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Using Pedagogy to Improve Learning and Instruction

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This article presents concrete methods facilitating retention and understanding, as well as satisfaction with—and efficiency of—the learning experience. At first blush, it appears relevant to instructors for both preliminary and fellowship exams. But it is also relevant to candidates who engage in self-study. Finally, it is relevant to professional actuaries learning new material as they drive through their career path.

PEDAGOGY THEORY—EXECUTIVE FUNCTION

Most candidates are familiar with the Marzano pedagogical hierarchy,[1] according to which all subject matter is classified into one of four levels: (1) knowledge retrieval, (2) comprehension, (3) analysis and (4) knowledge utilization. Each successive level is more challenging than those that precede it. Each level is in turn explained by sublevels; for example, material is classified in the analysis level if it involves classification, generalization, specification, error correction or matching.

Recently, after reviewing the hierarchies of Bloom, Marzano, Gagne, Van Hiele, Anderson and others, I found a very simple unification of these theories that is much easier to use and implement: a pedagogic activity is higher level if it involves two or more distinct parts of the mind.[2] Psychologists use the term *executive function* (EF) to refer to that part of the mind that is used when two or more other mind components are simultaneously active.

The following examples illustrate the variety of applications of EF to learning.

Example 1: The Rule of Four

Deborah Hughes-Hallet initiated a calculus reform movement based on the rule of four.[3] This rule says that every calculus problem, concept and illustration should involve four distinct brain activities: verbal, algebraic-formal, geometric-visual and computational. For example, it is not enough to teach the first derivative test to locate extrema, one must also teach how to recognize extrema in both a graph and computational table, as well as learn the verbal cues requiring extrema for their solution. Here, superior pedagogy is achieved through the use of four distinct brain areas requiring EF.

Example 2: Multicomponent Problems

My syllabus always declares that all problems will be multicomponent; there will be no plug-in or drill problems. For example, instead of the drill problem, How much does $1,000 accumulate to in 3 years at an annual effective rate of 3%? I instead may use the following:

> $1,000 is deposited in an account earning 3% for 3 years; the accumulated value is deposited in another account earning 4% for 4 years. Calculate the actuarial equivalent level effective rate that would allow $1,000 upon deposit to accumulate in 7 years to the same amount accumulated in the 4% account.

Here, superior pedagogy is achieved through a multicomponent problem requiring separate brain areas, or EF, for the two (or more) subproblems to be solved.[4]

Example 3: The Trail Making Test

Figure 1 (see page 17) contains miniature versions of the Part A and Part B tests of the trail making test (the actual test uses 25 items versus the six shown in the figure).[5] An examiner presents a blank test to an examinee. The test has two parts, as shown. The examiner then provides a pencil and instructs the examinee to connect the numbers and letters available so as to create a sequential trail. Figure 2 shows a completed trail making test.

Figure 1
A Blank Trail Making Test Given to an Examinee Who Must Create Trails

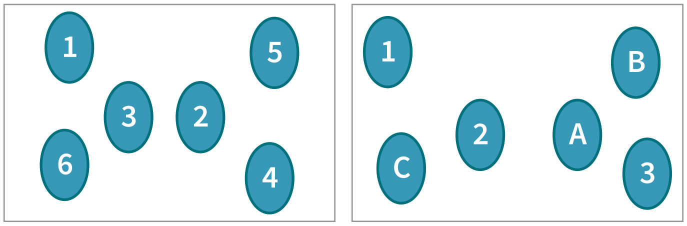
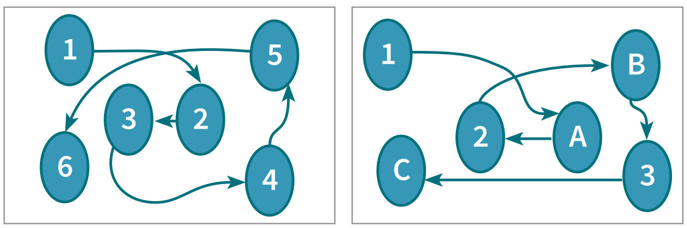


Figure 2
The Completed Trail Making Test



In administering the test, the examiner first asks the examinee to create a trail for the Part A test and times (e.g., in seconds) how long this takes. This process is repeated for the Part B test. The examiner assesses the examinee by subtracting the time needed to complete Part A from the time needed to complete Part B.

Although all people can easily complete the two tests (i.e., create the trails), the Part B test uses two parts of the brain (the part for letters and the part for numbers) while the Part A test only uses one. Hence, the Part B test always takes longer than the Part A test. Psychologists and neurologists use this simple-appearing test to diagnose brain injury (e.g., after a stroke or car accident) and to assess the chances of recovery. I often use this test to show that simple improvements, such as adding another brain area (letters and numbers), can actually have significant impact although they may appear inconsequential.

We continue the illustrative examples with material from the preliminary actuarial exams.[6] The next two examples demonstrate how continually approaching all subject matter with multiple brain areas, such as the rule of four, facilitates discovery of the most elegant approach. In the next three examples, we replace traditional algebraic proofs with punchy, crisp, instant proofs that do not require tedious manipulations.

Example 4: Achieving Simplicity Using a Geometric Approach

Table 1 presents a purely geometric argument proving both the formula for an n-year annuity immediate certain and the geometric series sum formula. The interpretation of Table 1 should be clear. An investor deposits \$1 at time $t = 0$ in a bank account earning at effective rate i and withdraws that \$1 at time $t = n$. The present value (PV) of the investor's transactions are $1 - v^n$. The bank, seeing the \$1 in the account, deposits an amount \$$i$ of interest at the end of every period. We can calculate the PV of the bank's transactions in two ways: (1) It is the sum of the discounted values of each interest deposit; and (2) since it is a sequence of end-of-period payments of i, definitionally, it is an annuity of amount i. A no-arbitrage argument requires the investor's and bank's PV to be equal, instantly leading to a non-algebraic proof of the formula for the annuity immediate and sum of geometric series.

Table 1
Activity of an Investor and a Bank in an Account Earning Effective Rate i

Time $t =$	0	1	2	…	n	Present Value
Investor	1				−1	$1 - v^n$
Bank		i	i		i	$i(v + v^2 + \ldots + v^n) = i a_{\overline{n}\rvert i}$

Example 5: Live Application of the Theory in This Article

Many of the principles of this article (e.g., the rule of four, the idea of addressing multiple modalities when teaching) are known. One goal of this article is to encourage readers to take these familiar ideas and apply them to current textbooks, handouts and monographs to achieve improvements in pedagogical approach.

As an exercise, the reader is invited to apply the same proof method presented in Table 1 to the Society of Actuaries (SOA) study note regarding interest rate swaps,[7] with a goal of obtaining an elegant simple proof that the fixed payment of an m-year deferred n-year swap is given by $(P_m - P_{m+n})/(P_{m+1} + P_{m+2} + \ldots + P_{m+n})$, where P_k is the price of a 0-coupon bond with maturity 1 at time k.

Example 6: Achieving Simplicity Using a Calculator Approach

In the following example, the use of a calculator approach—versus an algebraic approach—provides an elegant, punchy and simple solution to a traditional loan-refinancing problem.

> A borrower takes out a 15-year loan for \$400,000, with level end-of-month payments, at an annual nominal interest rate of 9% convertible monthly. Immediately after the 36th payment, the borrower decides to refinance the loan at an annual nominal interest rate of j, convertible monthly. The remaining term of the loan is

Table 2
Solution to the Loan Problem Using the BA II Plus TV Line

N	I	PV	PMT	FV	Comment
15*12	9/12	–4	CPT	0	Original loan
12*12	Keep	CPT	Keep	Keep	OLB_{36}
Keep	CPT	Keep	Last row – 0.0040988	Keep	Refinanced loan

Note: All numbers in the problem were divided by 100,000. OLB = outstanding loan balance.

> kept at twelve years, and level payments continue to be made at the end of the month. However, each payment is now $409.88 lower than each payment from the original loan. Calculate *j*.[8]

The traditional approach to solving this problem would be to write down equations and then calculate (perhaps manually). By using the BA II Plus Time Value (TV) line, an elegant solution is accomplished with only 10 keystrokes, as shown in Table 2.

The interpretation of Table 2 should be clear. PV, N, I, PV, and FV represent the price, number of periods, effective annual rate, periodic payment and any extra one-time terminal payment, respectively. The first row of Table 2 corresponds to the more familiar $4 = Ra_{\overline{180}|0.75\%}$. A feeling of greater familiarity with equations versus the TV table reflects a teaching style emphasizing algebraic over other modalities of solution. The TV table is well suited for a quick solution of all loan problems.

PEDAGOGY THEORY—GOAL SETTING

In my review and unification of the educational hierarchies, EF is one of four pillars of pedagogy, two of which deal with subject matter content. The EF pillar corresponding to the rule of four has already been discussed. The other pillar dealing with subject-matter content is goal setting (GS) referring to the best method to subdivide complex problems into a sequence of subgoals. Good GS requires three attributes. Subgoals should be (1) clear and specific, (2) achievable in a reasonable amount of time, and (3) challenging. The literature speaks about the GS paradox: although increasing challenge seems to delay achievable timely, studies show that such increases actually increase learning and performance.[9]

Readers who have taken or taught preliminary exams will recognize the following examples as difficult for candidates. These problems are difficult precisely because candidates do not know where to begin or what to do. We can succinctly formulate this difficulty as follows: while candidates have been taught subject matter, they have not been taught GS methods for these types of problems. In the examples that follow, we will only describe the high-level GS steps. Indeed, presenting formulas would not be challenging and hence not meet GS criteria. It is precisely by giving specific but high-level subgoals that learning increases.

Example 7: Multi-Rate and Payment Problems

A standard way to make a problem with a "plug-in" solution challenging is to replace parameters with pairs of parameters (operating at different times). For example, in interest and mortality theory a single interest rate can be replaced by two interest rates (operating at different times); a single payment can be replaced by two payments (operating at different times). Candidates who memorize formulae can initially be bewildered by such a problem, not knowing how to begin a solution.

> Calculate the present value of a loan paid back by four end-of-year payments of 10 following by three end-of-year payments of 15. Assume the annual effective interest rates are 1.5% for the first two years, 1% for the next three years and 2% for the last two years.

Such multi-interest, multi-payment problems are common in several preliminary exams. When I give such problems for homework, I will simply give a high-level tip of *how* to create subgoals: "Break the problem into subproblems, each of which has one: (1) interest rate, (2) cash flow (scheme), and (3) money growth method." This tip immediately suggests breaking up the problem into computing the PV of four loans: (1) a two-year loan of 10 at effective rate 1.5%, (2) a two-year loan of 10 at 1%, (3) a one-year loan of 15 at 1%, and (4) a two-year loan of 15 at 2%.The PV of the problem loan is then the sum of the discounted values of these four loans.

Example 8: Reinvestment Problems

Reinvestment problems naturally have several components, each with different parameters. The candidate can typically solve any particular component but fails to solve the entire problem precisely because of a lack of organizational tools to properly set subgoals.

Table 3
A Complete Set of Four Methods for Calculating BV and OLB

Method Name	Description	When Used
Prospective	PV future payments	If you know *n* and *i*
Retrospective	CV loan – AV payments	If you know *P* and *i*
BV_1–BV_2	Buy at BV_1, receive coupons and sell at BV_2	If you don't know *n* or *P*
Spreadsheet method	$I_t = i * OLB_{t-1}$, $R = I + P$, $OLB_{t-1} - P_t = OLB_t$	Line by line

> An amount P is paid by an investor for a 10-year, \$10 coupon bond, with 1,000 redemption value yielding 4.5%. As each coupon is received, half is deposited into an account earning 5%, while the remainder is pocketed as profits. Calculate the overall yield to the investor over the 10 years.

Here is the GS for this problem: (1) Create a separate problem for each distinct interest rate; (2) identify each cash flow in the problem as an inflow, outflow or intermediate flow (e.g., the \$5 half-coupons initially deposited in the 5% account are intermediate cashflows since it is the accumulated value of this account at time $t = 10$, which is the inflow to the investor); and (3) create a summary timeline with all inflow and outflow cash flows. The solution of the equation of value for this summary timeline provides the solution for the reinvestment problem. This tip applies to all reinvestment problems which, as noted earlier, are typically difficult because students lack proper GS tools.

Example 9: A Fellowship Approach (Pros and Cons of Multiple Methods)

Certain topics in the preliminary examination syllabi are challenging because multiple methods exist. Doing many examples with each method does not by itself achieve pedagogical mastery. Rather, a proper approach is to use the GS methods of fellowship-exams: What are the possible methods? What are the pros and cons? Which method is best in this problem? Such a fellowship-examination approach is challenging, an important prerequisite for proper GS.

> A five-year bond with quarterly coupon payments of 2.5 has book values (BV) of 970.95 and 980.44 at times $t = 2$ and $t = 3$, respectively. Calculate i, P, C, and r.

As indicated, solving this problem by formulas, although clear and timely achievable, would not be challenging and hence would not meet GS criteria. In solving this problem, I use a fellowship-exam approach: (1) Determine the methods of calculating outstanding balance, (2) identify the pros and cons of each method, (3) decide which method is appropriate to this problem, and (4) calculate the values. Answers to parts 1 and 2 are presented in Table 3.

The prospective and retrospective methods are not useful in this problem because neither n, P, nor i are known. Therefore, we must use the BV_1–BV_2 approach. The following equations can be used to solve for i and P: $970.95 = 2.5a_{\overline{4}|i} + 980.44v_i^4$; $P = 2.5a_{\overline{8}|i} + 970.95v_i^8$. Similar equations can then be used to solve for C and r. The use of a fellowship-exam approach (which always emphasizes proper GS) is useful in several otherwise difficult problem domains.

A pedagogic activity is higher level if it involves two or more distinct parts of the mind.

USING EF AND GS TOGETHER

EF and GS can and should be combined. We can illustrate this with the topic of distributions in the Probability exam syllabus. EF suggests summarizing this information in a rectangular database array format. Each row in the database would contain one distribution. The first three sets of columns, which frequently do occur in this database form in textbooks, contain: (A) distribution name and parameters; (B) associated functions (f, F, P and s); and (C) statistics (moments, central moments, percentiles, moments with caps and deductibles). Notice that, for example, (B) is a *set* of four columns. We would add four more sets of columns: (D) sums of random variables (RV); (E) relationships between distributions; (F) generating functions (PGF, MGF and products); (G) computational implications (e.g., values for E[X] indicate evaluation of integrals that may have independent value in certain problems). The database is a *visual* aid for dealing with formal algebraic relationships. The next example illustrates this approach.

Example 10: Database Approach to Distributions

An actuary determines that the claim size for a certain class of accidents is a random variable, X, with moment-generating function $M_X(t) = (1 - 2{,}500t)^{-4}$ Calculate the standard deviation of the claim size for this class of accidents.[10]

We show here how to use the visual aid of the database to establish GS steps:

1. Determine column sets relevant to the problem solution. *Solution*: The problem gives an MGF corresponding to category (F) of columns.

2. Look through the (F) columns to find a functional form similar to $1/(1 - 2{,}500t)^4$. *Solution*: Without the exponent of 4, the function $1/(1 - 2{,}500t)$ resembles the exponential distribution MGF. (Note: We assume the database is incomplete; otherwise, we could directly look up the gamma distribution MGF.)

3. To deal with the exponent of 4, look through the individual columns in the (F) category of generating functions. *Solution*: If the database is set up properly, one column would give the product formula for the MGF of sums of random variables.

4. Go back to the exponential row and look up either the category (D) columns (sums of random variables) or the category (E) columns (relationships between distributions) to deal with the sum. *Solution*: A sum of four identically distributed and independent exponential random variables is gamma distributed. We also obtain related parameters.

5. Go to the row with the gamma distribution to the category (C) columns. *Result*: Calculate the variance, and hence the standard deviation, of the gamma distribution to answer the question.

We again emphasize that the visual aid, as well as the GS steps of tracing a path in rows and columns, is a high-level description that still requires candidate work (so it is challenging) but is clear and achievable timely. Hence, this tip fulfills GS criteria and helps students.

CONCLUSION

This article has shown how to use EF and GS with specific illustrative examples. Challenging problems should always address multiple brain areas. Good GS should establish clear subgoals, each one achievable timely, yet challenging the candidate with more work than just plugging in. GS can be assisted by using alternate brain areas. We believe these techniques, when properly applied, can enrich the learning and instruction experience and performance. We encourage readers to apply these principles to the material they learn and teach. ■

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ENDNOTES

1 Robert J. Marzano, *Designing a New Taxonomy of Educational Objectives* (Thousand Oaks, CA: Corwin Press, 2000).

2 Russell J. Hendel, "Leadership for Improving Student Success through Higher Cognitive Instruction," in *Comprehensive Problem-Solving and Skill Development for Next-Generation Leaders*, ed. Ronald A. Styron Jr. and Jennifer Styron (Hershey, PA: IGI Global, 2017), 230–54.

3 The rule of four was originally the rule of three. See, for example, Oliver Knill, "On the Harvard Consortium Calculus," last updated February 14, 2009, *http://www.math.harvard.edu/~knill/pedagogy/harvardcalculus/*.

4 Russell J. Hendel, "Dr. Hendel's FM Lecture Notes," 2017 and 2019, *http://www.rashiyomi.com/math/DrHendelsFMLectureNotes.pdf*.

5 "Trail Making Test (TMT) Parts A & B," *http://apps.usd.edu/coglab/schieber/psyc423/pdf/IowaTrailMaking.pdf*.

6 Throughout this presentation I assume the reader is familiar with (1) the material on the Probability and Financial Mathematics preliminary SOA examinations and (2) the BA II plus Texas Instrument (TI) calculator. The SOA website (*www.soa.org*) has both syllabi and standard textbooks. The TI website presents the BA II plus guidebook (*https://education.ti.com/en/guidebook/details/en/ADF11FB65B284B6195B0A7E9502784BA/baiiplus*).

7 Jeffrey Beckley, "Interest Rate Swaps," Education and Examination Committee of the Society of Actuaries Financial Mathematics Study Note, 2017, *https://www.soa.org/globalassets/assets/Files/Edu/2016/edu-2016-fm-25-17-interest-rate-swaps.pdf*.

8 Society of Actuaries, "Exam FM Sample Questions," Problem #75, *https://www.soa.org/globalassets/assets/Files/Edu/2017/exam-fm-sample-questions.pdf*.

9 Supra note 2.

10 Society of Actuaries, "Exam P Sample Questions," Problem #57, *https://www.soa.org/globalassets/assets/Files/Edu/edu-exam-p-sample-quest.pdf*.