prime}=P-C-E-\left(V_{1}-V_{0}\right)$, where
$P=$ Premium:
$C=$ All benefits, other than dividends, paid to policyholders;
$E=$ Expenses, commissions, and taxes other than FIT;
and $V_{1}$ and $V_{0}$ are mean reserves at the ends of the current and previous calendar years.

Note that in $B^{\prime}$, interest paid on nonpension deferred annuities is limited to the amount defined in new section $805(f)$.

The Fraser marginals can be derived from the above formula for $T$. Let

$$
K=\left(\frac{i^{N P}}{i^{v}} v^{N^{p}}+\frac{i^{p}}{i^{r}} v^{P}+b^{\prime}\right) .
$$

Then

$$
\begin{aligned}
m^{\wedge T} & =r h K ; \\
m^{r} & =r[1-(1-h) K]=m^{N T}+r(1-K) ; \\
m^{I} & =h m^{T}+(1-h) m^{N T}=r h, \quad \text { where } \quad I=I^{r}+I^{N T} .
\end{aligned}
$$

Also,

$$
\begin{aligned}
m^{N P} t_{\Lambda} & =m^{P} t_{h}=r(1-h) t_{k} ; \\
m^{G^{\prime}} & =m^{P}=-m^{c}=-m^{\left(v_{1}-v_{0}\right)}=-m^{B^{\prime}}=-m^{8 \mid \text { sic) factor }}=-m^{x}=r ; \\
m^{B^{\prime}} & =-r h ; \\
m^{D^{\mu}} & =-a_{1} r ; \\
m^{D^{N P}} & =-a_{2} r ; \\
m^{N} & =-0.03 a_{2} r ; \\
m^{H} & =m^{A}=0 .
\end{aligned}
$$

In matters involving decisions as to investment in taxable versus nontaxable securities, $m^{T}$ and $m^{N T}$ are applicable; these are functions of $K$. Otherwise, $m^{\prime}=r h$, which is independent of $K$, can be used, without the need to separate taxable and nontaxable investment income.

| $D^{N P}+0.03 N>X$ |  |  |
| :---: | :---: | :---: |
| Fraser Situation | (i-1 | I' |
| $\begin{aligned} & \mathrm{A} \\ & \mathrm{~B} \\ & \mathrm{C} \\ & \mathrm{D} \end{aligned}$ | $\begin{aligned} & G-I<\left[a_{1} D^{r}+a_{2}\left(D^{* P}+0.03 N\right]^{*}\right. \\ & {\left[a_{1} D^{\rho}+a_{2}\left(D^{\mathrm{NP}}+0.03 N\right)\right]^{*}<G-I<D-X } \\ & D-X<G-I<D \\ & D<G-I \end{aligned}$ | $\begin{aligned} & G-\left[a_{1} D^{P}+a_{2}\left(D^{N P}+0.03 N\right)+X\right]^{* *} \\ & \quad I-X \\ & G-D \\ & y_{2}(G-D+D) \end{aligned}$ |

Note.-If $X+a_{1} D^{\rho}+a_{2}\left(D^{N P}+0.03 N\right)>D$, so that the election is not made, then the bracket * becomes 0 and the bracket ** becomes $D$, as in the 1959 law.

| $D^{* P}+0.03 N<X$ |  |  |
| :---: | :---: | :---: |
| Fraser Situation | G-l | TI |
| $\begin{aligned} & \mathrm{A} \\ & \mathrm{~B} \\ & \mathrm{C} \\ & \mathrm{D} \end{aligned}$ | $\begin{aligned} G-l & \left.<l a_{1} D^{P}+\left(1+a_{2}\right)\left(D^{N P}+0.03 N\right)\right]^{*} \\ {\left[a_{1} D^{P}+\left(1+a_{2}\right)\left(D^{N r}+0.03 N\right)\right]^{*} } & <G-I<D-X \\ D-X & <G-I<D \\ D & <G-I \end{aligned}$ | $\begin{aligned} & G-\left[a_{1} D^{P}+\left(1+a_{2}\right)\left(D^{\mathrm{NP}}+0.03 N\right)\right]^{* *} \\ & \quad I-X \\ & G-D \\ & 1 /(G-D+I) \end{aligned}$ |

Note.-If $a_{1} D^{\mu}+\left(1+a_{2}\right)\left(D^{N P}+0.03 N\right)>D$, so that the election is not made, then the bracket * becomes 0 and the bracket ** becomes $D$, as in the 1959 law.

The 818(c) factor on CRVM policies, newly defined in TEFRA. is derived by differencing $T$, expressed in marginal format, with respect to the difference between NLP and CRVM reserves, and substituting the elective approximation for the excess of NLP over CRVM reserves:

818(c) factor $=-Y\left[\left(F_{1}-V_{1}\right)-\left(F_{0}-V_{0}\right)-1 / 2(1-h) i\left(F_{0}+F_{1}-V_{0}-V_{1}\right)\right]$, where

$$
\begin{aligned}
Y & =0.021 \text { for CRVM permanent policies issued before April 1, } 1982 \\
& =0.019 \text { for CRVM permanent policies issued after March 31, } 1982 \\
& =0.005 \text { for CRVM term policies for terms of more than fifteen years ; }
\end{aligned}
$$

$F$ and $V$ are aggregate face amounts and mean reserves on CRVM policies in force at the ends of the previous and current calendar years, and $i$ is the average rate of interest in CRVM reserves in the calendar year.

The contributions of CRVM policies in their first policy year tend to cause the 818(c) factor to reduce the tax in aggregate. However, the 818(c) term, when reduced to the policy level, is complex, as shown in Section III.

## APPENDIX II

## DERIVATION OF NEW FORMULA (4) FOR THE GENERALIZED DIVIDEND

Formula (1) from my previous paper ([1], p. 318) follows:

$$
\begin{align*}
f_{n} V_{n}= & \left(P_{n}+f_{n-1} V_{n-1}\right)\left(1+i^{\prime}\right) \\
& +\left(\pi_{n}-P_{n}\right)\left(1+i^{\prime}\right)-\left[E_{n}^{\prime}\left(1+i^{\prime}\right)+E_{n}^{\prime \prime}\left(1+i^{\prime} / 2\right)\right] \\
& -q_{n-1}^{\prime}\left[F_{n}\left(1+i^{\prime} / 2\right)-f_{n} V_{n}\right] \\
& -w_{n-1}^{\prime}\left(C_{n}-f_{n} V_{n}\right)  \tag{1}\\
& -(F I T) \\
& +G_{n}+R_{n} \\
& -D_{n}\left[1-m q_{n-1}^{\prime}+\left(i^{\prime} / 2\right) q_{n-1}^{\prime}(1-m)\right],
\end{align*}
$$

where $f_{0} V_{0}=-(1-r) E_{0}$ assuming that TEFRA is in effect at issue.
Setting $f_{n} V_{n}=V_{n}+S_{n}$, setting (FIT) equal to its value in Section II,
and solving for the dividend, we obtain the new formula (2), ignoring the higher-order term involving $i^{\prime} q_{n-1}^{\prime}$ :

$$
\begin{aligned}
(1- & m q_{n-1)}^{\prime} D_{n} \\
= & \left(P_{n}+V_{n-1}\right)\left(1+i^{\prime}\right)-V_{n} \\
& +\left(\pi_{n}-P_{n}\right)\left(1+i^{\prime}\right)-\left[E_{n}^{\prime}\left(1+i^{\prime}\right)+E_{n}^{\prime \prime}\left(1+i^{\prime} / 2\right)\right] \\
& -q_{n-1}^{\prime}\left[F_{n}\left(1+i^{\prime} / 2\right)-V_{n}\right] \\
- & w_{n-1}^{\prime}\left(C_{n}-V_{n}\right) \\
& -r\left\{\left(\pi_{n}-E_{n}^{\prime}-E_{n}^{\prime \prime}\right)+h i^{\prime}\left(\pi_{n}+V_{n-1}+S_{n-1}-E_{n}^{\prime}-1 / 2 E_{n}^{\prime \prime}-1 / 2 q_{n}^{\prime}{ }_{1} F_{n}\right)\right. \\
& \quad-\left[V_{n}\left(1-q_{n-1}^{\prime}-w_{n-1}^{\prime}\right)-V_{n-1}\right]-q_{n-1}^{\prime}\left[F_{n}-(T D)_{n}\right] \\
& \quad-w_{n}^{\prime}\left[C_{n}-(T D)_{n}\right]-a\left(1-m q_{n-1}^{\prime}\right) D_{n}-a\left(q_{n-1}^{\prime}+w_{n-1}^{\prime}\right)(T D)_{n} \\
& +1 / 2(1-h) i\left(V_{n-1}+P_{n}+V_{n}\right) \\
& \quad-(818(\mathrm{c}) \text { factor on CRVM policies })\} \\
+ & G_{n}+R_{n} \\
- & {\left[S_{n}\left(1-q_{n-1}^{\prime}-w_{n-1}^{\prime}\right)-S_{n-1}\left(1+i^{\prime}\right)\right] . }
\end{aligned}
$$

Now, by introducing the expression for $B_{n}$ from Section III and collecting terms, we can derive formula (4) for $D_{n}$ as it appears in Section III.

## REFERENCES

1. Cody, D. D. "An Expanded Financial Structure for Ordinary Dividends," TSA, XXXIII (1981), 313-38.
2. Fraser, J. C. "Mathematical Analysis of Phase 1 and Phase 2 of 'The Life Insurance Company Income Tax Act of 1959,'" TSA, XIV (1962), 51-117.

## DISCUSSION OF PRECEDING PAPER

## PAUL SARNOFF:

Mr. Cody is to be commended for presenting this interesting and informative examination of the operation of an ordinary dividend formula under TEFRA. The application of the marginal tax rates under TEFRA to each individual policy has the nice actuarial property that the sum of the individual policy tax charges reconciles with the aggregate tax charge upon the company for the year.

As the author indicates in Section III of the paper, the treatment of acquisition expense differs between policies issued before 1982 and those issued in 1982 and later. Policies in the latter class receive a credit in the calculation for the federal tax effect that results from deducting acquisition expense under TEFRA in the year that expense is incurred. As that expense is recovered out of the future income stream, tax charges would be generated under the dividend formula applicable to future years. On the other hand, for policies issued before 1982, no such credit would be accorded for acquisition expense, since such expense was incurred in a preTEFRA year. Recovery of previously incurred acquisition expense would give rise to dividend formula tax charges in future dividend years to which TEFRA is applicable.

This prompt rebate to the 1982 policyholder of the federal income tax effect of acquisition expense has the advantage of a favorable effect upon net cost illustrations of newly issued policies. The corresponding charge against the earnings of prior years' issues enables the company to make such rebates without impairing its financial position. The method does create the possibility, however, that replacement of pre 1982 policies by more recent policies would become attractive, or at least more attractive than formerly.

The method also creates the potential for a customer relations problem. The fact that newly issued policies would have a significantly more favorable net cost than the earlier issues may be difficult to explain. This type of problem can arise when members of one family, or employees or owners of a single firm, buy policies in adjacent calendar years and find it difficult to understand why such large differences in cost exist. I question whether the nice property that the sum of the individual tax charges reconciles with the total charge to the company is adequate justification that the method is fair, equitable and practical.

In the case of a 1982 policy, that policy's funds would be credited, under this method, with an amount equal to that policy's (negative) contribution to that year's federal income tax. What is the source of the money for that credit? It is my view that the source is not the federal government, since federal taxation is not normally a two-way street. The source of the money must be a charge that is made against some other class of policyholders. In other words, the pre 1982 policyholders are being charged not only for the tax on the operations of their own policies, but also to make a contribution to the 1982 policies. This might be rationalized on the grounds that when the 1982 policy has been in force as long as those early policies are currently in force, the 1982 policies will be called upon to contribute corresponding charges. That, of course, assumes that the provisions of TEFRA continue in effect from 1982 on, and that the 46 percent corporate tax rate continues unchanged throughout the period. At the time of this writing Congress is considering permanent tax legislation that would replace TEFRA and modify the 46 percent corporate tax rate, by a special deduction, to bring it effectively to the 34.5 percent level. But even if the given federal income tax formula and tax rate were to remain fixed and unchanged, I seriously question whether it is equitable and proper for an insurance company to make these transfers from one class of policyholders to another. The fundamental principle of life insurance has long been to provide insurance ot each class of policyholders at as close to actual cost as possible. Such transfers seem contrary to that equitable principle.

The matter may be simplified by considering a life insurance company that has been subject to stable rates of new business, lapse, mortality, investment income and expense for a long enough period that it may be considered a stationary model. The company's federal income tax under TEFRA may be simplistically described as a tax on 22.5 percent of the dividends paid to policyholders. The acquisition expense incurred by policies issued in 1982 balances exactly with the recovery of prior years' acquisition expense. Does it really follow that, despite this balance, the current year's issues would receive a large credit and the prior years' issues should have a large tax charge? Is the fact that the government determines the aggregate tax on the company in a certain way an adequate justification for the company to take money from old policyholders and give it to new policyholders?

## (AUTHOR'S REVIEW OF DISCUSSION) <br> DONALD D. CODY

Mr. Sarnoff has raised interesting points about the allocation of federal income tax to different policy classes. In both of my papers on the gener-
alized dividend financial structure, I have adopted the principle of charging costs directly to different policy classes as precisely as possible. This tends to assure optimum pricing in the free market, and it minimizes effects of changing distribution of business. Additionally, it enhances cost control because net costs are immediately affected by changes in expenditures and practices. This result is not assured if costs in one class are absorbed in greater or lesser degree by other classes. The pure concept, of course, is modified by the formulated allocation of indirect and overhead expenses, by averaging investment-year method (IYM) interest rates with portfolio rates (the $\alpha$-factor) to reduce replacements, and in other ways identified in the papers.

The precise allocation of federal income tax by marginal factors has been a feature of my thinking for the marketing and control reasons already mentioned. As Mr. Sarnoff notes, in essence this approach makes the insurance company an intermediary between the policyholder and the federal government. This is not a new concept, for example, lower federal income taxes charged to pension policies have long been a feature of pricing in all companies.

The above procedures for charges are equitable in my view, assuming that they are applied consistently and uniformly across all classes. Mr. Sarnoff questions the suggested handling of TEFRA credits to acquisition expenses on policies issued after December 31, 1981, noting that no such credits were available on earlier issues in Phase 1 companies, thereby creating a discontinuity in dividends on policies issued in 1981 and 1982. (The $818-\mathrm{c}$ credits and charges involve the same problem for policies issued in March and April, 1982.) While I do not agree that equity is impaired within my own generalized dividend structure, I agree with Mr. Sarnoff's privilege to define equity otherwise. I completely agree with the desirability of spreading the jump in credits on acquisition expenses if replacement dangers loom large, just as I have recommended modifying IYM interest rate credits. I would note, however, that rapidly changing interest rates and competition from unbundled products with direct IYM credits and income tax charges already have caused large replacement problems within and among companies. TEFRA acquisition expense credits, including the strange $818-\mathrm{c}$ credit, are only part of the problem. I am indebted to Mr. Sarnoff for highlighting this important matter.

Mr. Sarnoff noted that the simplistic value of the TEFRA tax in the idealized steady state situation (level sales and stable interest, expense, claim and termination rates, and so on) is $X$ percent of the dividends paid. The value of $X$ can be derived from my formula for the Complete $(F I T)_{n}$ Function
by substituting the approximate value of the dividend $\left(\mathrm{D}_{n}\right)$ and ignoring terms of lesser magnitude as follows:

$$
\begin{aligned}
(F I T)_{n} & \approx \frac{(1-a) r}{1-a r}\left[D_{n}+(F I T)_{n}\right] \\
& \approx \frac{(1-a) r}{1-r}\left[D_{n}\right]=X D_{n}
\end{aligned}
$$

For nonpension policies in a mutual company, $X=19.17$ percent (rather than Mr. Sarnoff's 22.5 percent). In a stock company, $X=12.78$ percent (one-third lower).
One purpose for writing this second paper was to show how precise TEFRA effects could be derived algebraically using the generalized dividend financial structure. Any contribution method dividend formula then can be adjusted using the general results with any modifications that seem desirable, including those discussed by Mr. Sarnoff.

I have found that the new federal income tax bills now being considered by Congress produce a tax formula which is quite similar to that under TEFRA. The resulting generalized dividend formula is also similar to that of my paper. It includes one new term, however, involving the increase in the excess of statutory reserves over tax law reserves during the policy year and another new term involving a ratable allocation of the differential earnings amount to the dividend or to some grading basis.


[^0]:    ${ }^{1}$ A short description of [1] may be found in RSA. VIII, No. 2 (Orlando, 1982), 444-48.

