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A GUIDE TO QUANTIFYING C-3 RISK

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INTRODUCTION

The C-3 risk is the risk of loss faced by a financial intermediary, such as a life insurance company, because of changes in either the level of interest rates or the shape of the yield curve.

The term C-3 was coined by the Trowbridge Committee on Valuation and Related Matters, which noted in 1979 that the risks faced by insurance companies could usefully be classified as C-1, the risk of asset failure; C-2, the risk of pricing insufficiency; and C-3.

The C-3 risk comprises the mismatch risk and the disintermediation risk. The mismatch risk, described by Redington [5], arises when there is a mismatch in timing between asset and liability cash flow streams, the consequences of which are either asset reinvestment or disinvestment transactions in an uncertain future interest rate environment. The disintermediation risk arises when options are available to either asset or liability clients that expose the intermediary to financial antiselection.

Actuarial literature of the last decade refers frequently to the C-3 risk, and much work has been done in the field of interest rate modeling. The purpose of this paper is to provide a guide to a practicing actuary in quantifying the C-3 risk for a company or a block of business. This paper is designed primarily for valuation actuaries who must decide on interest rates for valuation purposes and for corporate actuaries who must decide on adequate surplus levels.

OUTLINE OF A C-3 MODEL

Quantifying C-3 risk requires a model that can simulate the future cash flow for a block of business under different interest rate scenarios. Each simulation should proceed sequentially from interval to interval either until all liabilities have been discharged or until some earlier point in time when all assets have been exhausted. At each simulation date the assets and liabilities should be valued, the surplus position assessed, and a determination made of the initial additional asset required to avoid insolvency. A measure of the C-3 risk on a simulation is obtained by comparing the starting asset required to avoid a state of insolvency during the simulation to the initial imputed value of the liabilities. By performing the simulation under a number of different scenarios, the actuary obtains a probability distribution of the

C-3 risk and can make provision for it in either additional reserves or appropriated surplus at a desired confidence level.

The main ingredients of the C-3 model illustrated in this paper are:

1. A model for generating plausible future interest rate scenarios
2. The liability cash flow vector
3. The asset cash flow vector
4. Logic for imputing values to flows
5. The reinvestment strategy to be applied when asset cash flow exceeds liability cash flow, generating excess assets for investment
6. The disinvestment strategy to be applied when liability cash flow exceeds asset cash flow, causing a cash shortage for meeting current obligations
7. Call logic to simulate cash flow changes arising as clients exercise options prompted by a drop in interest rates (Examples are bond calls and mortgage prepayments.)
8. Put logic to simulate cash flow changes arising as clients exercise options prompted by a rise in interest rates (Examples are cash surrenders and new policy loans).

In each simulation the logic proceeds sequentially from one interval to the next. The spacing of the intervals might be monthly, quarterly, or annually. For many purposes yearly intervals will be appropriate, but if the cash flow is highly seasonal, a yearly model may give a poor estimate of the C-3 risk. For the model illustrations in this paper simulation dates are yearly. At each simulation date the model first determines whether and to what extent call or put options are to be exercised and adjusts the asset and liability cash flows accordingly. The model then applies asset cash flow occurring at the simulation date to settle liability payments falling due at the simulation date. If at the simulation date the asset cash flow exceeds the liability flow, the excess asset flow is reinvested according to the reinvestment strategy. If at the simulation date the asset cash flow is less than the liability flow, the shortfall is met by the sale of future asset cash flow according to the disinvestment strategy. The results in either event are a reduction to zero at the simulation date of both the asset and the liability cash flow and appropriate changes in future asset cash flow.

Appendix 1 shows the asset and liability cash flow vectors and the initial valuation factors. The flows were arbitrarily created, and the initial valuation factors are derived from the initial yield curve. The Macaulay duration and convexity factors are shown as well; these indicate a significant degree of mismatch between the asset and liability flows in the illustration.

Although one hundred simulations were performed in the analysis, the C-3 requirements of only ten of them are listed in the illustration. For these ten simulations the C-3 requirements vary from $320 to $49,520. The mean C-3 requirement for the 100 simulations is $17,187, or 2.7 percent of liabilities.

The C-3 requirements at different confidence levels, determined from a ranking of the results of all the simulations, are shown as well. For example, the C-3 requirement at the 50 percent confidence level is $13,375 and at the 99 percent confidence level is $60,335.

Shown as well are the valuation rates of interest if the reserves are intended to incorporate an amount to cover the C-3 risk at a particular confidence level. These were obtained by solving for the internal rate of return with the present value of the liability flow set equal to the initial present value increased by the amount required to cover the C-3 risk at the given confidence level.

Appendixes 2 and 3 show calculation details for two of the simulations. Simulation 10, which appears in Appendix 2, illustrates events on a decreasing track. Simulation 90, which appears in Appendix 3, illustrates events on an increasing track.

INTEREST GENERATION MODEL

The interest generation model designed for this paper (referred to as IGM) is a stochastic model designed to generate a spectrum of plausible future inflation rate tracks, long-term (ten-year) interest rate tracks, and short-term (three-month) interest rate tracks. After the tracks were generated, they were ranked according to the sum of the long-term rates and short-term rates that were simulated. The result is that low-numbered tracks are associated with low-interest-rate simulations and high-numbered tracks are associated with high-interest-rate simulations. See Appendix 4 for statistical information on 100 simulated interest rate tracks. Appendixes 2 and 3 show full details on tracks 10 and 90. See Appendix 5 for mathematical details.

Inflation Rates

IGM assumes that interest rates are influenced by inflation rates. Inflation depends on global and national economic forces and, as recent history shows, can range over a wide spectrum. The model requires four parameters to simulate inflation: the starting rate, the maximum and minimum plausible rates of inflation, and the inflation drift component. In each simulation the model takes the inflation rate for a random walk with steps that are normally distributed with a mean of zero and a standard deviation equal to the drift component, subject to the condition that the walk cannot go outside the minimum and maximum barriers.

Long-Term Rates

IGM requires four parameters to simulate long-term interest rates: the starting rate, the long-term goal, the long-term trend component, and the long-term drift component. In each simulation the model takes the long-term interest rate for a random walk in which each step has two components. One component is stochastic and adds to the logarithm of the long-term rate a normally distributed random number with mean zero and standard deviation equal to the long-term drift component. The second component is deterministic and moves the long-term interest rate towards a moving target equal to the sum of the inflation rate and the long-term goal, the gap being closed by a fraction equal to the long-term trend component. The long-term rate might be likened to a drunk chasing a moving inflation rate.

Short-Term Rates

IGM requires five parameters to simulate the ratio of short-term rates to long-term rates: the starting ratio, the ratio goal, the short-term trend component, the short-term drift component, and the maximum plausible ratio. In each simulation the model takes the ratio on a random walk in which each step has two components. One component is stochastic and adds to the logarithm of the ratio a normally distributed random number with mean zero and standard deviation equal to the short-term drift component. The second component is deterministic and moves the ratio towards the ratio goal, the gap being closed by a fraction equal to the short-term trend component. The process is modified to prevent the ratio from exceeding the maximum plausible ratio. The short-term rates are readily computed given the long-term rate and the ratio of the short-term rate to the long-term rate.

Comment

Adding a random amount to the logarithm of a variable is equivalent to multiplying the variable by a positive random amount. Under such a process, the random variable never becomes negative, a desirable attribute when simulating interest rates.

Yield Curves

By simulating the long-term rate (ten years) and the short-term rate (three months), one gets two points on the yield curve. Other points are required for the C-3 model. While a linear interpolation might be used, I found that a cubic curve gives a better representation of past yield curves, with parameters of the cubic curve determined by the spread between the three-month rate and the ten-year rate.

Assignment of Parameters

To generate a set of plausible future-interest-rate scenarios, the parameters should be assigned in a credible manner. For the illustrations in this paper, a number of principles were followed in assigning parameters.

For past-interest-rate data, the *Report on Canadian Economic Statistics* [1] was used.

The starting values of the inflation rate, the long-term rate, and the short-term rate are those for 1987. The maximum inflation rate of 15 percent reflects a belief that the government can prevent and will not tolerate a higher rate. The long-term goal of 4 percent reflects the belief that the real rate of interest adjusted for inflation is relatively stable and in the 3–4 percent range. The minimum inflation rate of minus 4 percent recognizes the occurrence of negative inflation rates in the past and sets a floor for the long-term rate of 0 percent. The short-term ratio goal was set at 0.726 to reflect the average historical ratio. The short-term ratio maximum was set at 1.3, because higher ratios appear implausible.

The trend and drift factors were derived by using a trial-and-error process on past data. The drift component for each past year can be imputed if the trend and goals are stipulated. The experiment was repeated for different stipulations until a drift component with zero mean was obtained.

Comments on Interest Models

Much of the recent literature on interest rate modeling deals with binomial and diffusion models. These are more sophisticated than the type of interest rate simulation described above and are geared more to the needs of an investment actuary responsible for asset-liability matching. A good survey of interest rate models has been prepared by Sharp [6].

THE LIABILITY CASH FLOW VECTOR

The liability cash flow vector is the net expected cash outflow by interval for benefits, expenses, and premiums. The elements of the vector may be positive or negative, but the general thrust will be positive. Negative liability flow is equivalent to positive asset flow. The liability cash flow must be determined by the actuary from an analysis of the block of business with appropriate assumptions for mortality, lapses, expenses, and so on.

THE ASSET CASH FLOW VECTOR

The asset cash flow vector is the net expected cash inflow by interval for the assets supporting the block of business. The asset cash flow must be determined by an analysis of the underlying assets. Each bond and mortgage in the portfolio can be depicted as a cash flow stream ending at its maturity. For equity-type assets the cash flow stream may reflect both expected income while the asset is held and expected proceeds on future sale.

LOGIC FOR IMPUTING VALUES TO CASH FLOWS

At any time the marketplace defines a market value for risk-free investments maturing at different future dates. From this information the yield curve then prevailing can be established. This principle has been well enunciated by Milgrom [3].

From the yield curve, valuation factors can be derived for determining the present value of future payments. The imputed value of the assets is derived by applying the valuation factors to the asset flow, and the imputed value of the liabilities is derived by applying the same valuation factors to the liability flow.

For the illustrations in this paper the initial yield curve applicable at the beginning of each simulation ranges from an effective yearly rate of 10.31 percent for a three-month horizon to 10.52 percent for a ten-year horizon.

The valuation factors derived from the initial yield curve are shown in Appendix 1. As the yield curve changes, the valuation factors change accordingly. For the detailed calculations shown in Appendixes 2 and 3, tables of the valuation factors at each simulation date are shown.

An advantage of using imputed values for the C-3 analysis is consistency in the valuation of both asset and liability cash flows. The imputed value of the assets is likely to be higher than the market values because the marketplace recognizes the value of options available to clients and any inherent C-1 risk.

If a block of business is in a surplus position, the imputed value of the assets should exceed the imputed value of the liabilities; the reverse is true of a block of business in a deficit position. In the illustrations in this paper, the imputed values of the assets and liabilities are equal to $643,394 at the start of the simulation, implying a block of business initially in a neutral position.

REINVESTMENT STRATEGY

At simulation dates when asset cash flow exceeds liability cash flow, the net amount must be reinvested. The model should reflect the company's reinvestment strategy. In the illustrations excess asset cash flow is invested in pure discount bonds maturing at the earliest dates when anticipated cash flow is negative. The price of such bonds is determined from the yield curve simulated to be in effect on the date of the excess cash flow.

DISINVESTMENT STRATEGY

At simulation dates when asset cash flow is less than liability cash flow, there is a need to sell assets to meet current cash requirements. In the illustrations the assets to be sold are selected from those maturing at the earliest dates when excess cash flow is anticipated. The proceeds for such bonds are determined from the yield curve simulated to be in effect on the date of the cash shortage.

CALL LOGIC

When interest rates fall, borrowers have a financial incentive to prepay their loans and to refinance them at the more favorable rates then prevailing. In doing so, they purchase for its call value an asset with an enhanced current value. In the illustrations the fraction of an asset that is called depends on the excess of its current imputed value over its call value and on two parameters: the call threshold parameter and the call intensity parameter. The call threshold parameter is the minimum differential that must exist before call activity commences, and the call intensity parameter recognizes the magnitude of reponse to a particular differential. The mathematical expression for the call fraction is given in Appendix 5. Setting the call parameters is likely to be subjective and should be done with investment department expertise, taking into account the availability of the call options and the predisposition of the asset clients to use them.

While call options tend to be associated with asset clients, there are situations in which they become available to liability clients. An example occurs when liability clients have the right to make additional voluntary deposits at a guaranteed interest rate.

In the illustrations the call fraction is computed independently for the assets maturing at each future date. Call values were computed as 105 percent (the call penalty parameter) of the discounted value of the liability cash flow at 10 percent (the call interest parameter). The call values for reinvested assets are computed with the same call penalty parameter, but the call interest parameter is replaced by the interest rates current at the time of reinvestment. The parameters to be assigned by the actuary should be appropriate for the block of business being analyzed.

PUT LOGIC

When interest rates rise, policyholders have a financial incentive to surrender policies or to borrow on their security. In doing so they sell at a favorable price future cash flow that has diminished in value. In the illustrations the fraction of the liability flow that is surrendered depends on the excess of the cash surrender value over its current imputed value and on two parameters: the put threshold parameter and the put intensity parameter. The put threshold parameter is the minimum differential that must exist before put activity commences, and the put intensity parameter recognizes the magnitude of reponse to a particular differential. The mathematical expression for the put fraction is given in Appendix 5. Setting the put parameters is likely to be subjective and should take into account the predisposition of policyholders to respond to interest rate changes.

For the illustrations cash values were computed as 95 percent (the put penalty parameter) of the discounted value of the liability cash flow stream at 6 percent (the put interest parameter). The parameters should be appropriate to the block of business being analyzed.

REFERENCE TO OPTION PRICING THEORY

In referring to calls and puts, the above sections employ some of the language of option pricing theory, but the theory itself is not used. The presence of inertia in the exercise of prepayment, loan, and surrender options makes the application of option pricing theory in an insurance setting difficult for the problem addressed in this paper. Clancy has provided a good description of option pricing theory and its applications [2].

CONCLUSION

The model described in this paper is designed to help an actuary analyze and quantify C-3 risk. A critical ingredient in the process is an interest generation model that can generate a spectrum of plausible future yield curves changing through time in both level and shape. Some sensitivity analysis can be performed by using a set of postulated interest rate tracks, but such an approach has limitations if answers are desired at specified confidence levels.

REFERENCES

1. CANADIAN INSTITUTE OF ACTUARIES. *Report on Canadian Economic Statistics*. Ottawa, Ont.: 1988.
2. CLANCY, R.P. "Options on Bonds and Applications to Product Pricing," *TSA* XXXVII (1985): 97–152.
3. MILGROM, P.R. "Measuring the Interest Rate Risk," *TSA* XXXVII (1985): 241–302.
4. NAYLOR, T.H., BALINTFY, J.L., BURDICK, D.S., AND CHU, K. *Computer Simulation Techniques*. New York: John Wiley & Sons, 1968.
5. REDINGTON, F.M. "Review of the Principles of Life-Office Valuations," *Journal of the Institute of Actuaries* LXXVIII (1952): 286–340.
6. SHARP, K.P. "Stochastic Models of Interest Rates," *Transactions of the 23rd International Congress of Actuaries, Helsinki, Finland, 11-16 July 1988*, 5 (1988): 247–62.

APPENDIX 1

ILLUSTRATION OF THE C-3 MODEL

TABLE 1A

CALL AND PUT PARAMETERS

Call Threshold Factor	1.05
Call Intensity Factor	10.00
Call Penalty Factor	1.05
Call Interest Factor	10.00
Put Threshold Factor	1.05
Put Intensity Factor	1.00
Put Penalty Factor	0.95
Put Interest Factor	6.00

TABLE 1B

Cash Flow Picture at Time 0

Time	Cash Flows			Valuation Factor
	Asset	Liability	Net	
1	14,681	234,883	−220,202	0.91534
2	222,279	22,727	199,552	0.83858
3	119,303	83,855	35,448	0.76825
4	27,516	142,699	−115,183	0.70332
5	48,081	14,051	34,030	0.64312
6	82,423	30,805	51,618	0.58720
7	148,956	79,730	69,226	0.53532
8	64,739	40,295	24,444	0.48733
9	64,193	340,575	−276,382	0.44316
10	281,780	10,380	271,400	0.40278
PV	643,394	643,394	0	

Note: The valuation factors are derived from the initial interest rate track.

Duration and convexity measures

	Asset	Liability
Duration	5.26	4.44
Convexity	37.29	30.07

TABLE 1C

C-3 Requirements on Some of the Tracks

Track	Requirement
10	13,375
20	16,593
30	3,483
40	320
50	1,200
60	13,832
70	25,402
80	13,334
90	49,520
100	34,462
Mean of 100	17,187

TABLE 1D

C-3 Requirements and Valuation Interest Rates

Confidence	C-3 Need	Valuation Interest Rate	Reserve
0%	$ 0	9.362%	$643,394
50	13,375	8.860	656,769
70	22,238	8.538	665,632
90	37,883	7.986	681,277
99	60,335	7.231	703,729

APPENDIX 2

CALCULATION DETAILS UNDER TRACK 10

TABLE 2A

INTEREST TRACK DETAILS

Time	Long Rate	Short Rate
0	9.520	9.310
1	8.946	8.481
2	8.330	7.717
3	8.756	8.309
4	6.936	4.911
5	6.924	4.429
6	6.415	4.820
7	5.112	4.548
8	4.918	5.001
9	4.743	6.166
10	4.937	4.255

TABLE 2B

INTEREST DISCOUNT FACTORS

Time	1	2	3	4	5	6	7	8	9	10
0	0.915	0.839	0.768	0.703	0.643	0.587	0.535	0.487	0.443	0.403
1		0.922	0.849	0.782	0.719	0.661	0.607	0.556	0.509	0.465
2			0.928	0.860	0.796	0.736	0.681	0.628	0.579	0.533
3				0.923	0.852	0.786	0.724	0.667	0.613	0.563
4					0.949	0.893	0.836	0.781	0.731	0.684
5						0.952	0.896	0.839	0.784	0.733
6							0.951	0.898	0.846	0.795
7								0.956	0.913	0.871
8									0.954	0.912
9										0.947

Values show present value at row time of one payable at column time.

TABLE 2C

EFFECTIVE INTEREST RATES

Time	1	2	3	4	5	6	7	8	9	10
0	9.25	9.20	9.19	9.20	9.23	9.28	9.34	9.40	9.46	9.52
1		8.49	8.52	8.55	8.59	8.63	8.68	8.74	8.80	8.87
2			7.77	7.84	7.90	7.95	8.00	8.05	8.11	8.17
3				8.32	8.34	8.36	8.40	8.44	8.49	8.55
4					5.38	5.85	6.17	6.36	6.47	6.53
5						5.04	5.64	6.04	6.28	6.42
6							5.17	5.51	5.75	5.90
7								4.59	4.64	4.69
8									4.85	4.72
9										5.63

Values show effective interest rate between row time and column time.

TABLE 2D

TIME 1

There is no call activity at time 1
There is no put activity at time 1

Picture at Time 1

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
1	−220,202	1.000000	0	0	0	0
2	199,552	0.921702	0	22,727	22,727	21,694
3	35,448	0.849152	0	83,855	83,855	72,767
4	−115,183	0.781819	−115,183	27,516	142,699	21,707
5	34,030	0.719242	25,445	39,496	14,051	28,325
6	51,618	0.661020	51,618	82,423	30,805	53,737
7	69,226	0.606807	69,226	148,956	79,730	88,286
8	24,444	0.556299	24,444	64,739	40,295	34,882
9	−276,382	0.509230	−276,382	64,193	340,575	31,444
10	271,400	0.465367	271,400	281,780	10,380	125,477
PV	3,533		3,533	486,778	483,245	

Deficit at time 1:	−3,533
Discount factor to time zero:	0.91534
Indicated C-3 requirement:	−3,234
Sell assets maturing at time:	2 3 5

TABLE 2E

TIME 2

Call Activity at Time 2

Year	Old Asset	Call Factor	PV-New	Call Percentage	New Asset	New Flow
2	22,727	1.050000	1.000000	0.00	78,867	56,140
3	83,855	0.954545	0.927863	0.00	83,855	0
4	27,516	0.867769	0.859873	0.00	27,516	−115,183
5	39,496	0.788881	0.796074	0.00	39,496	25,445
6	82,423	0.717164	0.736364	0.00	82,423	51,618
7	148,956	0.651967	0.680540	0.00	148,956	69,226
8	64,739	0.592698	0.628330	9.62%	58,508	18,213
9	64,193	0.538816	0.579423	22.40*	49,812	−290,763
10	281,780	0.489833	0.533493	32.38	190,527	180,147
PV	545,899				541,108	1,550

*Note: $0.2240 = 1 - \exp -10 \left[\frac{0.579423}{0.538816} - 1.05\right]$

There is no put activity at time 2

Picture at Time 2

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
2	56,140	1.000000	0	0	0	0
3	0	0.927863	0	83,855	83,855	80,043
4	−115,183	0.859873	−49,894	92,805	142,699	82,825
5	25,445	0.796074	25,445	39,496	14,051	31,158
6	51,618	0.736364	51,618	82,423	30,805	59,111
7	69,226	0.680540	69,226	148,956	79,730	97,114
8	18,213	0.628330	18,213	58,508	40,295	34,678
9	−290,763	0.579423	−290,763	49,812	340,575	26,840
10	180,147	0.533493	180,147	190,527	10,380	93,326
PV	1,550		1,550	518,381	516,831	

Deficit at time 2:	−1,550
Discount factor to time zero:	0.83858
Indicated C-3 requirement:	−1,300
Reinvest assets now maturing to time:	4

TABLE 2F

TIME 3

There is no call activity at time 3
There is no put activity at time 3

Picture at Time 3

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
3	0	1.000000	0	0	0	0
4	−49,894	0.923209	−49,894	92,805	142,699	89,795
5	25,445	0.852014	25,445	39,496	14,051	34,273
6	51,618	0.785847	51,618	82,423	30,805	65,022
7	69,226	0.724236	69,226	148,956	79,730	106,826
8	18,213	0.666785	18,213	58,508	40,295	38,145
9	−290,763	0.613165	−290,763	49,812	340,575	29,523
10	180,147	0.563095	180,147	190,527	10,380	102,659
PV	1,615		1,615	468,821	467,206	

Deficit at time 3: −1,615
Discount factor to time zero: 0.76825
Indicated C-3 requirement: −1,241

TABLE 2G

TIME 4

Call Activity at Time 4

Year	Old Asset	Call Factor	PV-New	Call Percentage	New Asset	New Flow
4	92,805	1.050000	1.000000	0.00	206,556	63,857
5	39,496	0.954545	0.948927	0.00	39,496	25,445
6	82,423	0.867769	0.892521	0.00	82,423	51,618
7	148,956	0.788881	0.835705	8.93%	135,652	55,922
8	58,508	0.717164	0.781373	32.65	39,403	−892
9	49,812	0.651967	0.730799	50.79	24,511	−316,064
10	190,527	0.592698	0.684045	64.70	67,261	56,881
PV	540,780				525,675	−11,960

There is no put activity at time 4

Picture at Time 4

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
4	63,857	1.000000	0	0	0	0
5	25,445	0.948927	25,445	39,496	14,051	37,701
6	51,618	0.892521	51,618	82,423	30,805	71,524
7	55,922	0.835705	55,922	135,652	79,730	107,013
8	−892	0.781373	0	40,295	40,295	28,990
9	−316,064	0.730799	−229,638	110,937	340,575	82,299
10	56,881	0.684045	56,881	67,261	10,380	39,865
PV	−11,960		−11,960	382,976	394,936	

Deficit at time 4: 11,960
Discount factor to time zero: 0.70332
Indicated C-3 requirement: 8,412
Reinvest assets now maturing to time: 8 9

TABLE 2H

TIME 5

Call Activity at Time 5						
Year	Old Asset	Call Factor	PV-New	Call Percentage	New Asset	New Flow
5	39,496	1.050000	1.000000	0.00	65,685	51,634
6	82,423	0.954545	0.952035	0.00	82,423	51,618
7	135,652	0.867769	0.896084	0.00	135,652	55,922
8	40,295	0.790556	0.838667	10.29	36,150	−4,145
9	110,937	0.788429	0.783648	0.00	110,937	−229,638
10	67,261	0.651967	0.732757	52.25	32,118	21,738
PV	409,537				406,499	−16,616

There is no put activity at time 5

Picture at Time 5						
Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
5	51,634	1.000000	0	0	0	0
6	51,618	0.952035	51,618	82,423	30,805	78,677
7	55,922	0.896084	55,922	135,652	79,730	117,714
8	−4,145	0.838667	0	40,295	40,295	32,229
9	−229,638	0.783648	−168,184	172,391	340,575	138,032
10	21,738	0.732757	21,738	32,118	10,380	20,940
PV	−16,616		−16,616	392,448	409,063	

Deficit at time 5:	16,616
Discount factor to time zero:	0.64312
Indicated C-3 requirement:	10,686
Reinvest assets now maturing to time:	8 9

TABLE 2I

TIME 6

Call Activity at Time 6						
Year	Old Asset	Call Factor	PV-New	Call Percentage	New Asset	New Flow
6	82,423	1.050000	1.000000	0.00	92,643	61,838
7	135,652	0.954545	0.950883	0.00	135,652	55,922
8	40,295	0.874675	0.898266	0.00	40,295	0
9	172,391	0.851288	0.845642	0.00	172,391	−168,184
10	32,118	0.717164	0.795077	44.37	17,868	7,488
PV	418,925				417,815	−21,257

There is no put activity at time 6

Picture at Time 6						
Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
6	61,838	1.000000	0	0	0	0
7	55,922	0.950883	55,922	135,652	79,730	129,486
8	0	0.898266	0	40,295	40,295	35,245
9	−168,184	0.845642	−95,059	245,516	340,575	211,684
10	7,488	0.795077	7,488	17,868	10,380	12,814
PV	−21,257		−21,257	387,010	408,267	

Deficit at time 6:	21,257
Discount factor to time zero:	0.58720
Indicated C-3 requirement:	12,482
Reinvest assets now maturing to time:	9

TABLE 2J

TIME 7

Call Activity at Time 7						
Year	Old Asset	Call Factor	PV-New	Call Percentage	New Asset	New Flow
7	135,652	1.050000	1.000000	0.00	141,591	61,861
8	40,295	0.957984	0.956108	0.00	40,295	0
9	245,516	0.917819	0.913226	0.00	245,516	−95,059
10	17,868	0.788881	0.871481	42.14	10,339	−41
PV	413,962				413,340	−24,985

There is no put activity at time 7

Picture at Time 7						
Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
7	61,861	1.000000	0	0	0	0
8	0	0.956108	0	40,295	40,295	38,602
9	−95,059	0.913226	−27,320	313,255	340,575	290,294
10	−41	0.871481	−41	10,339	10,380	8,156
PV	−24,985		−24,985	333,610	358,594	

Deficit at time 7:	24,985
Discount factor to time zero:	0.53532
Indicated C-3 requirement:	13,375
Reinvest assets now maturing to time:	9

TABLE 2K

TIME 8

Call Activity at Time 8						
Year	Old Asset	Call Factor	PV-New	Call Percentage	New Asset	New Flow
8	40,295	1.050000	1.000000	0.00	40,365	70
9	313,255	0.985725	0.953705	0.00	313,255	−27,320
10	10,339	0.867769	0.911838	0.78	10,258	−122
PV	348,476				348,472	−26,096

There is no put activity at time 8

Picture at Time 8						
Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
8	70	1.000000	0	0	0	0
9	−27,320	0.953705	−27,246	313,329	340,575	308,857
10	−122	0.911838	−122	10,258	10,380	8,902
PV	−26,096		−26,096	308,177	334,273	

Deficit at time 8:	26,096
Discount factor to time zero:	0.48733
Indicated C-3 requirement:	12,717
Reinvest assets now maturing to time:	9

TABLE 2L

TIME 9

There is no call activity at time 9
There is no put activity at time 9

Picture at Time 9					
Time	Old Flow	PV Factor	New Flow	New Asset	New Liability
9	−27,246	1.000000	−17,534	0	17,534
10	−122	0.946729	−10,380	0	10,380
PV	−27,361		−27,361	0	27,361

Deficit at time 9:	27,361
Discount factor to time zero:	0.44316
Indicated C-3 requirement:	12,126
C-3 requirement:	13,375

APPENDIX 3

CALCULATION DETAILS UNDER TRACK 90

TABLE 3A

INTEREST TRACK DETAILS

Time	Long Rate	Short Rate
0	9.520	9.310
1	9.916	8.774
2	10.533	9.913
3	13.423	9.289
4	16.332	12.013
5	14.764	15.928
6	13.062	16.752
7	14.209	13.040
8	13.208	12.774
9	12.977	9.460
10	12.686	9.607

TABLE 3B

INTEREST DISCOUNT FACTORS

	1	2	3	4	5	6	7	8	9	10
0	0.915	0.839	0.768	0.703	0.643	0.587	0.535	0.487	0.443	0.403
1		0.918	0.839	0.765	0.696	0.634	0.577	0.525	0.477	0.431
2			0.909	0.826	0.749	0.679	0.615	0.557	0.503	0.454
3				0.906	0.805	0.709	0.623	0.549	0.485	0.429
4					0.884	0.766	0.657	0.563	0.483	0.416
5						0.866	0.756	0.662	0.579	0.506
6							0.865	0.764	0.681	0.607
7								0.883	0.776	0.681
8									0.887	0.786
9										0.906

Values show present value at row time of one payable at column time.

TABLE 3C

EFFECTIVE INTEREST RATES

	1	2	3	4	5	6	7	8	9	10
0	9.25	9.20	9.19	9.20	9.23	9.28	9.34	9.40	9.46	9.52
1		8.99	9.20	9.36	9.46	9.54	9.59	9.64	9.70	9.79
2			9.97	10.04	10.10	10.15	10.20	10.26	10.31	10.37
3				10.38	11.45	12.14	12.56	12.76	12.83	12.86
4					13.16	14.28	15.01	15.44	15.65	15.72
5						15.46	15.03	14.76	14.63	14.60
6							15.55	14.39	13.67	13.28
7								13.26	13.48	13.64
8									12.78	12.79
9										10.37

Values show effective interest rate between row time and column time.

TABLE 3D

TIME 1

There is no call activity at time 1

Put Activity at Year 1

Year	Old Asset	Old Liability	PV-New	New Asset	New Liability
1	14,681	234,883	1.000000	−1,164	229,910
2	222,279	22,727	0.917544	222,279	22,246
3	119,303	83,855	0.838549	119,303	82,080
4	27,516	142,699	0.764634	27,516	139,678
5	48,081	14,051	0.696490	48,081	13,754
6	82,423	30,805	0.634148	82,423	30,153
7	148,956	79,730	0.577205	148,956	78,042
8	64,739	40,295	0.524987	64,739	39,442
9	64,193	340,575	0.476685	64,193	333,364
10	281,780	10,380	0.431455	281,780	10,160
PV	697,609	698,487		681,764	683,698

Cash value: 748,359
Percentage surrendered: 2.12

$$\text{Note: } 0.0212 = 1 - \exp - \left[\frac{748359}{698487} - 1.05\right]$$

Picture at Time 1

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
1	−231,074	1.000000	0	0	0	0
2	200,033	0.917544	0	22,246	22,246	21,235
3	37,223	0.838549	0	82,080	82,080	71,226
4	−112,162	0.764634	−112,162	27,516	139,678	21,707
5	34,327	0.696490	10,894	24,647	13,754	17,676
6	52,271	0.634148	52,271	82,423	30,153	53,737
7	70,914	0.577205	70,914	148,956	78,042	88,286
8	25,297	0.524987	25,297	64,739	39,442	34,882
9	−269,171	0.476685	−269,171	64,193	333,364	31,444
10	271,619	0.431455	271,619	281,780	10,160	125,477
PV	−1,934		−1,934	451,854	453,788	

Deficit at time 1: 1,934
Discount factor to time zero: 0.91534
Indicated C-3 requirement: 1,771
Sell assets maturing at time: 2 3 5

TABLE 3E

TIME 2

There is no call activity at time 2

Put Activity at Year 2

Year	Old Asset	Old Liability	PV-New	New Asset	New Liability
2	22,246	22,246	1.000000	-16,040	20,683
3	82,080	82,080	0.909321	82,080	76,313
4	27,516	139,678	0.825835	27,516	129,864
5	24,647	13,754	0.749270	24,647	12,787
6	82,423	30,153	0.679219	82,423	28,034
7	148,956	78,042	0.615196	148,956	72,559
8	64,739	39,442	0.556677	64,739	36,671
9	64,193	333,364	0.503130	64,193	309,943
10	281,780	10,160	0.454040	281,780	9,446
PV	481,969	485,325		443,683	451,227

Cash value: 544,946
Percentage surrendered: 7.03

Picture at Time 2

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
2	-36,723	1.000000	0	0	0	0
3	5,767	0.909321	0	76,313	76,313	72,844
4	-102,349	0.825835	-102,349	27,516	129,864	23,877
5	11,860	0.749270	0	12,787	12,787	10,088
6	54,389	0.679219	21,126	49,160	28,034	35,256
7	76,397	0.615196	76,397	148,956	72,559	97,114
8	28,068	0.556677	28,068	64,739	36,671	38,370
9	-245,750	0.503130	-245,750	64,193	309,943	34,588
10	272,333	0.454040	272,333	281,780	9,446	138,025
PV	-7,544		-7,544	423,000	430,545	

Deficit at time 2: 7,544
Discount factor to time zero: 0.83858
Indicated C-3 requirement: 6,327
Sell assets maturing at time: 3 5 6

TABLE 3F

TIME 3

There is no call activity at time 3

Put Activity at Year 3

Year	Old Asset	Old Liability	PV-New	New Asset	New Liability
3	76,313	76,313	1.000000	20,959	68,130
4	27,516	129,864	0.905977	27,516	115,939
5	12,787	12,787	0.805151	12,787	11,416
6	49,160	28,034	0.709027	49,160	25,028
7	148,956	72,559	0.623049	148,956	64,779
8	64,739	36,671	0.548585	64,739	32,739
9	64,193	309,943	0.484611	64,193	276,709
10	281,780	9,446	0.428863	281,780	8,433
PV	426,668	443,718		371,314	396,139

Cash value: 516,231
Percentage surrendered: 10.72

Picture at Time 3

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
3	−47,171	1.000000	0	0	0	0
4	−88,424	0.905977	−88,424	27,516	115,939	26,265
5	1,371	0.805151	0	11,416	11,416	9,907
6	24,132	0.709027	0	25,028	25,028	19,744
7	84,177	0.623049	37,701	102,479	64,779	73,494
8	32,000	0.548585	32,000	64,739	32,739	42,208
9	−212,516	0.484611	−212,516	64,193	276,709	38,047
10	273,346	0.428863	273,346	281,780	8,433	151,827
PV	−24,825		−24,825	303,184	328,009	

Deficit at time 3: 24,825
Discount factor to time zero: 0.76825
Indicated C-3 requirement: 19,072
Sell assets maturing at time: 5 6 7

TABLE 3G

TIME 4

There is no call activity at time 4

Put Activity at Year 4

Year	Old Asset	Old Liability	PV-New	New Asset	New Liability
4	27,516	115,939	1.000000	−39,046	97,562
5	11,416	11,416	0.883732	11,416	9,607
6	25,028	25,028	0.765755	25,028	21,061
7	102,479	64,779	0.657360	102,479	54,511
8	64,739	32,739	0.563099	64,739	27,549
9	64,193	276,709	0.483359	64,193	232,848
10	281,780	8,433	0.416333	281,780	7,097
PV	308,932	343,473		242,371	289,029

Cash value: 419,923
Percentage surrendered: 15.85

Picture at Time 4

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
4	−136,608	1.000000	0	0	0	0
5	1,810	0.883732	0	9,607	9,607	9,170
6	3,967	0.765755	0	21,061	21,061	18,276
7	47,969	0.657360	0	54,511	54,511	43,002
8	37,189	0.563099	0	27,549	27,549	19,757
9	−168,655	0.483359	−168,655	64,193	232,848	41,852
10	274,683	0.416333	83,737	90,834	7,097	53,837
PV	−46,658		−46,658	144,809	191,467	

Deficit at time 4: 46,658
Discount factor to time zero: 0.70332
Indicated C-3 requirement: 32,816
Sell assets maturing at time: 5 6 7 8 10

TABLE 3H

TIME 5

There is no call activity at time 5

Put Activity at Year 5

Year	Old Asset	Old Liability	PV-New	New Asset	New Liability
5	9,607	9,607	1.000000	-34,565	8,071
6	21,061	21,061	0.866070	21,061	17,694
7	54,511	54,511	0.755805	54,511	45,797
8	27,549	27,549	0.661651	27,549	23,145
9	64,193	232,848	0.579141	64,193	195,625
10	90,834	7,097	0.505806	90,834	5,962
PV	170,396	225,716		126,224	189,633

Cash value: 276,318
Percentage surrendered: 15.99

Picture at Time 5

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
5	-42,636	1.000000	0	0	0	0
6	3,367	0.866070	0	17,694	17,694	16,890
7	8,714	0.755805	0	45,797	45,797	39,741
8	4,404	0.661651	0	23,145	23,145	18,259
9	-131,432	0.579141	-131,432	64,193	195,625	46,037
10	84,871	0.505806	25,125	31,087	5,962	20,268
PV	-63,409		-63,409	118,153	181,562	

Deficit at time 5: 63,409
Discount factor to time zero: 0.64312
Indicated C-3 requirement: 40,780
Sell assets maturing at time: 6 7 8 10

TABLE 3I

TIME 6

There is no call activity at time 6

Put Activity at Year 6

Year	Old Asset	Old Liability	PV-New	New Asset	New Liability
6	17,694	17,694	1.000000	895	16,445
7	45,797	45,797	0.865438	45,797	42,563
8	23,145	23,145	0.764200	23,145	21,511
9	64,193	195,625	0.680934	64,193	181,814
10	31,087	5,962	0.607270	31,087	5,541
PV	137,605	211,845		120,806	196,888

Cash value: 237,947
Percentage surrendered: 7.06

Picture at Time 6

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
6	−15,550	1.000000	0	0	0	0
7	3,233	0.865438	0	42,563	42,563	40,629
8	1,634	0.764200	0	21,511	21,511	18,667
9	−117,620	0.680934	−117,620	64,193	181,814	50,641
10	25,546	0.607270	6,603	12,144	5,541	8,710
PV	−76,082		−76,082	104,361	180,443	

Deficit at time 6: 76,082
Discount factor to time zero: 0.58720
Indicated C-3 requirement: 44,675
Sell assets maturing at time: 7 8 10

TABLE 3J

TIME 7

There is no call activity at time 7

Put Activity at Year 7

Year	Old Asset	Old Liability	PV-New	New Asset	New Liability
7	42,563	42,563	1.000000	41,483	42,352
8	21,511	21,511	0.882923	21,511	21,405
9	64,193	181,814	0.776474	64,193	180,912
10	12,144	5,541	0.681339	12,144	5,514
PV	119,675	206,505		118,595	205,481

Cash value: 217,857
Percentage surrendered: 0.50

Picture at Time 7

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
7	−869	1.000000	0	0	0	0
8	107	0.882923	0	21,405	21,405	20,432
9	−116,719	0.776474	−116,719	64,193	180,912	55,705
10	6,631	0.681339	5,493	11,007	5,514	8,683
PV	−86,887		−86,887	76,242	163,129	

Deficit at time 7: 86,887
Discount factor to time zero: 0.53532
Indicated C-3 requirement: 46,512
Sell assets maturing at time: 8 10

TABLE 3K

TIME 8

There is no call activity at time 8
There is no put activity at time 8

Picture at Time 8

Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Value
8	0	1.000000	0	0	0	0
9	−116,719	0.886691	−116,719	64,193	180,912	61,275
10	5,493	0.786004	5,493	11,007	5,514	9,551
PV	−99,176		−99,176	65,571	164,747	

Deficit at time 8: 99,176
Discount factor to time zero: 0.48733
Indicated C-3 requirement: 48,331

TABLE 3L

TIME 9

There is no call activity at time 9
There is no put activity at time 9

Picture at Time 9						
Time	Old Flow	PV Factor	New Flow	New Asset	New Liability	Call Valuation
9	−116,719	1.000000	−106,746	0	106,746	67,403
10	5,493	0.906059	−5,514	0	5,514	10,507
PV	−111,742		−111,742	0	111,742	

Deficit at time 9:	111,742
Discount factor to time zero:	0.44316
Indicated C-3 requirement:	49,520
C-3 requirement:	49,520

APPENDIX 4

ILLUSTRATIONS OF INTEREST RATE MODEL

TABLE 4A

INPUT PARAMETERS

Maximum Inflation Rate	15.000%
Minimum Inflation Rate	−4.000%
Inflation Drift Factor	3.580
Long-Term Trend Factor	0.073
Long-Term Drift Factor	0.108
Long-Term Goal	4.000%
Short-Term Trend Factor	0.144
Short-Term Drift Factor	0.275
Short-Term Ratio Goal	0.726
Short-Term Ratio Maximum	1.300
Initial Inflation Rate	4.170%
Initial Long-Term Rate	9.520%
Initial Short-Term Rate	9.310%
Number of Years	10
Number of Simulations	100
Number of Tracks	100

TABLE 4B

OUTPUT STATISTICS

	Year 2	Year 4	Year 6	Year 8	Year 10
Inflation Rates					
Minimum	−4.000	−4.000	−4.000	−4.000	−4.000
Mean	4.040	4.619	4.526	4.664	4.785
Standard Deviation	3.651	5.530	6.133	6.309	6.344
Maximum	15.000	15.000	15.000	15.000	15.000
20th Percentile	0.871	−0.822	−2.099	−1.919	−2.086
40th Percentile	3.134	2.853	2.231	2.078	2.355
60th Percentile	4.952	5.981	6.260	6.317	6.368
80th Percentile	6.989	9.794	10.915	11.270	11.661
Long-Term Rates					
Minimum	6.663	5.141	3.514	2.824	2.452
Mean	9.443	9.440	9.388	9.384	9.280
Standard Deviation	0.944	1.763	2.366	3.033	3.331
Maximum	12.972	17.784	18.568	22.774	21.532
20th Percentile	8.639	7.969	7.388	6.742	6.375
40th Percentile	9.170	8.860	8.558	8.383	8.126
60th Percentile	9.647	9.835	9.730	9.830	9.771
80th Percentile	10.211	10.778	11.291	11.671	12.056
Long-Term Rates Less Inflation					
Minimum	−5.061	−7.007	−7.100	−7.215	−8.031
Mean	5.403	4.821	4.861	4.720	4.495
Standard Deviation	3.541	4.976	5.153	5.167	5.171
Maximum	14.793	16.386	16.883	18.576	20.176
20th Percentile	2.333	0.276	−0.008	−0.210	−0.422
40th Percentile	4.497	3.334	3.413	3.672	2.913
60th Percentile	6.271	6.158	6.873	6.699	6.432
80th Percentile	8.418	9.769	9.989	9.389	9.099
Short-Term Rates					
Minimum	4.209	2.442	1.990	1.611	1.156
Mean	9.016	8.547	8.085	7.712	7.564
Standard Deviation	2.100	2.956	3.345	3.713	3.918
Maximum	15.939	23.119	22.075	29.606	27.524
20th Percentile	7.131	5.933	5.238	4.598	4.307
40th Percentile	8.296	7.409	6.726	6.178	5.832
60th Percentile	9.430	8.941	8.446	7.954	7.822
80th Percentile	10.949	11.076	10.509	10.442	10.455
Ratio of Short-Term to Long-Term Rates					
Minimum	0.516	0.358	0.288	0.274	0.288
Mean	0.955	0.904	0.859	0.819	0.811
Standard Deviation	0.203	0.255	0.264	0.267	0.273
Maximum	1.300	1.300	1.300	1.300	1.300
20th Percentile	0.764	0.669	0.608	0.569	0.553
40th Percentile	0.879	0.808	0.752	0.714	0.706
60th Percentile	1.000	0.957	0.913	0.862	0.852
80th Percentile	1.155	1.178	1.126	1.064	1.087

APPENDIX 5
TECHNICAL DETAILS

Normally Distributed Random Numbers

Given $R1$ and $R2$, two uniformly distributed independent random variates on the (0, 1) interval, then

$$X_1 = (-\ 2 \ln R_1)^{1/2} \cdot \cos 2 \pi R_2$$

$$X_2 = (-\ 2 \ln R_1)^{1/2} \cdot \sin 2 \pi R_2$$

are two random variates from a standard normal distribution [4].

Inflation Rate

Given the maximum inflation rate INFMAX, the minimum inflation rate INFMIN, and the inflation drift factor INFDRIFT, inflation rates INF are determined recursively as follows, starting with INFSTART at time 0:

$$A = \text{INF}\ (t - 1) + (\text{INFDRIFT})\ (\text{NRN})$$

$$\text{INF}(t) = \text{Max}[\text{INFMIN, Min}\ [A, \text{INFMAX}]]$$

where NRN is a normally distributed random number with mean zero and unit variance.

Long-Term Interest Rate (10 year)

Given the long-term trend factor LTREND, the long-term drift factor LDRIFT, and the long-term goal LGOAL, the long-term interest rates are determined recursively as follows, starting with LSTART at time 0:

$$B = \text{LINT}\ (t - 1) \cdot \exp\ (\text{LDRIFT})\ (\text{NRN})$$

$$\text{LINT}(t) = B + \text{LTREND}\ [\text{LGOAL} + \text{INF}(t) - \text{B}]$$

Ratio of Short-Term Interest Rate (3 months) to Long-Term Rate

Given the short-term trend factor STREND, the short-term drift factor SDRIFT, the short-term ratio goal SGOAL, and the maximum plausible ratio RMAX, the ratios of the short-term rates to the long-term rates are determined recursively as follows, starting with the initial ratio RSTART at time 0:

$$C = \text{RINT}(t-1) \cdot \exp\,[\text{SDRIFT}]\cdot[\text{NRN}]$$

$$\text{RINT}(t) = \text{Min}\,[\text{RMAX}, C + \text{STREND}\,(\text{SGOAL} - C)]$$

The short-term interest rates are then determined as follows:

$$\text{SINT}(t) = \text{RINT}(t) \cdot \text{LINT}(t)$$

Computation of Intermediate Yield Rates

Given the three-month rate and the ten-year rate at a point in time, intermediate rates $I(x)$ are obtained by a cubic interpolation given that D is the excess of the ten-year rate over the three-month rate. First, a linear interpolation is performed:

$$J(x) = I(0.25) + (x - 0.25) \times \text{D}.$$

Then

$$I(x) = J(x) + (x - 0.25)\,(x - 10)\,(a\,x + b)$$

where a and b are linear functions of D given by:

$$a = 0.002501 + 0.003611\,(D - 1.1606)$$

$$b = -\,[0.021536 + 0.03563\,(D - 1.1606)]$$

The equations and parameters were derived from historical data that indicated that a and b were highly correlated with D.

In the absence of data, rates for terms longer than ten years are assumed to equal the ten-year rate.

CALL Logic

CALL activity is invoked separately for each future asset payment if interest rates have dropped sufficiently. Let VN be the discounted value of the payment on the current yield curve and let VO be the computed call value. Let CMIN be the call threshold factor and CI be the call intensity factor. The fraction $F1$ of the asset that is redeemed for its call value is given by:

$$F1 = 1 - \exp - \text{CI}\,[\text{Max}\,(0, (\text{VN/VO}) - \text{CMIN})]$$

PUT Logic

PUT activity is invoked collectively for future liability payments if interest rates have increased sufficiently. Let VN be the discounted value of the liability flow on the current yield curve and let VO be the computed cash value. Let PMIN be the put threshold factor and PI be the put intensity factor. The fraction $F2$ of the liabilities that are surrendered is given by:

$$F2 = 1 - \exp - \mathrm{PI}\,[\mathrm{Max}\,(0, (\mathrm{VO/VN}) - \mathrm{PMIN})]$$

DISCUSSION OF PRECEDING PAPER

FRANK J. ALPERT:

Mr. Mereu is to be congratulated for preparing an instructive and helpful paper about evaluating C-3 risk. The investigation is becoming increasingly important, not only because of the magnitude of the potential losses and the difficulty of analysis but also because it is a central paradigm for other broad issues of management. I recommend the paper to both product actuaries and investment managers, as well as to corporate and valuation actuaries, because the decisions of the first groups will largely determine the extent of the risk.

Mr. Mereu has succinctly illustrated the elements that are essential for measuring C-3 risk:

- An objective measurement of the risk at each checkpoint, translated into capital or reserve requirements, derived from
- Cash-flow testing, through
- Multiple economic scenarios, recognizing
- Investment and disinvestment strategies and
- Options available to asset clients; and
- Company pricing strategies and
- Options available to liability clients.

All these except company pricing strategies are explicitly used in Mr. Mereu's paper, and we can assume that pricing strategies are implicitly involved in the PUT parameters used in the calculations.

The testing discussed in the paper involves arbitrary asset and liability cash flows, in which the incidence of payments is considerably less well-behaved than the typical block of insurance policies. This permits the reader to concentrate on the process rather than the product. But because the cash flows are arbitrary and there is an implied need for rebalancing each year, some of the relationships are less than obvious.

Mr. Mereu illustrates the C-3 risk as a risk of cash-flow mismatch and measures it by comparing the present values of the asset and liability cash-flow streams discounted at consistent interest rates. For the reasons outlined below, I prefer looking at year-by-year results.

The risk event we are trying to capture is the single interval (or separate intervals) in which the asset cash flow is less than the liability cash flow. If that occurs, the company will have losses and may have to supply capital to meet its obligations. The losses are real—there would be a real reduction in economic value—but they may not show up immediately in accounting figures.

On the other hand, if the asset cash flow exceeds the liability cash flow in every interval in the scenario, then the company will show profits and will have no need for additional capital, and the present value of the net cash flow will always be positive. Conversely, a negative net present value signals that one or more future intervals will have negative cash flow.

In addition, other advantages of looking at single years rather than present values are as follows:

- The net present value can be positive and still have intervals with negative cash flows.
- A single-year measure can be coupled with a risk measure of solvency, requiring that the assets be bigger than the reserves on a year-by-year basis.
- In some models, today's present values can be obtained easily, but present values at future dates are difficult to obtain without a significant increase in running time.

For management use, the results of the scenario testing are summarized by degree of capital and remaining risk, as shown in the table below. This provides substantial information in a compact and understandable form.

HYPOTHETICAL SPDA

Capital As Percentage of Premium	ROE	Percentage of Years with Positive	
		Cash Flow	Solvency
3	17.0%	73.7%	79.5%
5	13.3	80.0	88.9
10	10.0	86.0	98.7
15	8.7	88.1	100.0

As shown in the Appendix, the two procedures are equivalent and provide equivalent judgments under the assumption that at the valuation date the cash flows will be rebalanced.

Nevertheless, I recognize that actuaries are comfortable using present values and will continue to discount cash-flow streams, partly because discounted cash flow comes up in so many other contexts. For example:

- The present value of future profits is demonstrably equal to the present value of future net cash flows.

- The economic value of the enterprise can be defined as the present value of future profits. For stock companies, at least, this is the function that management should be striving to maximize.
- The market value of the assets is by definition the present value of the expected cash flow, discounted at current market interest rates. The corollary is that maximizing market value will enhance the economic value of the enterprise, because it will increase the present value of asset cash flows and net cash flows. Thus, it appropriate to manage the assets for total return within the established constraints.
- Many companies use duration measures as an operating rule. The underlying justification is that duration is a first approximation to the change in present value of a cash-flow stream when interest rates change; therefore, keeping asset durations close to liability durations will minimize changes in the net present value of assets less liabilities.

Perhaps the moral is that we seek statistics on single years, but plan profits on present values.

Multiple Interest Rate Scenarios

Like Mr. Mereu, we use multiple, randomly generated, interest rate scenarios. I think this is the best approach. I agree that deterministic scenarios are limited if the actuary is interested in results at various confidence levels. Moreover, a deterministic set may not display all the risky scenarios. It could be said that every adverse event in economic history was a surprise at the time it happened.

Others who are much more expert in economics than I am have written extensively about how to randomly generate yield curves and interest rate scenarios. There are recommended assumptions for starting points, volatility and drift; there are recommended distributions; and sometimes it is required that there be no risk-free arbitrage. All these points are important.

I would just add two other observations: Whatever method is used, the actuary should review the resulting scenarios for reasonableness and economic sense; and depending on the purpose of the testing, the actuary may want to include differentials in other economic parameters on some random or distributed basis. In particular, the default rates and inflation should vary with the scenario conditions.

The necessity of review was brought out in a recent study we did. Great care had been taken in setting the initial rates, the shape of the yield curve, the probability distribution of short-term and long-term rates, the probability of inversion, and the overall minimum and maximum rates. Nevertheless, on detailed examination about 30 percent of the scenarios were discarded because they were economically inconsistent. It is noteworthy that the rejected scenarios did not significantly affect the year-by-year means or variances of the overall set—they were found only by a computerized review of each scenario year by year.

The use of multiple year-by-year scenarios provides the opportunity to vary other economic parameters at the same time. For example, inflation rates should vary with interest rates. Depending on the purpose of the test, inflation could be directly derived from the Treasury rate in a deterministic approach or randomly related to the Treasury rate to more accurately reflect reality.

Default Risk

Buff and others have demonstrated that the C-1 risk of loss of asset value must be considered in direct connection with the C-3 interest rate risk. Accordingly, the same multiple scenarios that are used to test the C-3 risk should also be used to test the C-1 risk. The initial default rate for each asset obviously depends on its quality rating. But from there, the default rates should change with the interest rates in the scenario, with more than linear increases for both high interest rates and lower quality. In our studies, we have used an exponential formula of the form

$$x = \text{excess of 90-day T-bill rate over } 7\tfrac{1}{2}\%$$
$$\text{Default} = \text{Base Rate} \times K^x$$

The factors and adjusted default rates produced by this formula are shown below:

	Investment Grade			
	AA2	A	Baa	B
Factor K	1.05	1.09	1.125	1.125
Base Default Rate at 7.5% or less	0.0015	0.0017	0.0020	0.0250
Default Rate at				
10%	0.0017	0.0021	0.0027	0.0336
12½%	0.0019	0.0026	0.0036	0.0451
15%	0.0022	0.0032	0.0048	0.0605

Other Risks

Scenario testing can also be used to measure the effect of C-2 risks such as increased mortality or lapse. This can be advantageous in producing a combined measure of risk in a single evaluation, rather than constructing separate studies.

Asset and Liability Options

The two scenarios illustrated in the paper allow the reader to follow the logic of the calculations. The examples have either liability put options or asset call options, but not both at the same time. In real life, we can expect both options to be exercised at the same time in at least some circumstances—one example would be deferred annuities with a bailout provision: A drop in interest rates would allow both the assets to be called and the annuities to be surrendered without penalty.

A model that allows both types of options to be exercised at the same time probably must include assumptions on the order of processing or allow for an iterative evaluation at each interval, or both. In any event, the actuary evaluating the C-3 risk should be aware of how the structure of the model may affect the results.

Summary

This excellent paper can be a guide for anyone interested in evaluating the C-3 risk. It succinctly illustrates the important elements and suggests ways of performing the calculations and displaying the results. I have made additional suggestions about the use of year-by-year results, the necessity of reviewing randomly generated scenarios, and a technique for simultaneous testing of the C-1 risk by adjusting the default rates.

For those who are interested in further explorations within Mr. Mereu's model, additional tables and analyses are available at my *Yearbook* address.

APPENDIX

Let $\text{disc}(n,n+t)$ = the discount factor between n and $n+t$.

$$\$1 \text{ at } n+t = \text{disc}(n,n+t) \text{ at } n$$
$$\text{ACF}(r) = \text{asset cash flow at } r$$
$$\text{LCF}(r) = \text{liability cash flow at } r$$
$$\text{PVACF}(s) = \Sigma\ \text{ACF}(s+j) \times \text{disc}(s,s+j)$$
$$\text{PVLCF}(s) = \Sigma\ \text{LCF}(s+j) \times \text{disc}(s,s+j).$$

Assume:

the evaluation is at the end of interval n

$PVACF(n) < PVLCF(n)$ (otherwise there is no risk)

all $ACF(r) = LCF(r)$ except $ACF(n+t) < LCF(n+t)$

(the portfolio has been rebalanced to meet the liability cash flows to the extent possible).

In the paper, the C-3 risk is measured by:

$$\text{disc}(0,n) \times [PVACF(n) - PVLCF(n)]$$
$$= \text{disc}(0,n) \times [ACF(n+t) - LCF(n+t)] \times \text{disc}(n,n+t)$$

since all other years drop out

$$= [ACF(n+\text{t}) - LCF(n+t)] \times \text{disc}^*(0,n+t)$$

where disc* is a composite discount factor from the initial yield curve and the yield curve at time n. This differs from the calculation of the present value of the single year only in the discount rate:

$$[ACF(n+\text{t}) - LCF(N+t)] \times \text{disc}(0,n+t).$$

In theory, discounting should be at the successive rates used in the scenario, so that the value at $t=0$ would be:

$$[ACF(n+t) - LCF(n+t)] \times \text{disc}(0,1) \times \text{disc}(1,2) \times \text{disc}(2,3) \ldots .$$

SARAH L. CHRISTIANSEN:

Professor Mereu is to be complimented for writing such a readable paper on a current topic of great importance. It could serve as a basic guide to many actuaries. He clearly put a great deal of effort into his model, which is based on Canadian data. However, I have a few comments with respect to the model, most of which pertain to his interest-rate-generating mechanism (IGM).

Regulation 126 Requirements

In the U.S. much of the C-3 testing is done to satisfy New York Regulation 126 as well as to meet internal purposes. New York requires that projections be continued until the major portion of insurance cash flows is gone from the contractual obligation on the valuation date [Section 95.9(c)].

New York Regulation 126 sets various horizons depending on the product type, which may easily exceed 10 years for such products as single-premium whole life insurance. For annuities in payment New York Regulation 126 Section 95.9 suggests a time horizon of 20 years or longer.

Thus, it is important that:

1. There be rates for times greater than 10 years. U.S. data suggest that rates for 15, 20, and 30 years are generally different from the 10-year rate.
2. The IGM project reasonable scenarios for 20 or 30 years.

Relationship to Inflation

Projected interest rates should be the rates that the company expects to earn on its assets. In general, this would include a spread over Treasuries. Rarely would the long-term rate be less than the inflation rate, especially by the significant amounts that Professor Mereu indicates in his Table 4B. It is even rarer historically for the short-term rate to be less than the inflation rate.

By using Mereu's Appendix 5 (Technical Details), more simulations of interest rate scenarios were run with both a 10-year and a 30-year projection period.

The following table summarizes the number of occurrences and the percentage of times that IGM produced rates that were less than inflation.

	Rates Less Than Inflation	
	10-Year Projection Period	30-Year Projection Period
Number scenarios	400 (4000 curves)	200 (6000 curves)
Long-term rates	366 (9.15%)	747 (12.45%)
Short-term rates	49 (1.225%)	475 (7.92%)
Both	31 (0.78%)	358 (5.96%)

These simulations lead to the conclusion that the process tends to get out of hand when extended beyond the 10-year scope or, to use Professor Mereu's rather graphic description, the moving inflation rate gets away from the drunk.

Bounds on Rates

A question of reasonableness is whether there should be a minimum value for the ratio of short- to long-term interest rates. U.S. data seem to indicate that a ratio of 0.6 would serve as a reasonable minimum, and that in fact was used in the above calculations for the 30-year projection period.

The question of when is an interest rate scenario reasonable can be answered in part by looking at the rates that it projects. For the U.S., I would consider that all rates could be between 3 percent and 25 percent. Political considerations are likely to prevent interest rates from exceeding 25 percent. The U.S. national debt and health care inflation are likely to continue to keep rates above 3 percent. Although Canada does not have a problem with the national debt, Canada does have pressure on the National Health Insurance; thus Canadian rates would also be unlikely to be less than 3 percent. Also, Canadian interest rates are subject to pressure from the exchange rate in an attempt to maintain parity with the U.S. dollar. Hence, Canadian interest rates tend to be higher rather than lower than the U.S. counterparts. The political pressures and a stable Canadian economy lead to the conclusion that 25 percent is a ceiling for Canadian rates. New York Regulation 126 in fact requires that rates be between 4 percent and 25 percent.

Professor Mereu's IGM procedure tends to produce rates that are both too high and too low. The following table is a summary of the typical results using 100 scenarios.

	Number of Scenarios with	
	10-Year Projections	30-Year Projections
Rates between 3% and 25%	55	31
Rates below 3%	40	53
Rates above 25%	5	16
Minimum	0.67	0.59
Maximum	28.99	36.98
Minimum ratio	0	0.6

Due to the greater variability of the short-term rates relative to the long-term rates, it is almost always the short-term rate for which the problem occurs. Unfortunately there tend to be long sequences of rates that are too low (and occasionally too high), so that a simple "take the maximum (minimum) after the rates are calculated" would tend to stick at the 3 percent (25 percent) level.

Parameter Modifications

At the cost of moving away from historically determined parameters, the input parameters were changed. The goals were to keep a tighter rein on inflation, reduce to about 1 percent the frequency of short-term rates that were less than inflation, to give more power to the mean reversionary process, and to produce more reasonable results.

Reasonable changes in the parameters accomplished all the goals except that of producing reasonable results. Using parameter changes alone, reasonable 30-year scenarios resulted 50 percent to 60 percent of the time, which was an improvement, but not sufficient. In order to minimize the stickiness that arises from putting barriers on the rates, it was preferable to compare rates at each point rather than after the scenario was completed.

The following table shows parameter changes that were made while attempting to maintain reasonableness.

Parameter	From	To	Comments
Maximum Inflation	15%	12%	U.S. data showed only 2 years with a rate between 12% and 15% from 1926 to 1987.
Minimum Inflation	−4%	−2%	U.S. data showed only 1 year between these two values (and 4 years during the Great Depression where inflation $< -2\%$).
Long-Term Trend	0.073	0.33	Increase power of mean reversionary process.
Short-Term Trend	0.144	0.35	Prevent process from getting out of hand.
Long-Term Goal	4%	5.5%	These factors are intertwined. In order to keep short-term rates above the minimum, the long-term goal times minimum ratio should be above the minimum rate. The ratio goal is the expected ratio.
Ratio Goal	0.729	0.85	
Minimum Ratio	—	0.65	
Minimum Rate	—	3%	
Maximum Rate	—	25%	

These parameter settings produced the following results relative to inflation.

Projection period	30 years
Number of scenarios	100 (3000 curves)
Long-term rates less than inflation	89 (2.97%)
Short-term rates less than inflation	20 (0.67%)
Both less than inflation	20 (0.67%)

The short-term drift parameter controls how rapidly or smoothly the short-term ratio and rates will change and also affects the proportion of inverted curves. The higher the drift factor, the greater the proportion of inverted curves and the more violent the yearly changes and hence more stickiness at 3 percent.

Professor Mereu's method for determining intermediate rates produces positively shaped yield curves (nondecreasing), inverted curves (which contain a small bump and are not monotonically decreasing), and some bowed curves (where the curve is nearly level). These shapes are very nice; however, it would be desirable to have 20-year rates. Because Professor Mereu commented in the discussion on Mr. Jetton's paper "Interest Rate Scenarios" [*TSA* XL (1988): 423–39], what would be his reaction to solving Mr. Jetton's formula for the 10-year rate in terms of one-year and 20-year rates and for the 20-year rate in terms of 10-year and one-year rates?

At the risk of belaboring the obvious, cash-flow projections should be done separately for each class of assets or liabilities with different payment patterns or put and call characteristics, and then summed. Within classes more accurate parameters could be determined because of a greater degree of homogeneity in behavior. Taken to its logical extreme, the most accurate (and time-consuming) projections would be done on a seriatim basis.

Duration Calculations

Professor Mereu mentions and calculates Macauley duration and convexity but does not discuss their use or significance. Macauley duration is used as a proxy for the percentage change in the present value for a given change in interest rates. When the durations of the assets and liabilities are matched, their present value then would change by the same amount, "immunizing" the company against changes in interest rates. As such, the Macauley duration is used as a proxy for the normalized first derivative of present value with respect to interest. Convexity as usually used is a normalized proxy for the second derivative of present value with respect to interest. However, they are appropriate proxies only in the case that the cash flows are not interest-rate-sensitive.

Note that

$$PV = \sum_t cf_t \, (1 + i)^{-t}$$

and thus

$$\frac{dPV}{di} = \sum_t \left[\left(\frac{dcf_t}{di} \right) (1 + i)^{-t} + (-t) \, cf_t \, (1 + i)^{-(t+1)} \right]$$

by the product rule; that is,

$$\frac{dPV}{di} = \sum_t \left(\frac{dcf_t}{di}\right)(1 + i)^{-t} - (1 + i)^{-1} \sum_t t\, cf_t\, (1 + i)^{-t}$$

and the usual definition of Macauley duration is

$$\sum_t t\, cf_t\, (1 + i)^{-t}$$

which is a proxy for the

$$\frac{dPV}{di}\, PV$$

only if the first term is 0, which implies

$$\frac{dcf_t}{di} = 0$$

which is to say that the cash flows are not themselves interest-rate-sensitive.

Option pricing models calculate duration and convexity as normalized proxies for first and second derivatives by evaluating the present value at three interest rates, i_o, $i_o + \epsilon$, and $i_0 - \epsilon$ for some fixed, small ϵ such as 0.50. They use approximations to the two-term Taylor series expansion for PV at i_o, assuming that the remainder term is small enough to ignore. After solving the resulting 2 by 2 system of equations, the results are normalized.

[Taylor series $PV(i_o + x)$

$$= PV(i_o) + PV'(i_o)\, x + (1/2)PV''(i_o)x^2 + (1/6)PV'''\, (\xi)\, x^3]$$

where ξ is between i_o and $i_o + x$.

In the case in which the cash flows are not interest-rate-sensitive, both techniques give the same results (provided that ϵ is sufficiently small). However, in the case in which the cash flows are interest-rate-sensitive, actuaries may be deriving a false sense of security from having Macauley durations matched. Convexities in particular appear to differ widely between the two methods.

Professor Mereu's investment technique is to minimize negative cash flows at the earliest future time and is not an immunizing strategy. It would be helpful to know how both techniques compare with respect to the C-3 reserve requirement and perhaps also with respect to return on investment.

In conclusion, Professor Mereu has presented us with a basic model to determine C-3 reserve requirements, which can be customized to fit the needs of many actuaries.

(AUTHOR'S REVIEW OF DISCUSSION)

JOHN A. MEREU:

I thank Frank Alpert and Sarah Christiansen very much for their excellent discussions, which I believe greatly enhance my paper.

Both of the discussants have constructed and tested the model discussed in the paper.

Mr. Alpert, an actuary experienced in the asset management field, has made a number of good observations on alternate ways of presenting results and recognition of other risks. He also has suggested culling out unreasonable interest scenarios.

He has carried out some interesting experiments with the model. He notes that there may still be a significant amount of C-3 risk, even if initial cash flows are matched, arising from the presence of call and put options.

He also notes that if the C-3 requirement is added to the initial assets, a mysterious shortfall can still develop if the investment strategy for such excess asset is the same as for the other assets. The model essentially assumes that such excess assets are invested in a long-term instrument maturing at the termination of the contract.

Dr. Christiansen has suggested a number of ways for improving the interest rate model results. By adjusting the driving parameters, she has reduced the number of unreasonable scenarios that are generated.

I agree with her suggestion that a realistic extension of the model beyond 10 years would be desirable. I did not have ready access to the history of 20-year rates. The history should be analyzed to determine how 20-year rates compare to 10-year rates and 3-month rates.

In conclusion, I am happy to have provided a framework in which further improvements for measuring the C-3 risk can be developed.