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THE RISK OF ASSET DEFAULT

REPORT OF THE SOCIETY OF ACTUARIES C-1 RISK TASK FORCE OF THE COMMITTEE ON VALUATION AND RELATED AREAS

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ABSTRACT

This paper summarizes default experience on bonds from 1900 through 1987. Although the information is more extensive for publicly traded bonds, the paper includes some information on private placements, some of which is not otherwise available. It should constitute an appropriate reference for defaults and default losses.

INTRODUCTION

This study attempts to describe the changes in bond values that are attributable to default risks. The most noted change would be the drop in value immediately after the default announcement is made. Two other changes are as important, although not as sudden, as the loss after default. The first is the gradual decline in market value as default approaches. According to older studies, this decay starts several years before actual default takes place. The second change is the increase in value from the date of default to the date of final extinguishment. This paper attempts to discuss all three of these movements of price.

1. IMPORTANCE OF BONDS TO THE LIFE INSURANCE INDUSTRY

Traditionally, bonds, and especially publicly traded bonds, have constituted a major component of the assets of life insurance companies. While bonds have been important to life insurance companies, life insurance companies have been important to bonds. Because life insurance companies are one of the two long-term intermediaries (the other is pension funds), they constitute a major portion of the total market for bonds. As of the end of 1987, life insurance companies had $406 billion, or 45 percent of their invested assets of $905 billion, in corporate securities [14]. Of the $1099

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billion of corporate and $82 billion of foreign bonds outstanding, 33 percent was held by life insurance companies and 4.6 percent by other insurance companies. In addition, 25 percent, or $293 billion, was held by private and state and local pension funds. Together they constitute more than 62 percent of the holdings of the bonds outstanding. The only other large holders of bonds are foreign entities, in the amount of $158 billion; this large amount is a recent phenomenon [5].

Of the total of $98.3 billion increase in investment holdings for life insurance companies during 1987, $63.7 billion (64.8 percent) was in corporate securities. The relative proportions invested in privately placed, as opposed to publicly traded, bonds vary depending on the current stage of the business cycle of the economy.

Bonds clearly have been—and still are—the largest portion of the investment portfolios of most life insurance companies, and this situation is likely to continue.

2. SOURCES OF DATA

The task force studied bonds issued by United States corporations. Although there are occasional offerings of bonds of foreign governments and corporations domiciled in other countries (especially Canada), they are too small and infrequent to allow statistical analysis of the experience. In addition, United States life insurance companies have never had substantial holdings of such bonds. The vast majority of the bonds in life insurance company portfolios are the issues of commercial and industrial corporations in the United States.

The default and return characteristics of these obligations have been extensively studied. The classic study was prepared by the National Bureau of Economic Research and published as a series of books written by W. Braddock Hickman. There were three such volumes. The first was *The Volume of Corporate Bond Financing since 1900* [8], the second was *Corporate Bond Quality and Investor Experience* [9], and the third was *Statistical Measures of Corporate Bond Financing since 1900* [10]. The first volume was published in 1953 and the last in 1960. These studies covered all bond issues of more than $5 million during that period and a 10 percent sample of all smaller issues. All results were carried through 1943 or to the prior extinction of the issue. The National Bureau again investigated the problem of default on bonds for 1944–1965, and the work was published in *Trends in Corporate Bond Quality* by Thomas R. Atkinson in 1967 [3]. Elizabeth

T. Simpson acted as assistant to both Hickman and Atkinson, so the studies are therefore likely to be consistent.

The National Bureau then proceeded to studies of other matters, while the study of default rates on bonds was taken up seriously on Wall Street. In December 1978, Smith Barney published a study written by Jeffrey Hill of the default experience of the years 1966–1977 [11]. John D. Fitzpatrick and Jacobus T. Severiens published a useful article in the summer of 1978 [7]. This was followed up by a series of studies written by Edward Altman, who has annually updated the experience [1]. He has become the successor to Hickman and Atkinson.

The quality of the statistics varies considerably. The Hickman study covers the longest time and the longest preparation; however, it covered only a sample of the smaller issues. The other studies seem to have been done more quickly but probably had better access to information because they were done soon after the fact. They should be expected to have better quality and more data. However, the general level of economic activity is so much higher now than prior to 1943 that many issues could have existed and been privately placed with little record of the event. A representation of consistency in quality or even in the nature of the data cannot be made. At least one serious inconsistency exists. Hickman and Atkinson considered bonds to have defaulted if the issuer forced an exchange. Later authors do not consider these transactions to be defaults. First Boston has done a separate study on the costs to investors of these exchanges, and information on exchanges is included in the recent study by Asquith, Mullins, and Wolff [2].

The nature of the experience itself creates some problems. Experience on default of corporate bonds has improved over the period. Hickman could study about 1,200 defaults that occurred during the 45-year period. Atkinson only had 120 during the following 20 years; the Hill study covered only 143; and the Altman study included 188 for the period 1970–1986. Because the Hill study and the Altman study overlap for eight years, the total number of defaults studied after Hickman was about 300, or about 25 percent of the number in Hickman's study. In addition, among the defaults of the later studies, there was still a high proportion of railroad bonds. There would seem to be little of predictive value that can be learned from the study of railroad bonds, because there are not many issues and the characteristics of their problems are well known. The point is that while we may use the total data to determine the likelihood of default during a specific period, if we wish to consider characteristics of bonds that default we may have to go back to the Hickman data.

Except as noted, all experience is for straight bonds with fixed interest, stated maturity date, and no conversion rights or warrants, etc. Straight bonds have always been the dominant form of bond available; they are the only form on which we have substantial data. To the extent we have data on these variants, the results, as discussed by Atkinson, are consistent with those of straight bonds.

3. RATES OF DEFAULT

3.1 Definition of Default

For this study a definition of default is necessary. Default includes not only a failure to pay principal or interest promptly but also any significant contract modification. This last part of the definition of default is important because one way in which banks and insurance companies avoid the stigma attached to overt defaults is to "restructure" the loan or direct placement, thereby avoiding the unpleasantness of showing a default of principal or interest in their statements. Of the 549 large issues defaulting in the Hickman study [10, Table 229], 80 percent of those returning to good standing by 1943 did so by exchange. Of the 53 defaulting on principal payment, almost 90 percent of those returning to good standing by 1943 did so by exchange. Of those 549, 105 never experienced a failure to pay on time because the contracts were modified before a failure took place. Of those 105, 67 were restructured before maturity, while the remainder were restructured at maturity.

Particularly among private placements, the proportion of defaults by restructuring rather than by failure to make payments promptly has been higher in recent years. The Hill and Altman studies cover publicly traded bonds only. They do not consider restructuring as a default. This would have excluded 105 defaults of the Hickman study, or 23 percent of the defaults on principal or interest. This probably should not vitiate use of the results, but does indicate that the default rates would be somewhat higher if the older definition had been used. A recent study [2] shows that exchanges from 1977–1986 were 25 percent of the amounts of current definition defaults. This is consistent with the 23 percent from the Hickman studies.

Defaults do not occur suddenly and without warning. Although a "Johns Manville" is always possible, the majority of defaults take place among corporations generally understood to have problems. Such problems can be made known by the agencies in the form of lowered ratings and by the market itself in terms of lower prices and therefore higher interest rates on such obligations. Of the 716 large issues that defaulted in the Hickman study, five years before default 256 were investment grade (top four classes); 152 were below investment grade; and 308 were unrated. Two years before default, 225 were investment grade; 323 were below investment grade; and 168 were unrated. One year before default, 158 were investment grade (two-thirds in the lowest investment grade); 435 were below investment grade; and 190 were unrated. For the 440 smaller issues that defaulted five years before default, 63 were investment grade; 84 were below investment grade; and 293 were unrated. One year before default, 49 were investment grade (half in the lowest grade); 201 were below investment grade; and 190 were still unrated. The number of defaults since World War II is too small to allow firm statistical inferences. However, because of the improved availability of information and the many techniques for using financial data to simulate agency ratings, bonds that are currently rated investment grade have negligible probability of defaulting in the immediate future (unless there is a change in the capital structure, as in a leveraged buyout).

3.2 Aggregate Results

Figure 1 and Table 1 show the rates of default for all publicly traded bonds from 1900 through 1987. The numbers indicate a radical reduction in the percentage of outstanding bonds going into default after 1940. Figure 2 and Table 2 attempt to put this into perspective. The values in Table 3 are weighted by the total value of the bonds.

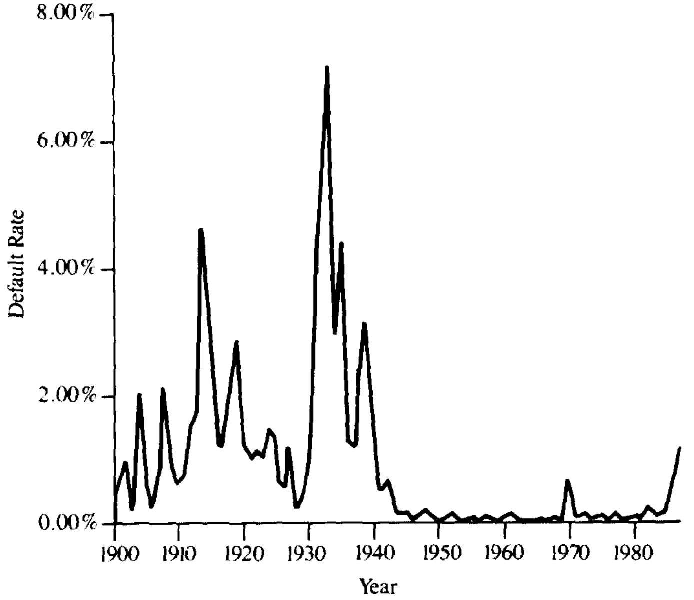
The differences in the average default rates, weighted or unweighted, are so large that we could, from inspection, conclude that the two major periods, 1900–1944 and 1945–1987, are so different that sampling error is not a possible explanation. To get a quantitative idea of the likelihood of such differences occurring by chance, we can compute the standard error of a ratio as described on page 29 of Cochran [4]. The standard errors in Table 3 are effectively weighted standard deviations of the mean default rates. The difference in the above weighted averages is

$$1.820 - 0.249 = 1.571.$$

The standard error of this difference is the square root of the sum of the squares of the two standard errors above, which is 0.290. The ratio of the difference to the standard error of the difference is 5.41. The use of the normal approximation leads us to conclude that the difference is not due to sampling error. The two samples are taken from different populations. The economy has changed.

FIGURE 1

DEFAULT RATES FOR STRAIGHT BONDS
1900–1987



The values for the decennial periods in Table 2 establish that, although there is considerable variation between periods, only when we cross the line of 1945 do we get the spectacular results in Table 3. The conclusion is that the nature of the default-creating process has changed since 1945. In more direct language, we are living in a different economic world.

3.3 Application of Beta Distribution

The beta probability distribution function is frequently used in actuarial and scientific work; it is appropriate for analysis of proportions. The values under the beta distribution can go only from 0 to 1, exactly as a proportion should. A beta distribution can be fitted by taking the mean and the standard deviation of the distribution of the data and solving for the parameters of the beta distribution.

A beta distribution was fitted to the data for 1900–1944 and 1945–1985, and the details of the distribution were calculated. It was then possible to

TABLE 1

RATES OF DEFAULT FOR ALL PUBLICLY TRADED BONDS
(DOLLARS IN THOUSANDS)
1900–1987

Year	Defaults	Outstanding	Default Rate	Year	Defaults	Outstanding	Default Rate
1900	$ 25.30	$ 5,913.9	0.428%	1945	$ 26.30	$22,130.0	0.119%
1901	36.70	6,468.0	0.567	1946	1.90	21,510.0	0.009
1902	78.00	7,785.1	1.002	1947	26.50	22,980.0	0.115
1903	15.70	8,600.2	0.183	1948	51.10	26,380.0	0.194
1904	194.40	9,297.3	2.091	1949	30.70	30,610.0	0.100
1905	58.50	10,059.1	0.582	1950	0.80	33,310.0	0.002
1906	24.50	10,836.1	0.226	1951	4.00	34,910.0	0.011
1907	90.90	11,743.7	0.774	1952	58.30	38,360.0	0.152
1908	271.80	12,538.3	2.168	1953	2.70	43,100.0	0.006
1909	116.10	13,187.4	0.880	1954	1.90	46,620.0	0.004
1910	83.30	13,712.2	0.607	1955	31.90	50,200.0	0.064
1911	102.30	14,206.2	0.720	1956	3.20	53,240.0	0.006
1912	225.20	15,037.8	1.498	1957	55.50	57,220.0	0.097
1913	265.40	15,735.7	1.687	1958	30.00	63,260.0	0.047
1914	746.40	16,073.0	4.644	1959	13.10	68,920.0	0.019
1915	571.90	15,981.9	3.578	1960	7.30	72,010.0	0.010
1916	193.10	16,169.2	1.194	1961	106.40	75,470.0	0.141
1917	206.20	16,736.6	1.232	1962	0.50	79,690.0	0.001
1918	359.30	17,215.5	2.087	1963	2.40	83,770.0	0.003
1919	491.40	17,126.4	2.869	1964	0.00	87,550.0	0.000
1920	205.30	17,142.2	1.198	1965	7.10	91,550.0	0.008
1921	179.50	17,798.3	1.009	1966	0.00	60,400.0	0.000
1922	213.50	18,764.2	1.138	1967	42.90	72,800.0	0.059
1923	197.10	19,524.9	1.009	1968	52.20	83,500.0	0.063
1924	303.40	20,551.1	1.476	1969	0.00	95,400.0	0.000
1925	292.30	21,644.4	1.350	1970	796.71	116,200.0	0.686
1926	125.40	22,313.1	0.562	1971	82.00	132,500.0	0.062
1927	284.00	23,870.4	1.190	1972	193.25	145,700.0	0.133
1928	57.10	25,744.4	0.222	1973	49.07	154,800.0	0.032
1929	96.80	26,556.1	0.365	1974	122.82	167,000.0	0.074
1930	228.10	26,712.0	0.854	1975	204.10	200,600.0	0.102
1931	940.20	28,065.5	3.350	1976	29.51	219,200.0	0.013
1932	1,352.70	27,839.5	4.859	1977	380.57	237,800.0	0.160
1933	1,901.40	26,468.0	7.184	1978	118.90	252,200.0	0.047
1934	710.40	24,430.3	2.908	1979	20.00	269,900.0	0.007
1935	1,055.90	23,741.7	4.447	1980	224.11	265,100.0	0.085
1936	288.50	22,572.9	1.278	1981	27.00	255,300.0	0.011
1937	253.40	22,682.8	1.117	1982	752.34	285,600.0	0.263
1938	620.20	21,916.7	2.830	1983	301.08	319,400.0	0.094
1939	698.90	22,040.7	3.171	1984	344.16	358,100.0	0.096
1940	420.60	21,473.1	1.959	1985	992.10	419,600.0	0.236
1941	106.80	21,348.1	0.500	1986	3,155.16	505,150.0	0.625
1942	145.30	20,842.1	0.697	1987	7,485.70*	648,000.0	1.155*
1943	82.10	20,638.3	0.398				
1944	34.50	22,340.0	0.154				

*1,340 and 0.207 excluding Texaco.

FIGURE 2

DEFAULT RATES FOR STRAIGHT BONDS
1940–1987

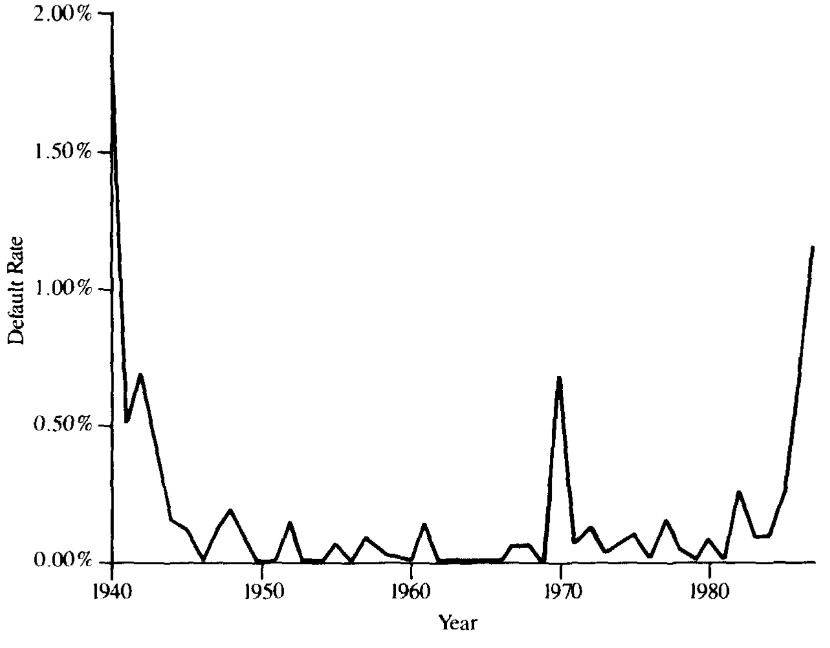


TABLE 2

DECENNIAL DEFAULT STATISTICS FOR ALL PUBLICLY TRADED BONDS

	Unweighted Default Statistics	
Period	Average Default Rate	Standard Deviation
1900–1909	0.890%	0.667%
1910–1919	2.012	1.241
1920–1929	0.952	0.403
1930–1939	3.200	1.843
1940–1949	0.425	0.550
1950–1959	0.041	0.048
1960–1969	0.028	0.044
1970–1979	0.132	0.191
1980–1987	0.321	0.363
1900–1944	1.650	1.472
1945–1987	0.119	0.213

TABLE 3

WEIGHTED DEFAULT STATISTICS FOR ALL PUBLICLY TRADED BONDS

Period	Weighted Default Statistics: Average Default Rate	Weighted Default Statistics: Standard Error
1900–1944	1.820%	0.272%
1945–1987	0.249	0.101

use a Chi-square test to establish how well the posited distribution fits the actual data. The Chi-square test works by comparing the actual number of data points within specified ranges with the number predicted by the assumed distribution. Table 4 shows the predicted and actual number of values within specified ranges for the years 1900–1944 and 1945–1985.

TABLE 4

BETA DISTRIBUTION RESULTS

Default Rate between		Expected Number of Years	Actual Number of Years
1900–1944			
0	0.464%	9	8
0.464%	0.948	9	10
0.948	1.58	9	13
1.58	2.61	9	5
2.61	100	9	10
1945–1985			
0	0.00525%	8.2	7
0.00525%	0.022	8.2	10
0.022	0.056	8.2	3
0.056	0.129	8.2	14
0.129	100	8.2	7

The value of Chi square for the earlier periods is 3.88 with two degrees of freedom. There is therefore about a 14 percent chance that the fitted distribution correctly represents the underlying data. Under usual statistical criteria this is not adequate to reject the proposed beta distribution as being inappropriate for the data. Examination of the discrepancies between the actual and expected values for this period indicates that the expected values are too low in the areas adjoining the center of the range. Because in risk studies we are concerned with the tails of the distribution, this distortion should not cause us to reject the distribution for risk studies.

When we examine the post-1945 period, the results are not so good. The value of Chi square is 8.15, and with two degrees of freedom there is only a 1.7 percent chance that the proposed distribution properly represents the underlying data. Examination of the details in Table 4 again gives us some comfort in the use of this distribution. The tails are approximately represented in the distribution. The actual distribution is flatter than the theoretical distribution. Because we would be using theoretical distributions to test the effects of the unlikely occurrence of very high default rates, the failure of the Chi square test, because of the way it failed, should not dissuade us.

3.4 Effects of the Economy

The evidence of the default rates themselves is that the periods before and after World War II constitute two different worlds of experience. No argument has been presented to the effect that the credit-worthiness of bonds has improved. If anything, the measures of financial strength in corporations have degenerated slightly over the last 40 years. The only plausible conclusion would seem to be that the sharp decline in default levels is the result of a more stable economy.

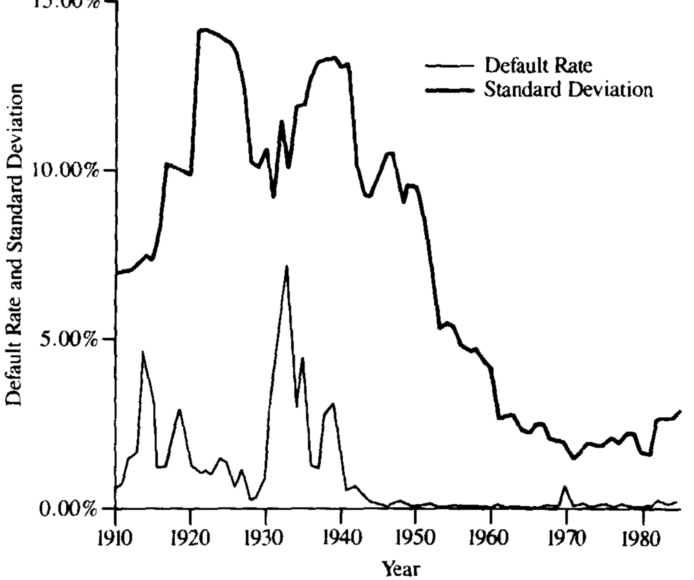
We can present several qualitative arguments on this subject. The first is simply that we have learned something about the control of the economy. We know enough not to kill world trade with a mad tariff bill; we know enough not to raise taxes during a depression; and we know enough not to reduce the money supply during a depression—actions that produced the collapse of the 1930s. One thing we do know is that the money supply should be increased during a depression, because that will cause inflation and thereby a transfer of wealth from the lender to the borrower. Bond defaults are not necessary to reduce the value of debt compared to the assets pledged for that debt.

The following graphs present some quantitative evidence of a more stable economy. Figure 3 shows the overall default rates previously presented, but also shows the standard deviation in the nominal growth rates in the economy for the previous ten years. The Gross National Product (GNP) figures have been supplied by the Department of Commerce. Although the two curves do not track very well, it is clear that after the war there was a considerable reduction in the instability of the economy.

A different view is contained in Figure 4. This shows default rates and the ten-year growth rates in the real economy. Note that the same phenomenon of stability in the economy is shown in the real sector as is shown for

FIGURE 3

DEFAULT RATES
STANDARD DEVIATIONS IN NOMINAL ECONOMIC GROWTH
1910–1987



the nominal GNP in Figure 3. In addition, we can also see that negative growth rates in the real economy corresponded to periods of higher default. This, of course, leads to the idea that there should be a strong correlation between defaults and happenings in the real and nominal economy.

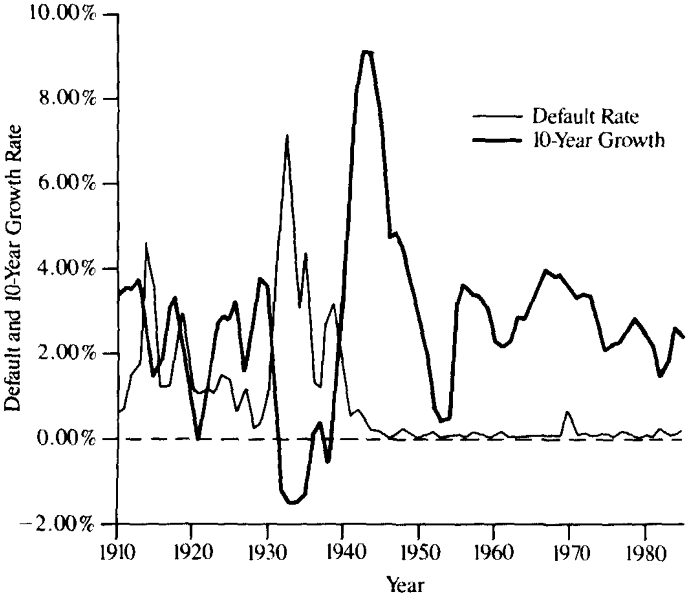
A final view of some causes of change in default rates is shown in Figure 5 and the statistical data below:

Variable		Coefficient	Standard Error	T-Stat
Variable 1 =	Nominal Annual GNP Growth Rate	−0.093257	0.018391	−5.0709
Variable 2 =	Real 3-yr. GNP Growth Rate	0.081354	0.033116	2.4566
Variable 3 =	Outstanding /Nominal GNP	0.065329	0.004162	15.6960
Variable 4 =	Nominal Growth Rate × 5-yr Standard Deviation	0.929599	0.170122	5.4643
Variable 5 =	3-yr Real Growth Rate × (Var 3)	−0.741979	0.138394	−5.3614
R^2 = 0.795	Adj.R^2 = 0.784	D.W. = 1.43		

The formula rates were developed by using regression analysis to represent the default rate in terms of growth rates in the real and nominal economy,

FIGURE 4

DEFAULT RATE
10-YEAR REAL GROWTH RATE
1910–1987



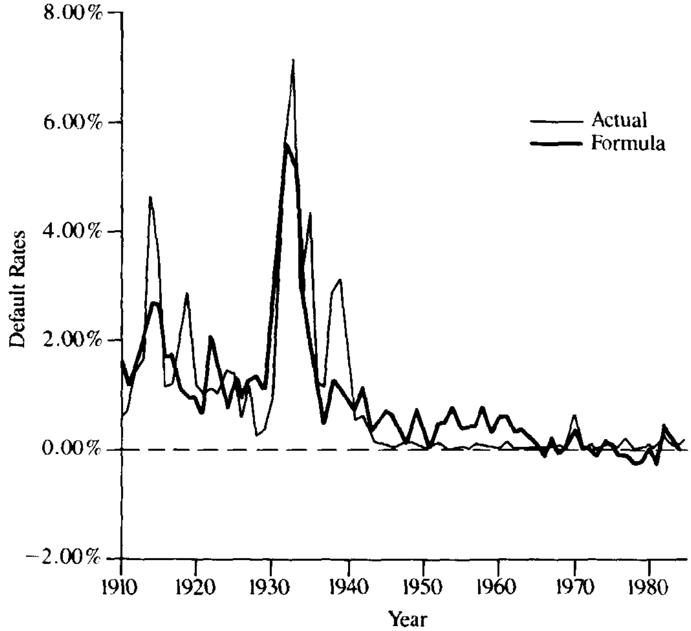
standard deviation in such growth rates, and the ratio of par values of bonds outstanding divided by the actual level of nominal GNP.

Although the Durbin-Watson statistic is low, and statistically significant, it is reasonably good for an economic series that would usually be assumed to have a high degree of autocorrelation. The R^2 statistics are high. The signs of the coefficients are plausible, considering the interplay between them and the fact that only variations of three independent series are used.

The point of this exercise is not to claim the development of a formula that predicts defaults. The purpose rather is to establish that the factors that could reasonably be expected to have influenced the level of default rates over the past 80 years have improved enough to explain the low default rates of the past 40 years. The regression results are obviously dominated by the high default rates of the 1930s. These high default rates can reasonably be expected not to recur unless we have a recurrence of (1) the high ratio of

FIGURE 5

DEFAULT RATES
ACTUAL VERSUS FORMULA
1910–1987



outstanding bonds to nominal GNP, (2) the higher values of the standard deviation of real and nominal GNP, and (3) the negative growth rates in both real and nominal GNP.

The great importance given in this regression to the effects of growth in the nominal GNP and the ratio of outstanding bonds to the nominal GNP deserves some comment. When there is accelerated growth in the nominal GNP, we are probably experiencing inflation. During an inflationary period real value is transferred from lenders to borrowers; bonds denominated in dollars should become easy to pay; and default should be easy to avoid. One thing our government has learned is how to stimulate the economy by creating inflation. The possibility of a recurrence of the conditions of the 1930s is not significant. This statement does not mean that new and different types of problems and governmental mistakes will not occur—only that we are unlikely to repeat the same mistakes with the same results.

Attempts to establish a relationship between default rates and the state of the economy, other than during the 1930s, were essentially unsuccessful. A more sophisticated approach may be successful. Verification of the credibility of such an approach probably would be difficult.

The Appendix shows the nominal and real GNPs for the period under study.

3.5 Incidence of Default

If an issue is destined to go into default, will the default take place early or late in its life? Our automatic assumption would be that default rates would increase with duration. Some in the finance field would argue that default will take place early or not at all. The argument there would be that a company should be expected to work its way out of its problems. If it can hold on for a few years, a bright new management can solve its troubles. Let us consider the experience. Table 5 is extracted from the Hickman study.

TABLE 5

DEFAULTS BY DURATION SINCE ISSUE

Category	Total Defaults	Less than 1.5 Years	1.5–2.5 Years	2.5–3.5 Years	3.5–4.5 Years	4.5–5.5 Years	5.5–6.5 Years	6.5–9.5 Years	9.5–14.5 Years	Over 14.5 Years
Large Issues	716	55	60	72	43	54	35	96	76	225
Small Issues	440	41	34	43	37	33	29	55	51	117
Total	1,156	96	94	115	80	97	64	151	127	342

There are not sufficient data to develop a mortality table for bonds. Hickman does give some information on the average life of a bond issue. The median period from offering to extinguishment for all bonds was about 14 years, or 60 percent of the period to original maturity. In another calculation average turnover rate was computed. For all bonds this rate was 5.8 percent per year, which implied an average life of 17 years. Because of the differences in the method of computation, this is consistent with the median figure. By using the average turnover, the relative portion of bonds outstanding for each duration implied a default rate for the first five years that was roughly twice that of later years.

The result is consistent with the idea that lower-grade bonds are the ones that default, but if they get through the bad period, the companies will prosper. It is also consistent with the fact that a high proportion of junk bonds were issued during the late 1920s, which were almost immediately exposed to the chilling economy of the 1930s.

The most recent studies by Altman [1] and Asquith [2] seem to show a strong pattern of defaults increasing with time since issue. Their work should be consulted for further information. The actuary should not be misled by early favorable default experience on a cohort of bonds. The default rates seem to increase, according to these studies, almost linearly with time. Therefore, early favorable experience on a growing fund of new issues is no evidence of superior management and provides no assurance of low default rates during a long holding period. The default rates in Table 6 must be considered in general, but certainly not total, agreement with Altman's work.

TABLE 6

MOODY'S DEFAULT RATES PER JULY 19, 1989 *Wall Street Journal*

Bond Rating	1 Year	5 Year	10 Year	15 Year
Aaa	0.00	0.2%	0.8%	2.1%
Aa	0.00	0.5	1.4	2.2
A	0.01%	0.5	1.4	2.7
Baa	0.16	1.6	3.7	5.9
Ba	1.56	8.3	14.2	18.9
B	6.69	22.3	29.3	32.9

Although use of the select approach to bond default is desirable for analysis of results, it is probably not necessary for setting surplus targets or risk charges for various classes of bonds. If a level of cumulative defaults is set for the life of a cohort of bonds, we can prudently assume that the rate is level over the period. It is far simpler for risk analysis to assume one beta distribution for each rating class rather than one for each year and rating class. There are simply not enough data to justify the more complex assumption. Finally, the use of an equivalent level rate is conservative, because defaults will be treated as occurring earlier than the referred-to studies imply.

Bond of a given rating class could have level default rates. For the pattern of increasing rates to appear, it would only be necessary for bonds to change rating classes. Because this does occur, an understanding of the phenomenon is important for understanding default experience. Here there are two different forces at work. The first is the simple random movements between ratings as one firm thrives and another firm falters. A 1983 publication by Prudential Bache written by Winslow Marston [15] records that among investment-grade issues during 1978–1982 there were 209 upgrades and 348 downgrades, while among lower-rated issues there were, during the same period, 142 upgrades and 121 downgrades. These changes in rating were mostly between the levels within investment grade or not investment grade.

The Hickman study shows a second force at work. Here a major change in the nature of the economy can cause wholesale lowering of ratings. Table 7 shows such carnage. Prior to 1928 the below-investment-grade bonds tended to be about 25 percent of the total by number and 15 percent by amount.

TABLE 7

ILLUSTRATION OF RATINGS CHANGES

Year	Investment Grade	Below Investment Grade	Unrated
	Number of Bonds Outstanding		
1928	4,137	1,464	649
1932	2,802	1,957	745
1936	1,929	2,040	585
1940	1,155	1,750	571
1944	801	962	916
	Bonds Outstanding, Amounts in Millions		
1928	$22,608	$ 3,016	$ 353
1932	20,353	8,140	521
1936	16,430	9,684	404
1940	14,291	10,537	533
1944	13,665	7,322	1,811

3.6 Defaults by Risk Class

Several risk classification systems have been used to rate bonds. The best known is the agency rating system. This classifies bonds into a series of rating classes of which only the top group is characterized as investment grade. The Standard & Poor's categorization is AAA, AA, A, and BBB as investment grade, and BB, B, CCC, etc., as below investment grade. D is reserved for defaulted bonds. The investment class for Moody's are Aaa, Aa, A, and Baa, with the other classes being below investment grade.

Market ratings are also used, and according to this system, if a bond offers a surprisingly high yield in the marketplace, it is because there is a higher risk to the offering. A third system has been used in which the various state legislatures set up lists of bonds that are deemed safe for investment by sophisticated financial institutions. The state lists have almost vanished, but the other two systems still exist, and a new one has developed. The National Association of Insurance Commissioners (NAIC) has created it own system for rating bonds, especially to provide for insurance company investments in bonds not rated by the services. The NAIC "No*" category corresponds to average below-investment-grade bonds, and "No**" corresponds to below-average below-investment-grade bonds. Each system has advantages for the bond buyer concerned about risk. The agency ratings have the best record for long-term predictions of default, while a high yield offered in the marketplace is a good indication of high risk in the near future. The NAIC ratings fill the obvious gap for which they were intended.

Harold Fraine [6] shows the following (Table 8) for all bonds issued during the period of the Hickman study. It is the total default rate for the entire lifetime of the bond. The corresponding Altman study information, also shown, is for ten years following issue. Agency rating is for original issue date. Also shown is an annual equivalent level default rate based on a multiple decrement table, Altman total rates, and assuming that exchanges are equal to one-fourth of the Altman default rates.

TABLE 8

VARIOUS RATINGS AT ISSUE AND THEIR DEFAULT EXPERIENCE

Agency Rating	Fraine Rate	Altman Rate	ELR
I	5.9%	0.13%	0.02%
II	6.0	2.46	0.3
III	13.4	0.93	0.1
IV	19.1	2.12	0.2
V	41.1	6.64	0.7
VI–IX	50.1	31.91	4.
Unrated	28.6		
All	17.3		

The Asquith [2] study shows that the 12-year default rate for junk issued in 1977 was 35 percent and the 11-year total rate for issues of 1978 was 34 percent. Because the data base used in [2] was slightly different than that of Altman, the results can be regarded as confirming Altman's.

The equivalent level rate (ELR) shown in Table 8 for the Altman class VI–IX would produce a 15-year total default rate of 40 percent. An ELR of

6 percent would produce a 15-year total default rate of around 50 percent. The Hickman study average life might be around 15 years.

Fraine also provides data (Table 9) showing the relationship between market rating at issue and default rate over the lifetime of the issues. Market rating is the excess of promised yield on an issue less the promised yield on the lowest-yielding issue then available.

TABLE 9

MARKET RATING AT ISSUE AND DEFAULT EXPERIENCE

Market Rating	Default Rate
Under 0.5%	10.5%
0.5–1.0	13.9
1.0–1.5	18.9
1.5–2.0	23.8
2.0–2.5	27.8
Over 2.5	39.1

Although both tables support the argument that there are real differences in quality of bonds that can be recognized by the market at the time of issue, Table 9 also contains some other information. If we contrast Tables 8 and 9, we note that a consistent strategy of buying bonds with the highest-yielding market ratings would produce lower defaults than buying low agency rated bonds. At least part of the explanation is in the reasonably favorable default experience of the unrated bonds. There are no bonds unrated from market rating.

The various studies of Altman provide the current experience on junk bond defaults. Because this work has been criticized by both the proponents and opponents of such investments, it might provide a basis for balanced judgment. Table 10 shows the basic data and a few statistics.

Beta distributions were fitted to these data, and Table 11 shows three such distributions. The first is based on the 1971–1986 data, the second on the 1970–1986 data, and the third on twice the 1970–1986 data. The high rate for 1970 was caused by the Penn Central default and might be considered an anomaly.

Table 11 deserves some explanation. Consider the first two columns, the first line. This line says that based on the experience for 1971–1986 there is a 15 percent chance that in any year the default ratio will be below 0.5 percent. The second line says that there is a 36 percent chance that the

TABLE 10

ALTMAN JUNK BOND EXPERIENCE

Year	Junk Out	Percentage of Total	Defaults ($ millions)	Default Rate
1970	$ 6,996.0	6.02%	$ 796.7	11.39%
1971	6,643.0	5.01	82.0	1.23
1972	7,106.0	4.88	193.3	2.72
1973	8,082.0	5.22	49.1	0.61
1974	11,101.0	6.65	122.8	1.11
1975	7,720.0	3.85	204.1	2.64
1976	8,015.0	3.66	29.5	0.37
1977	8,479.0	3.57	380.6	4.49
1978	9,401.0	3.73	118.9	1.26
1979	10,675.0	3.96	20.0	0.19
1980	15,126.0	5.71	224.1	1.48
1981	17,362.0	6.80	27.0	0.16
1982	18,536.0	6.49	577.3	3.11
1983	28,233.0	8.84	301.1	1.07
1984	41,700.0	11.64	344.2	0.83
1985	59,078.0	14.08	992.1	1.68
1986	92,985.0	18.41	3,155.8	3.39
1987	136,952.0	21.13	7,485.5	5.47

Junk Bond Default Rates

	1970–1987	1971–1987
Average Rate	2.40%	1.87%
Standard Deviation	2.62	1.49

default ratio will be below 1.0 percent, etc. The period covered is 16 years, and 15 percent of 16 years is 2.3. This is the expected number of occurrences of default rate below 0.5 percent shown in the third column. In the sample there were 3 years in which the default rate was below 0.5 percent, and this is the first line of the fourth column. The fifth, sixth, and seventh columns show the same information as columns two, three, and four, but using the experience including the year 1970. The last column illustrates the impact of doubling both the mean and standard deviation of the distribution. Because it is not based on any data, only the cumulative distribution function is given.

3.7 Rating Class—Pictures of History

While the previous sections have attempted to describe the experience by using numbers and statistical methodology, this section uses only graphs. There are two reasons for this: First, graphs provide a quick capsule of the experience, which may be more compelling than the dry data. Second, we

TABLE 11

PROPOSED CURVES TO PREDICT DEFAULT

Default Rate	1971–1986 Cumulative Distribution	1971–1986 Expected No.	1971–1986 Actual No.	1970–1986 Cumulative Distribution	1970–1986 Expected No.	1970–1986 Actual No.	Twice 1970–1986 Cumulative Distribution
up to 0.5%	15%	2.3	3	28%	4.8	3	19%
1.0	36	5.8	5	43	7.4	5	29
1.5	56	8.9	10	54	9.2	10	37
2.0	70	11.3	11	62	10.6	11	44
2.5	81	12.9	11	69	11.7	11	49
3.0	88	14.0	13	74	12.7	13	54
3.5	92	14.8	15	79	13.4	15	59
4.0	95	15.2	15	82	14.0	15	62
4.5	97	15.5	16	85	14.5	16	66
5.0	98	15.7	16	88	14.9	16	69
5.5	99	15.8	16	90	15.3	16	72
7.0	100	16.0	16	94	16.0	16	79
10.0				98	16.6	16	87
12.0				99	16.8	17	91
20.0							98
25.0							99

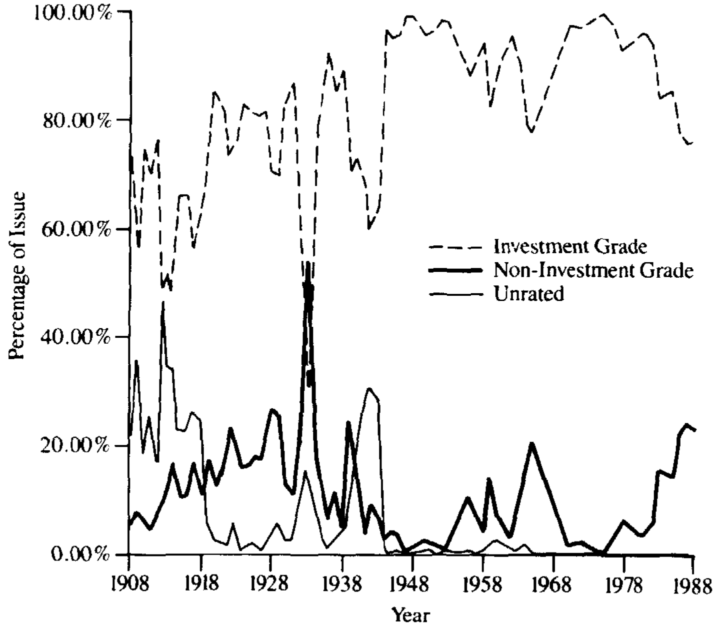
are emboldened to make some assumptions about unavailable data. Specifically, the percentage of junk bonds outstanding was interpolated from 1944 through 1970, and the percentage of new issues in the junk bond category was interpolated from 1966 through 1970. For the Hickman study years defaults were attributed to the junk class if that was their rating one year previous to default.

The rating system started in 1908. The peaks of non-investment-grade issues were in 1914, 1919, 1928, 1933, 1939, 1965, and 1987. The private placement market could have involved a large amount of non-investment-grade issues, and it would not be appropriately recognized in Figure 6. The peak in 1987 is consistent with a number of previous cyclical peaks. Not all of these ended in disaster. The peaks in 1933, 1939 and 1965 did not bring poor results for investors. The one in 1928 did.

The peaks of the 1930s were caused both by new issues in the non-investment-grade category and by the downgradings previously discussed. The current level is above the historic average of about 15 percent but is hardly startling.

FIGURE 6

ISSUED AMOUNTS BY QUALITY GRADE
1908–1988



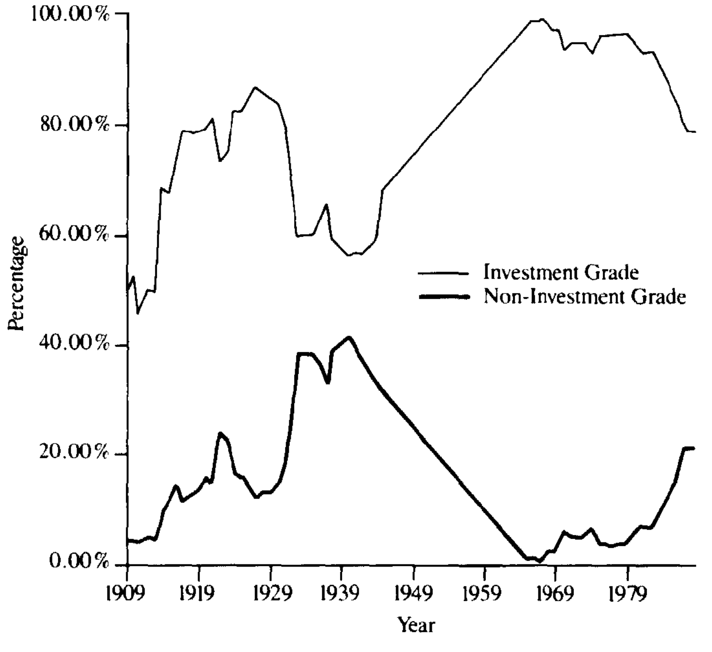
The default rates on junk bonds do not seem entirely inconsistent with those of the Hickman study period. The average for the entire period of the Hickman study was 3.5 percent, compared to about 2.5 percent in recent years. Although the rates for the period from 1945 to 1970 seem extremely low, this may be due to the lack of data on direct placements. Note that the effects of increasing default rates with duration since issue seem to have been overwhelmed by the effects of changes in the volatility of the economy.

Perhaps the most important information revealed by Figure 8 is that the default rate on investment-grade bonds has become substantially zero. Investment-grade bonds constitute 80 percent of the market. Even though the default rates on junk bonds are flirting with the rates of the Hickman period, the default rate for all bonds can maintain the very low levels discussed previously.

Total bond default rates are down from the Hickman period because investment-grade rates are down, but junk default rates are at about the same

FIGURE 7

PERCENTAGE OUTSTANDING BY QUALITY RATING
1909–1987



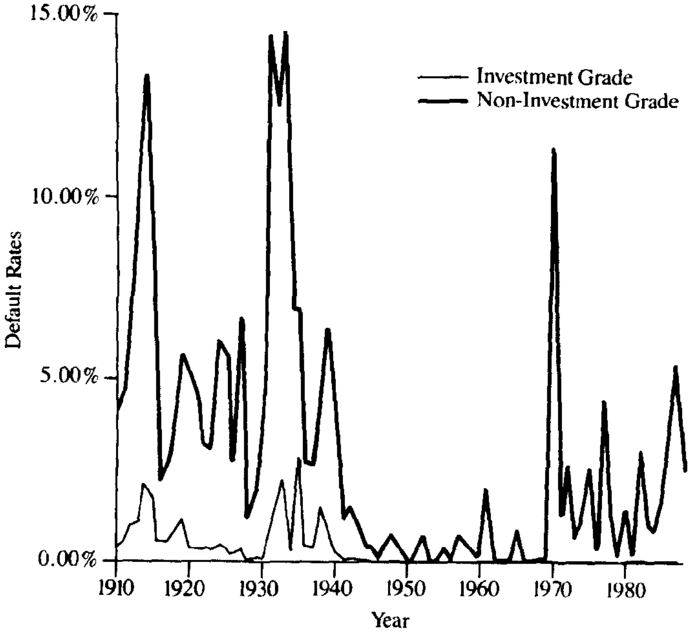
level. We can only speculate on the reasons for this pattern. The decrease in volatility in the real economy seems to be expressed through the investment-grade default rates rather than through the junk default rates. It may simply be that the agency ratings have improved in accuracy and in timeliness. If this were true, every bond would be reclassified into the junk category before it defaulted. Except for the occasional "Johns Manville," this seems to have been the case. There seems no other convenient explanation for the above-noted pattern.

4. EXPERIENCE PRIOR TO DEFAULT

While the losses at default are well known, it may not be so clearly recognized that much of the loss of a default actually comes before the formal event. The markets realize that the credit of the borrower has become suspect, and the price of the obligation declines. At about the same time, the

FIGURE 8

DEFAULT RATES BY QUALITY FOR OUTSTANDING 1940–1989



agencies also react to borrower problems by reducing the ratings on deteriorating credits. Table 12, from Vol. 3 of the Hickman study (Table 137), shows the rating progression of large issues that eventually defaulted. The same pattern was also evident in the prices and yields (Table 143); see Table 13.

A simpler version of this type of information is available from the Altman data base on bonds defaulting 1971–1986. The average price for these bonds on January 1 of the year of default was about 60, and the average price at the end of the month of default was about 36. Of the total loss on the date of default, from 100 to 36, almost two-thirds existed at the beginning of the year.

Variations can also occur in market price that are not directly related to default, but only to the risk of default. The markets will require higher returns if the perception is that risk of default has increased. In addition, the

TABLE 12

NUMBER OF LARGE ISSUES DEFAULTING AND RATING AT ISSUE

Time before Default	Agency Rating									
	I	II	III	IV	V	VI	VII	VIII	IX	No Rating
Five Years	24	48	78	106	75	57	15	4	1	308
Two Years	13	22	69	121	154	117	37	14	1	168
One Year	9	14	41	94	167	156	85	18	9	123

TABLE 13

NUMBER OF LARGE ISSUES DEFAULTING AND CURRENT YIELD

Time before Default	Current Yield, %									
	>4.5	4.5–4.9	5.0–5.4	5.5–5.9	6.0–6.9	7.0–7.9	8.0–9.9	10.0–11.9	12.0–14.9	<15
Five Years	38	85	61	54	88	50	32	17	9	10
Two Years	22	41	72	54	131	74	81	40	32	36
One Year	14	26	51	33	78	85	90	68	55	119

volume of bonds of various quality coming to market can affect the prices of outstanding obligations.

These factors can be compressed into the spread of promised yield on junk bonds versus Treasuries. When the spread changes, the relative prices of risky bonds change with respect to high-quality bonds. Figure 9 illustrates this effect.

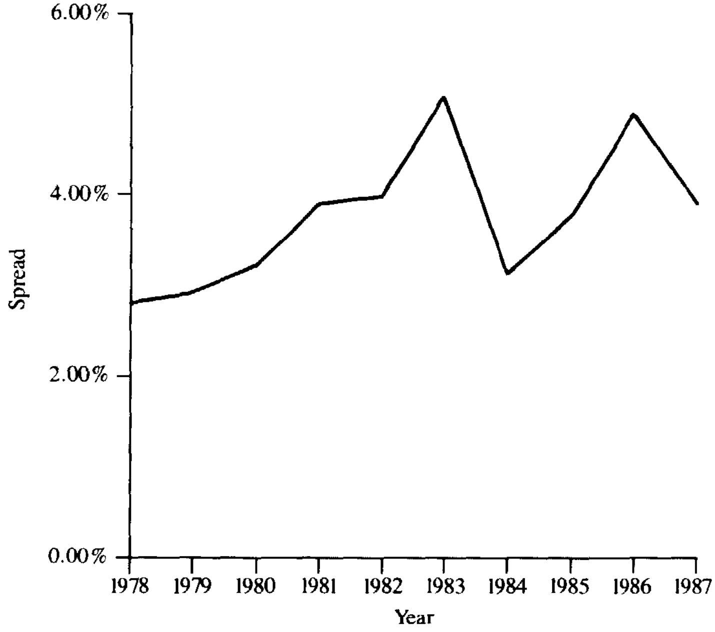
Year	Spread
1978	2.81%
1979	2.94
1980	3.23
1981	3.89
1982	3.98
1983	5.04
1984	3.10
1985	3.57
1986	4.90
1987	3.91

5. EXPERIENCE AFTER DEFAULT

The experience on bonds after default is as fascinating as any other taboo subject. Insurance companies are supposed to sell bonds before or *immediately* after they go into default. They are supposed to buy only good bonds

FIGURE 9

JUNK SPREADS OVER TREASURIES
1978–1987



that will not go into default. They certainly would never consider buying a bond already in default.

According to the Hickman study and considering only defaults of principal or interest, the average annual settlement rate, that is, the percentage of bonds in default that returned to good standing each year, was about 20 percent. This percentage holds for the entire period of the study and applies to industrials and public utilities as well as large and small issues. The only exception was railroads, which had slower settlements for all periods and sizes. The implied settlement periods were less than five years for all categories except railroads, which averaged 6.6 years. Industrials were relatively constant at a little over four years for both expansion and contraction phases of the economy. The other categories were about four years during expansion periods and about seven years during contraction periods. Railroads took ten years during contraction phases and almost six years during expansion phases. For all issues studied, the average period from default to return to good standing was 3.7 years, and the average recovery of value was 63 percent.

There are some other data available. A recent study of 57 defaulted direct placements of one large insurance company indicated that a default on this kind of investment is settled more rapidly. Defaults were settled within an average of 1.6 years, and the average recovery was more than 70 percent. Correlation and a least-squares fit between time for settlement and amount of settlement for these data are generally consistent with the Hickman data. The better recoveries on private placements seem to be the results of somewhat faster settlement.

Elliot Herskowitz prepared a unpublished study [12] for Professor Altman of the experience on debt of firms filing for bankruptcy during the period 1970–1982. The average settlement of these bankruptcies took place within three years. Available current experience then suggests that settlements now are taking place somewhat more rapidly than during the period of the Hickman study. However, the reduction in time does not seem very great. An assumption that 30 percent of defaults would be settled within six months, another 30 percent within the following two years, another 30 percent within the following two years, and the remainder within an additional two years would probably be a reasonably conservative way to visualize the settlement path speeded up by current conditions.

Perhaps the most interesting part of the Hickman study deals with the actual financial results of having invested during the period only in bonds that eventually went into default. Considering for all large issues only the bonds that eventually went into default, the promised yield was 6.4 percent, and the actual yield from issue to extinguishment was 2.3 percent. For only the issues of 1925–1931 was there a negative return, and that was only -0.1%. For the small issues sample the promised yield was 7.8 percent, and the realized yield for all issues was 2.4 percent. Only the issues of 1920–1924 showed a negative return, and that was only -0.3%.

We can look at these investments in terms of their real value. From 1925 to 1935 the cost of living (GNP deflator) went down 20 percent. From 1975 to 1985 the cost of living went up 88 percent. It seems plausible that an investment in the defaulting issues of the 1920s would have produced a better result, in real terms, than an investment in government bonds in the 1970s.

From the above results we must suspect that some rapid changes in value were taking place during the period from issue to default and default to extinguishment. The surprising thing is the strange stability in these interim results. The Hickman study indicates that at default bonds sell at about 40 percent of par value. The values were lower for defaults during the Depression and higher for other periods. In the Altman study the value used was 41 percent, based on more recent experience. For returns after default and based on the price at default, the Hickman study shows 20 percent compounded annually, as does the Herskowitz study and the results of the private placement study. Unfortunately, further examination of the Hickman report dispels the idea that investing in defaulted bonds is an automatic opportunity. Large-issue defaults prior to 1930 had only a 6.4 percent yield to extinguishment, while those of 1930–1943 had a 26.4 percent annual yield. The difference in price at default was the reason. During the later period the average price was 34, compared to 61 for the earlier period. As we should expect, returns after default are a function of the price and the value received. This is as a difficult a determination now as it was then.

The Herskowitz study makes clear one additional point. The favorable return results after default are the composite of widely differing results on individual debt securities. There are a few "home runs" that provide very large returns. Without those few very favorable results the remainder of the portfolio would be quite unsatisfactory.

6. DIVERSIFICATION

Diversification is always considered an important factor in investment portfolio management. Diversification should be on several different levels: by company, by industry, and by geographical region. Although there is no evidence on the importance of geography, the importance of company diversification seems clear. There is always a chance that any company can have its obligations go into default, exposing the holder to a loss of 60 percent or more of the investment.

It was possible to investigate the importance of industry diversification by using the Hickman data. The data are broken up into 32 minor subdivisions of the three major groups: rails, utilities, and industrials. Further, data showing the returns during 11 four-year periods are also detailed. From all these data we can draw the conclusion that diversification by industry is of some importance, but that diversification does not reduce risks to the same degree as it can in other contexts. The reasons are as follows:

1. Default rates in the major industries are correlated 40 percent with each other and 80 percent with the total market (correlation coefficient).

2. For all the returns over the entire period, there is twice as much variation on account of the period during which the investment was tracked as on account of the differences in minor industry.
3. Within major industry divisions, there is no striking difference in returns or standard deviation in returns for different groups.

The evidence shows that diversification by industry helps, but that it does not begin to make up for the losses that can be caused by being in low-grade bonds during a bad patch in the economy. This conclusion is at variance with the arguments of the investment bankers who are proponents of junk bonds. They generally argue that diversification can avoid the worst results of low-grade-bond investing.

7. RECOMMENDATION

We consider the extent of the risk of asset default in terms of the possible levels of loss on investment-grade bonds and junk bonds separately.

There have been almost no defaults or losses on bonds rated in investment grades. The level rate equivalent to the worst category in the Altman data presented in Section 3.5 was 0.3 percent per year. No year had a default rate as high as 1 percent. Experience in the years 1944–1970 had even lower defaults. If the average loss in that period was two-thirds of value, the loss of assets in the worst year would be about 0.6 percent of the assets. There is little justification for setting aside any surplus because of investment-grade bonds.

Junk bonds are far more interesting. Calculations with the Altman data base show that there is no correlation between the loss on default and the default rate in any of the last 16 years. Three calculations were prepared, shown as Tables 14, 15, and 16. Table 14 is the detailing of the beta distribution based on the experience on junk bonds for the years 1970–1987. Table 15 is the beta distribution of the prices on defaulted bonds at the end of the month of default for bonds that defaulted during the years 1970–1987. Table 16 is the beta distribution that has the same mean and standard deviation as one minus the product of the first distribution and one minus the second distribution. The third distribution then shows the probability of retaining specified portions of a beginning-of-year investment. Readers can make their own judgment of the level of risk in junk bonds.

TABLE 14

JUNK BOND DEFAULT RATES, FITTED TO A BETA DISTRIBUTION
BASED ON 1970–1986 EXPERIENCE

Cumulative Distribution	Default Rate	Cumulative Distribution	Default Rate	Cumulative Distribution	Default Rate
1%	0.000035	40%	0.00872	71%	0.0265
2%	0.000097	41%	0.0091	72%	0.0275
3%	0.000176	42%	0.00948	73%	0.0284
4%	0.000267	43%	0.00988	74%	0.0294
5%	0.00037	44%	0.0102	75%	0.0304
6%	0.000483	45%	0.0106	76%	0.0315
7%	0.000606	46%	0.0111	77%	0.0327
8%	0.000737	47%	0.0115	78%	0.0338
9%	0.000877	48%	0.012	79%	0.0351
10%	0.00102	49%	0.0124	80%	0.0364
11%	0.00117	50%	0.0129	81%	0.0378
12%	0.00134	51%	0.0134	82%	0.0392
13%	0.00151	52%	0.0139	83%	0.0408
14%	0.00168	53%	0.0144	84%	0.0424
15%	0.00187	54%	0.0149	85%	0.0441
16%	0.00206	55%	0.0154	86%	0.046
17%	0.00226	56%	0.016	87%	0.048
18%	0.00246	57%	0.0165	88%	0.0502
19%	0.00267	58%	0.0171	89%	0.0526
20%	0.00289	59%	0.0177	90%	0.0552
21%	0.00311	60%	0.0183	91%	0.0581
22%	0.00334	61%	0.019	92%	0.0613
23%	0.00358	62%	0.0196	93%	0.065
24%	0.00382	63%	0.0203	94%	0.0693
25%	0.00408	64%	0.021	95%	0.0743
26%	0.00433	65%	0.0217	96%	0.0804
27%	0.0046	66%	0.0224	97%	0.0883
28%	0.00487	67%	0.0232	98%	0.0994
29%	0.00515	68%	0.024	99%	0.118
30%	0.00544	69%	0.0248		
31%	0.00573	70%	0.0257		
32%	0.00603				
33%	0.00634				
34%	0.00666				
35%	0.00698				
36%	0.00731				
37%	0.00765				
38%	0.008				
39%	0.00836				

TABLE 15

SALVAGE ON JUNK BOND DEFAULT, FITTED TO A BETA DISTRIBUTION
BASED ON 1970–1986 EXPERIENCE

Cumulative Distribution	Default Price	Cumulative Distribution	Default Price	Cumulative Distribution	Default Price
1%	0.0819	40%	0.2695	71%	0.3726
2%	0.0991	41%	0.2726	72%	0.3765
3%	0.1112	42%	0.2757	73%	0.3806
4%	0.1209	43%	0.2789	74%	0.3847
5%	0.1292	44%	0.2819	75%	0.3889
6%	0.1365	45%	0.285	76%	0.3932
7%	0.1432	46%	0.2881	77%	0.3977
8%	0.1493	47%	0.2912	78%	0.4022
9%	0.155	48%	0.2943	79%	0.4069
10%	0.1603	49%	0.2975	80%	0.4117
11%	0.1654	50%	0.3006	81%	0.4167
12%	0.1703	51%	0.3037	82%	0.4218
13%	0.1749	52%	0.3069	83%	0.4272
14%	0.1793	53%	0.3101	84%	0.4327
15%	0.1836	54%	0.3133	85%	0.4385
16%	0.1878	55%	0.3165	86%	0.4446
17%	0.1919	56%	0.3197	87%	0.451
18%	0.1958	57%	0.323	88%	0.4577
19%	0.1997	58%	0.3263	89%	0.4649
20%	0.2034	59%	0.3296	90%	0.4725
21%	0.2071	60%	0.3329	91%	0.4807
22%	0.2108	61%	0.3363	92%	0.4897
23%	0.2143	62%	0.3397	93%	0.4995
24%	0.2178	63%	0.3432	94%	0.5104
25%	0.2213	64%	0.3467	85%	0.5229
26%	0.2247	65%	0.3502	96%	0.5375
27%	0.2281	66%	0.3538	97%	0.5553
28%	0.2314	67%	0.3574	98%	0.5787
29%	0.2347	68%	0.3611	99%	0.615
30%	0.238	69%	0.3649		0
31%	0.2412	70%	0.3687		
32%	0.2444				
33%	0.2476				
34%	0.2508				
35%	0.2539				
36%	0.2571				
37%	0.2602				
38%	0.2633				
39%	0.2664				
40%	0.2695				

TABLE 16

OVERALL RETAINED VALUE OF JUNK BONDS, FITTED TO A BETA DISTRIBUTION BASED ON 1970–1986 EXPERIENCE

Cumulative Distribution	Retained Value	Cumulative Distribution	Retained Value	Cumulative Distribution	Retained Value
1%	0.9194	40%	0.9873	71%	0.9963
2%	0.9323	41%	0.9878	72%	0.9965
3%	0.9399	42%	0.9882	73%	0.9967
4%	0.9453	43%	0.9886	74%	0.9969
5%	0.9495	44%	0.9889	75%	0.9971
6%	0.9529	45%	0.9893	76%	0.9972
7%	0.9558	46%	0.9897	77%	0.9974
8%	0.9583	47%	0.99	78%	0.9976
9%	0.9605	48%	0.9904	79%	0.9977
10%	0.9624	49%	0.9907	80%	0.9979
11%	0.9642	50%	0.991	81%	0.998
12%	0.9658	51%	0.9914	82%	0.9982
13%	0.9673	52%	0.9917	83%	0.9983
14%	0.9687	53%	0.992	84%	0.9985
15%	0.9699	54%	0.9923	85%	0.9986
16%	0.9711	55%	0.9925	86%	0.9987
17%	0.9722	56%	0.9928	87%	0.9989
18%	0.9733	57%	0.9931	88%	0.999
19%	0.9742	58%	0.9934	89%	0.9991
20%	0.9752	59%	0.9936	90%	0.9992
21%	0.976	60%	0.9939	91%	0.9993
22%	0.9769	61%	0.9941	92%	0.9994
23%	0.9777	62%	0.9944	93%	0.9995
24%	0.9784	63%	0.9946	94%	0.9996
25%	0.9792	64%	0.9949	95%	0.9997
26%	0.9799	65%	0.9951	96%	0.9997
27%	0.9805	66%	0.9953	97%	0.9998
28%	0.9812	67%	0.9955	98%	0.9999
29%	0.9818	68%	0.9957	99%	0.9999
30%	0.9824	69%	0.9959		
31%	0.983	70%	0.9961		
32%	0.9835				
33%	0.9841				
34%	0.9846				
35%	0.9851				
36%	0.9856				
37%	0.986				
38%	0.9865				
39%	0.9869				
40%	0.9873				

8. REAL ESTATE AND MORTGAGES

Real estate and mortgages constitute the other great class of life insurance company assets. Much less information is published on the experience with these types of assets, even though they constitute the bulk of the wealth of any nation. This point is not totally obvious. Most of the values in the balance sheets of corporations are in land and buildings. Further the bulk of the worth of average individuals is in their homes.

This note will not recommend specific surplus allocation for any of these types of assets if there is reasonable diversification. Geographical diversification is necessary for investment in home mortgages. Losses of 25 percent are easily possible on single investments in a specific local or regional market, but losses of this magnitude have not been experienced in the last 40 years in the national market.

An investment operation that specializes in local lending on homes is vulnerable to a downturn in that specific market. In favor of such a practice is the argument that the lender has greater familiarity with the market. This advantage is offset by a regular inability of the best local analysis to catch the dangers that can move the economy away from the local market. A choice by management of special commitment to the local area is not within the expertise of the actuarial profession. It is appropriate, however, for the actuary to point out the potential danger.

The values of real estate are sensitive to inflation, and the returns of real estate are therefore also sensitive. There are several simple reasons. One is that real estate and the costs of real estate are a large part of the Consumer Price Index (CPI), and therefore real estate rental and building expenses are part of the problem of inflation. In addition, as long as an old building is serviceable, its value must equilibrate with the cost of construction of a new building. This means that the value of an old building should be the depreciated replacement cost, so the real depreciation could be quite low.

There are academic-quality studies that support the theoretical positions. Ibbotson and Siegel [13] concluded that the returns on real estate correlate 85 percent with changes in the CPI. Meyer Melnikoff [16] reached about the same conclusion in his article.

The situation with respect to investment properties is somewhat more complex. Aside from buildings owned for home offices, the value of a property is not only a function of the property but also the leases negotiated for the rental of the property. If the property is not leased, there is no income and no value. However, leases, and particularly long-term leases, constitute a lien against the future income stream of a property. Ownership of a property with long fixed-rate leases, even net leases, is closer to holding a mortgage than equity.

A regional shopping mall is a convenient example of this phenomenon. The three or four anchor stores are usually leased to major retailers at fixed net rates for periods of 20 years or more. A change in the value of this property due to inflation does not quickly inure to the owner. This part of the property is not inflation-adaptive. On the other hand, the small boutiques have leases that allow annual changes in rates. This part of the package will produce income that is very sensitive to inflation and the general state of the economy.

The income stream from commercial real estate is a reflection of the lease structure of the properties. If the leases are short term, the income from the properties will adjust quickly to inflation. If there are longer fixed-rate leases, the adjustment will be slower. There can be no general rule about the income-producing ability of real estate except based on a detailed analysis of the leasing structure. Real estate professionals know this and instinctively try to get a reasonable balance. Studies have been made of the returns of the Prudential Prisa fund and the Equitable Prime Property fund, and the general conclusion is that a diversified fund will produce a total return of about 5 percent plus the inflation during the period of measurement. Because of the impact of leasing, this relationship works better over longer periods. A crucial part of this is that the major adjustment to inflation in the total return occurs through the increasing income on the properties and not just the profit on sale at the end of the period.

The implications of an analysis based on an assumption of an inflationary economy are interesting. If the value of real property goes up in step with inflation, then fixed-dollar loans made before a period of higher inflation will have no losses from defaults. However, fixed-rate loans made during a period of high inflation, in which anticipated high inflation has entered into the interest rate, are subject to total defaults if lower inflation reduces the value stream from the property assumed during the earlier period. This phenomenon was experienced by companies over the last few years.

To reiterate, the value of real property in dollars adjusts to the effects of inflation and, in fact, is part of the inflation. If low inflation has become embedded in the interest rates, then if inflation increases, the higher values of the underlying property protect the dollar value of the mortgage. A loss of real value to the lender has occurred. If a high rate of anticipated inflation is embedded in the interest rate and lower inflation actually develops, the underlying value of the property will not support the mortgage and a loss of value will occur in all such properties.

The loss of value referred to will generally not be in the form of a classic default. In the case of residential mortgages, there will be refinancing and repayment. In the case of commercial mortgages, there will be a renegotiation of terms and a reduction in rates. It is an error to assume that high-fixed-rate mortgages with no prepayment privilege will protect against loss of value if rates go down. If the underlying value adjusts to inflation, such mortgages must end up being renegotiated. Strangely, the best investment during a period of high anticipated inflation is government bonds, which cannot default and do not provide for call. During a period of low anticipated inflation, the best investment is short-term mortgages, variable rate loans, or equity. Long-term mortgages will always produce the lower of the rate at the time of issue and the current rate.

From the point of view of the risk of asset default the conclusions are as follows:

1. The economy is the same stable economy we have developed in more detail for bonds. Conventionally written mortgages should have the same low loss rates, say, 0.1 percent per year, as high-quality bonds.
2. Diversification, particularly in residential mortgages, can be a severe problem, but it is more of a management decision than one subject to actuarial expertise.
3. Real estate equity will produce returns that are related to inflation but subject to details of leasing and local legal restrictions.
4. Mortgages will produce the lower of the initial return and that of interest rates then current in the environment. During the transition from high to low inflation, the only prudent assumption is that all such mortgages are renegotiated over a period of three years. The renegotiation can be in the form of default or the form of prepayment and refinancing. This was the situation of the 1930s and it has recently recurred.

9. CONCLUDING REMARKS

Economies have always followed a pattern of expansion of credit followed by some sort of collapse that transfers wealth from lenders to debtors. In the 1930s there was a transfer from debtors to lenders of real wealth because of the decline in the cost of living. The necessary transfer from lenders to debtors was thereby increased and took the form of very high defaults. In the 1980s the transfer was accomplished by inflation. The argument developed above should be appropriate for an economy that can be expected to transfer wealth from lender to debtor by using inflation in the future.

Defaults could rise again. Massive structural dislocations are possible. Investors should regularly reconsider the returns and developing risks of all investments.

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APPENDIX

GNP DATA

Year	Nominal GNP	Real GNP	Year	Nominal GNP	Real GNP
1900	19.4	273.9	1944	211.4	1,380.8
1901	21.5	305.6	1945	213.4	1,354.8
1902	22.4	308.4	1946	212.4	1,096.9
1903	23.7	323.9	1947	235.2	1,066.7
1904	23.7	319.7	1948	261.6	1,108.7
1905	26.1	343.6	1949	260.4	1,109.0
1906	29.8	383.4	1950	288.3	1,203.7
1907	31.6	389.4	1951	333.4	1,328.2
1908	28.8	357.4	1952	351.6	1,380.0
1909	33.7	411.3	1953	371.6	1,435.3
1910	35.7	423.3	1954	372.5	1,416.2
1911	36.1	436.3	1955	405.9	1,494.9
1912	39.7	457.2	1956	428.2	1,525.6
1913	39.9	462.4	1957	451.0	1,551.1
1914	38.9	444.4	1958	456.8	1,539.2
1915	40.3	439.6	1959	495.8	1,629.1
1916	48.6	472.8	1960	515.3	1,665.3
1917	60.7	480.7	1961	533.8	1,708.7
1918	76.8	570.0	1962	574.6	1,799.4
1919	84.7	528.3	1963	606.9	1,873.3
1920	92.2	487.1	1964	649.8	1,973.3
1921	70.2	452.8	1965	705.1	2,087.6
1922	74.8	519.6	1966	772.0	2,208.3
1923	85.9	576.9	1967	816.4	2,271.4
1924	85.5	582.7	1968	892.7	2,365.6
1925	94.0	625.0	1969	963.9	2,423.3
1926	97.9	662.3	1970	1,015.5	2,416.2
1927	95.8	661.2	1871	1,102.7	2,484.8
1928	97.7	667.7	1972	1,212.8	2,608.5
1929	103.9	709.6	1973	1,359.3	2,744.1
1930	91.2	643.5	1874	1,472.8	2,729.3
1931	76.4	588.1	1975	1,598.4	2,695.0
1932	58.5	509.2	1876	1,782.8	2,826.7
1933	56.0	498.5	1977	1,990.5	2,958.6
1934	65.6	536.7	1978	2,249.7	3,115.2
1935	72.8	580.2	1979	2,508.2	3,192.4
1936	83.1	662.2	1980	2,732.0	3,187.1
1937	91.3	695.3	1981	3,052.6	3,248.8
1938	85.4	664.2	1982	3,166.0	3,166.0
1939	91.3	716.6	1983	3,401.6	3,277.7
1940	100.4	772.9	1984	3,774.7	3,492.0
1941	125.5	909.4	1985	3,992.5	3,573.5
1942	159.0	1,080.3			
1943	192.7	1,276.2			

DISCUSSION OF PRECEDING PAPER

DENNIS LAUZON:

The paper provides a valuable summary of default studies and some useful commentary on interpreting the data. In addition, the paper provides insights about the relation of defaults to economic conditions. For all this, I thank the authors.

However, I was perplexed by the report's recommendation that, "There is little justification for setting aside any surplus because of investment-grade bonds." The report did not provide a clear foundation supporting this recommendation. In fact, the report's estimated loss of 0.6 percent of assets in the worst year is a significant surplus requirement for many products and a significant allocation of capital for many companies. In addition, for most investment portfolios, the worst one-year loss would not represent adequate capital to support credit risk.

Surplus requirements for credit risk should consider the surplus consumed under various economic scenarios, the likelihood of those scenarios, and management's judgment about the balance to be struck between capital adequacy and capital efficiency. For example, by using the Hickman study and interpolation, the default losses during the Great Depression could be estimated as follows:

60% OF ESTIMATED DEFAULT RATES
BASED ON RATINGS 5 YEARS PRIOR TO DEFAULT
(ASSUMES 40% OF VALUE IS RETAINED ON DEFAULT)

	Class			
Year	I	II	III	IV
1930	0.0	0.0	0.0	0.2%
1931	0.0	0.0	0.1%	1.7
1932	0.0	0.4	1.4	2.4
1933	1.3%	1.6	8.8	2.7
1934	0.1	0.2	1.6	3.9
1935	2.3	2.3	1.7	3.7
1936	0.0	0.4	1.4	0.5
Present Value at 5%	2.8%	3.8%	12.1%	12.3%

A company holding a diversified mix of bonds at the start of 1930 would need 2.8 percent, 3.8 percent, 12.1 percent, and 12.3 percent to cover expected default losses on class I, II, III, and IV bonds, respectively, by the end of 1936.

To arrive at surplus requirements for a Great Depression scenario, these default losses should be adjusted for the following:

- Offsets for correlation with other risks
- Offsets for possible tax savings
- Additions for lack of diversification or reductions from assuming less railroad exposure
- Offsets for product margins
- Additions for guarantee fund assessments.

Great Depression scenario assumptions are probably too conservative. The report's assumptions that a fundamental economic change has taken place and that we will not let a 30 percent decline in real GNP happen again is probably correct. However, given the S&L mess, federal budget charades, and the pressure to leverage capital in a world with a great deal of capital needs, I am not as confident as the report that our economic knowledge will translate into economic wisdom.

Nevertheless, lower intensity scenarios, probably a 15 percent decline in real GNP, should be considered. The report's formula relating GNP and defaults could be used to produce default scenarios based on GNP assumptions. The following are more realistic and practical losses for investment-grade bonds during an adverse scenario:

Grade	AAA	AA	A	BBB
Losses	1%	2%	3.5%	5%

Again, to arrive at surplus requirements, these losses would be adjusted for risk correlation, tax offsets, and so on.

Capital requirements can be translated into needed spreads. For example, if Treasuries are earning 10 percent, income is taxed at 34 percent, and 15 percent is the target for after-tax return on capital, then s, the needed spread for a 1 percent capital requirement, is given by:

$$0.15 \times 0.01 = (1 - 0.34) \times \{[1.01 \times (0.1 + s)] - (1 \times 0.1)\}$$

or s would be about 13 basis points. With this additional spread, we are indifferent to investing liability funds in Treasuries or to putting liability funds plus 1 percent of capital in investments with a 1 percent capital requirement. In practice, the 13 basis would need to be augmented to cover any additional research expense, expected defaults, and the reduced liquidity provided relative to Treasury markets. In addition, the spread would have to account for any credit margins passed on in product pricing.

BENJAMIN W. WURZBURGER*:

The credit-related losses in any year can be written as the product $D \times L$, where D is the default rate and L is the loss rate per defaulted bond. (The loss rate L is thus 1 minus the retention rate on defaulted bonds.) Vanderhoof, Albert, Tenenbein, and Verni have concentrated on the analysis of the default rate D. Besides their analytical insights, they deserve special credit for compiling from various sources a time series on default rates, D, a time series that should serve as the standard data source for future researchers.

My note complements the paper by concentrating on the behavior of the loss rate, L. Both theory and empirical evidence suggest that L is positively correlated with D. This relation is important for the probability distribution of the product $D \times L$, credit-related losses, because it implies that a year of high defaults is also likely to display a high loss rate per default. While I do construct a new data series—a historical data series on loss rates on defaulted bonds—the foundation of this discussion can be found in the paper (page 573). The authors there observe that "the Hickman study indicates that at default bonds sell at about 40 percent of par value. The values were lower for defaults during the Depression and higher for other periods."

Section 1 briefly outlines the motivating theory, Sections 2–4 present and analyze the data, and Section 5 discusses the implications of the estimated relationship. Finally, Section 6 comments on the relevance of the junk bond evidence to loss rates.

1. Theory Suggests a Positive Correlation between L *and* D

Consider an insurer with some borderline-quality bonds and many average-quality bonds. In a reasonably good year, a few of the borderline bonds will be pushed just over the precipice into default, but because they were not pushed that far, the loss rates per defaulted bond should be small. In an adverse year, however, we can expect both a higher default rate and that some of the defaulting bonds will end up very much over the precipice, that is, a high loss rate, L. This argument could be formalized in terms of the Merton [4] model, which views defaults as arising from the equityholders'

*Dr. Wurzburger, not a member of the Society, is in the investment policy and research division at the John Hancock.

put option to put the firm to the bondholders and not repay the contractual debt.

2. Constructing Historical Data on Loss Rates, L

The data series on loss rates is unfortunately much less complete than the data on default rates. The authors do cite some work by Altman on the recent junk bond experience, but to try to get an L time series not restricted to junk bonds, I have gone back to the classic Hickman volume of statistical tables [2].

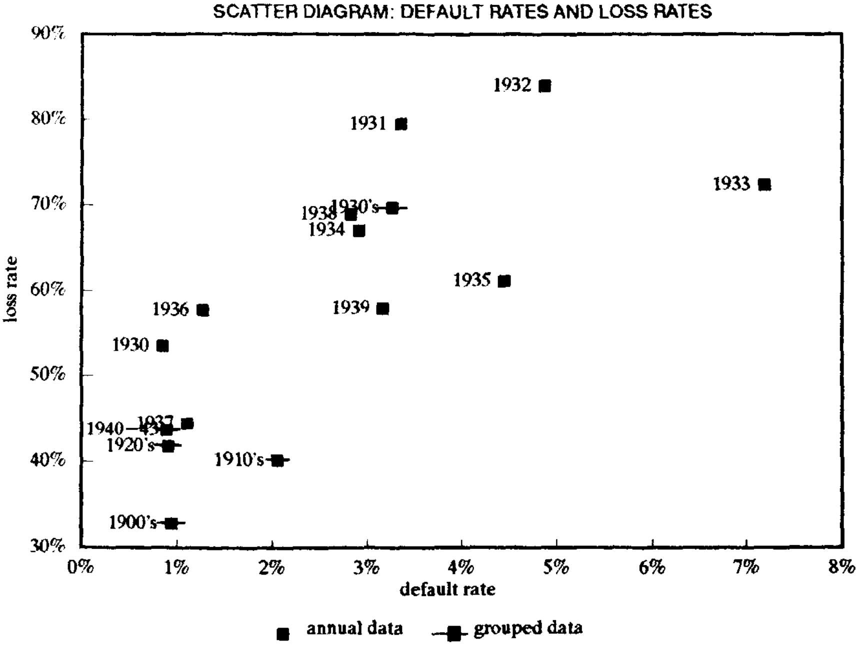
Hickman [2, Table 150, p. 268] presents a table classifying the defaulted issues by their market price at default. For example, Hickman reports that in the period 1900–1909, $0.0 of the defaulting issues (par value) had a market price at default of less than 10; $60.0 million of the defaulting issues had a market price of 10–19; and so on. By assuming that the issues in the 0–9 cell averaged 5 (that is, a recovery rate of 5 percent), in the 10–19 cell averaged 15, and so on, I can construct the average market price at default (the recovery rate) and hence the average loss rate on defaulted bonds. The Hickman data are restricted to "large issues" (issues with a total offering of more than $5 million.)

The Hickman cellular data on market price at default are on an annual basis for the years 1930–1939 and on a decennial (quadrennial) basis for the periods 1900–1909, 1910–1919, 1920–1929, and 1940–1943. (Henceforth, for simplicity, we refer to this as the "decennial data.") For comparison, we also list the annual D data for the years 1930–1939, as well as the corresponding decennial data on D. The decennial L data from Hickman are the sum of the dollar losses divided by the sum of the dollar defaults; the decennial D data are calculated as the sum of the dollar defaults (page 553, columns 2 and 3) divided by the sum of the decade's outstandings (column 3).

Year	Loss Rate, L	Default Rate, D
1900–1909	33%	0.95%
1910–1919	40	2.05
1920–1929	42	0.91
1930	54	0.85
1931	79	3.35
1932	84	4.86
1933	72	7.18
1934	67	2.91
1935	61	4.45
1936	58	1.28
1937	44	1.12
1938	69	2.83
1939	58	3.17
1930–1939	70	3.27
1940–1943	44	0.90

3. Tabular and Graphical Inspection of the Evidence

The data are graphed on the scatter diagram. A casual tabular or graphical inspection reveals that L is an increasing function of D. (The next section provides formal statistical evidence for the positive relation.) In particular, if we look at the annual data, we see that within the 1930s, the high D years (1932, 1933 and 1935) displayed higher L values than did the low (for the decade) years of 1930, 1936, and 1937. Similarly, if we look at the decennial data, we see that the high D decade (the 1930s) displayed a higher L value than the low D decades (the 1900s, the 1920s, and the 1940s). Thus, both the high-frequency intradecade data and the low-frequency interdecade data support the theoretical presumption that L is an increasing function of D.



A closer examination of the scatter plot also appears to reveal that the relationship is nonlinear, that L is a concave function of D. Preliminary regression analysis—adding a D^2 term to the explanatory variables—found a concavity term that was not quite significant at conventional levels (a t-statistic of 1.9), so we shall not include such a concavity term in our reported equations.

4. *Formal Statistical Evidence*

The following table reports three regressions to estimate the equation $L=c+hD$. The third regression, the grand one including both the intradecade and the interdecade regression, includes four decennial points, omitting the 1930–1939 summary statistic, which is already captured in the ten intradecade observations.

Sample	Constant	h	t-Statistic on h	R^2
10 Intradecade	0.51	4.1	2.5	0.44
5 Interdecade	0.27	11.4	2.8	0.72
10 Intra + 4 Inter	0.41	6.1	3.8	0.55

The t-statistics on h are all significant at conventional levels of significance. The t-statistic on the bottom-line equation is especially significant; a two-tailed test, 12 degrees of freedom $(10+4-2)$, 99 percent significance level, requires a t of only 3.1, and we found a t of 3.8.

Note that the intradecade regression shows a higher constant and a lower slope (h) than the interdecade regression. This is presumably reflective of concavity; the intradecade regression was run at high values of the independent variable, and the linear approximation (to a concave function) at high values of the explanatory variable will indicate a high constant and a lower slope. The grand estimate, the third equation, displays coefficients that are intermediate between the other regression results.

The third equation, $L=0.41+6.1D$, is the one we should presumably use for modeling the loss rates for various default scenarios.* It implies that in an environment of low default rates—say a D of 0.5 percent—one can anticipate a loss rate of $0.41+6.1\times 0.005=44$ percent, while in an environment of very high default rates—say a D of 5 percent—a loss rate of $0.41+6.1\times 0.05=71$ percent can be anticipated.

*The fact that our database includes grouped decennial data creates "aggregation bias," an index number problem; the classic work on this topic is Theil [5, especially Chapter 7]. Elimination of this bias generates the equation $L=0.41+5.9D$, a minimal modification of the estimated parameters. The estimate of the t-statistic on the D coefficient drops from 3.8 to 3.2, still significant at the 99 percent level. This last equation is unbiased; for an efficient estimator we ought to also give greater weights to the grouped data.

5. Implications of the Estimated Equation for the Dispersion of the Product, L × D

The coefficient of variation is a statistic that nicely summarizes the extent to which a probability distribution is spread out relative to its mean. (The coefficient of variation equals the standard deviation divided by the mean.) The authors' default data for 1900–1987 display a mean (unweighted) of 0.90 percent and a standard deviation of 1.31 percent, for a coefficient of variation of 1.46. So by neglecting our equation and assuming a constant value for L over the 1900–1987 sample, the variable $L \times D$ displayed an estimated coefficient of variation of 1.46.

As an exercise, we used the authors' D data and our estimated equation to simulate values of $L \times D$ over the same period 1900–1987. The simulated series displayed a coefficient of variation of 1.81. Recognition of the systematic relation between L and D significantly raises the estimate of the coefficient of variation of $L \times D$. A more complete analysis of the probability distribution of $L \times D$ would of course incorporate not only the systematic response of L to D but also the nonsystematic (residual or pure stochastic) variability in L.

6. Evidence about L *from the Junk Bond Market*

The authors do model (Table 15) the pure stochastic distribution of $1 - L$, the distribution of the retained value of *junk* bonds. Inasmuch as the authors' Table 15 data are apparently not based on an annual time series of junk bond L rates, it is not obvious that their procedure in Table 16, which combines the annual junk bond default rate distribution with their L rate distribution, is appropriate. In their final paragraph on page 574, the authors do refer to many new intriguing findings on junk bonds. Especially in view of recent developments in the junk bond market, this topic is too important to be relegated to a single short paragraph, and I hope the authors will take this opportunity to elaborate.

Finally, despite the importance of junk bonds, we should be wary about relating inferences from junk bond L experience to the aggregate bond universe. Hickman [3, p. 192] provides very strong evidence that the higher-quality bonds enjoy higher prices at default. While Altman [1, p. 916] does claim that there is virtually no correlation between the initial bond rating and the average price after default, his data show that the Aaa's and the Aa's do enjoy a *much* higher average price after default, about \$30 (per \$100) higher.

7. *Summary Remark*

Theory suggests and the historical data indicate that there is a very strong positive relation between the loss rate and the default rate. Probabilistic modeling that recognizes this relationship will display a much wider distribution for credit losses ($L \times D$) than models that treat L as an exogenous constant.

My thanks to David Allen for valuable suggestions and for writing the computer programs.

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(AUTHORS' REVIEW OF DISCUSSION)

IRWIN T. VANDERHOOF, FAYE ALBERT, AARON TENENBEIN, AND RALPH VERNI:

The authors thank Mr. Lauzon and Dr. Wurzburger for their comments on our work.

Dr. Wurzburger's comments are quite interesting and informative, and his hypothesis that the default rate is positively correlated with the amount of loss on default is seductive. It is true that if L and D are correlated, the dispersion of credit losses will be much larger than in the independence case. It is also true for the mean. If, for example, L and D are independent:

$$E(\text{CREDIT LOSS}) = E[L \times D] = E\{L\}E[D].$$

However, if L and D are positively correlated:

$$E(\text{CREDIT LOSS}) = E[L]E[D] + \rho\sigma_L\sigma_D$$

where ρ = correlation coefficient between L and D

σ_L = standard deviation of L

σ_D = standard deviation of D.

We considered the possibility of a positive correlation and tested for it, but were unable to determine evidence suggesting this relationship except during the decade of the 1930s. Indeed, Dr. Wurzburger's analysis in Section 2 of his comments is highly dependent on default experience during the Great Depression. If the 1930s decade is removed, the relationship disappears (of course there are only four points). These limited data seemed to us insufficient evidence to generalize, and so our paper does not address this area. It would be interesting to determine whether a positive correlation exists in the postwar years 1945 to the present.

The authors thank Mr. Lauzon for pointing out the confusing language in our recommendation. We hope the following will clarify our intent.

Investment-grade bonds had very few defaults, and the resulting losses were negligible. On the other hand, this alone did not convince us that *no* surplus ought to be set aside for investment-grade bonds. We did not mean to estimate losses of 0.6 percent of asset value on investment-grade bonds and then conclude this amount was small enough to ignore. A total of 0.6 percent of the original face of the issue has been adequate to provide for losses on all publicly traded bonds. So, if we can establish 0.6 percent as far in excess of anticipated default losses on investment-grade bonds in the future, then perhaps it is reasonable to set nothing specifically aside for investment-grade bonds. Also, the magnitude of losses on all publicly traded bonds seemed interesting to point out to the reader.

Mr. Lauzon's development relating capital requirements to required interest spreads on investments is helpful. We wholeheartedly agree with his assessment that surplus requirements depend heavily on judgments related to future economic scenarios.