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DURATION AND CONVEXITY FOR PENSION LIABILITIES

By Martin McCaulay

Martin McCaulay, FSA, EA, MAAA, FCA, is an actuary at the U.S. Department of Energy in Washington, D.C. He can be reached at *Martin.McCaulay@hq.doe.gov.*

Formulas for duration are good approximations for pension liabilities with small changes in interest rates. Considering the volatility in interest rates, it is more accurate to use duration with a convexity adjustment. In most cases, the convexity adjustment results in a lower duration for rate increases and a greater duration for rate decreases.

ESTIMATING CHANGES IN LIABILITIES

Similar to bonds, pension liabilities have an inverse relationship to interest rates. An interest rate decrease will increase liabilities, and an interest rate increase will decrease liabilities. The amount of the increase or decrease can be estimated using the duration of the liabilities. To apply the formula for duration to pension liabilities, for every 100 basis point (bp) change in interest rates, the liability changes by duration divided by 100 in the opposite direction.

The typical pension plan has a duration of about 15. Considering convexity, the typical pension plan has a duration that is less than 15 for interest rate increases and greater than 15 for interest rate decreases. The duration for active participants is typically longer than the duration for retired participants. The duration for the Normal Cost (NC) is typically longer than the duration for the Actuarial Accrued Liability (AAL).

MACAULAY DURATION

The original formula for duration that was developed in the year 1938 by Frederick Robertson Macaulay is a measure of a bond's weighted average cash flows, using yield (y), the time period (t), the number of time periods (n), the annual coupon payment (C), the maturity value (M), and the purchase price (P).

$$\text{Macaulay Duration} = \frac{\sum_{t=1}^{n} \frac{tC}{(1+y)^t} + \frac{nM}{(1+y)^n}}{P}$$

MODIFIED DURATION

Modified Duration is a measure of the sensitivity of a bond's price to interest rate movements. Modified Duration is the first derivative of how the price of a bond changes in response to interest rate changes. Taking the derivative and adjusting for the number of payments per year simplifies to the following relationship between Macaulay Duration with annual coupons and Modified Duration.

$$\text{Modified Duration} = \frac{\textit{Macaulay Duration}}{\left(1+ \frac{y}{\textit{Payments per Year}}\right)}$$

EFFECTIVE DURATION

Effective Duration is used to price bonds with options. Effective Duration approximates the slope of a bond's value as a function of interest rate movements taking the difference in the bond's value (V) for changes in the interest rate (i) by an equal amount (x = δi) in both directions, and dividing by twice the original value times the interest rate change in each direction.

$$\text{Effective Duration} = \frac{V_{i-x} - V_{i+x}}{(2)(V_i)(x)}$$

Pension liability duration is measured using the formula for Effective Duration, substituting the liabilities (L) for the bond's value (V).

$$\text{Duration} = \frac{L_{i-x} - L_{i+x}}{(2)(L_i)(x)}$$

Duration Example

Interest Rate	Liability
4%	$1,160,000
5%	$1,000,000
6%	$860,000

$$\text{Duration} = \frac{\$1{,}160{,}000 - \$860{,}000}{(2)(\$1{,}000{,}000)(0.01)} = \mathbf{15}$$

To apply the formula for duration to pension liabilities, for every 100 basis point (bp) change in interest rates, the liability changes by about 15% in the opposite direction. A 100 bp increase results in a new liability equal to 85% of the original liability. A 100 bp decrease results in a new liability equal to 115% of the original liability.

The liability change should be based on a compounded change rather than a simple change. Using compounding with a duration of 15, a 50 bp increase results in a new liability equal to 92.20% of the original liability, based on the square root of 85%. A 50 bp decrease results in a new liability equal to 107.24% of the original liability, based on the square root of 115%.

Estimates of duration also hold for yields outside of the corridor used to calculate the duration. Using the example above, a 200 bp increase results in a new liability equal to 72.25% of the original liability, based on the square of 85%. A 200 bp decrease results in a new liability equal to 132.25% of the original liability, based on the square of 115%.

CONVEXITY

The traditional formula for pension duration does not consider convexity. Convexity is equal to the second derivative of the change in liabilities for changes in cash flows. Interest rate decreases generally cause greater changes in liabilities than increases. Duration with a convexity adjustment can be used to provide a better estimate of the change in liability when there is significant volatility. Pension liability convexity can be approximated using a formula with the same variables as the formula for duration.

$$\text{Convexity} = \frac{L_{i-x} + L_{i+x} - (2)(L_i)}{(2)(L_i)(x^2)}$$

To include the convexity adjustment, the duration is adjusted by the convexity times the interest rate change.

Convexity Example

(using the liabilities above in millions)

$$\text{Convexity} = \frac{\$1.16 + \$0.86 - (2)\ (\$1.00)}{(2)(\$1.00)(0.01)^2} = \mathbf{100}$$

Illustrative Examples of Pension Duration with the Convexity Adjustment

The following formulas illustrate how making the convexity adjustment to duration results in a lower duration for rate increases and a greater duration for rate decreases.

$$\text{Duration} - \text{Convexity} = \frac{L_{i-x} - L_{i+x}}{(2)(L_i)(x)} - \left(\frac{L_{i-x} + L_{i+x} - 2(L_i)}{(2)(L_i)(x^2)}\right)(x)$$

$$= \frac{L_{i-x} - L_{i+x} - L_{i-x} - L_{i+x} + 2(L_i)}{(2)(L_i)(x)} = \frac{2(L_i) - 2(L_{i+x})}{(2)(L_i)(x)}$$

$$= \frac{(L_i) - (L_{i+x})}{(L_i)(x)} = \frac{1 - (\frac{L_{i+x}}{L_i})}{x}$$

$$\text{Duration} + \text{Convexity} = \frac{L_{i-x} - L_{i+x}}{(2)(L_i)(x)} + \left(\frac{L_{i-x} + L_{i+x} - 2(L_i)}{(2)(L_i)(x^2)}\right)(x)$$

$$= \frac{L_{i-x} - L_{i+x} + L_{i-x} + L_{i+x} - (2)(L_i)}{(2)(L_i)(x)} = \frac{(2)(L_{i-x}) - (2)(L_i)}{(2)(L_i)(x)}$$

$$= \frac{(L_{i-x}) - (L_i)}{(L_i)(x)} = \frac{(\frac{L_{i-x}}{L_i}) - 1}{x}$$

Duration with Convexity Adjustment Example

(using the liabilities above in millions)

To illustrate how the formula for duration with the convexity adjustment might be applied to pension liabilities, with a duration of 15 and a convexity of 100, the duration with the convexity adjustment would equal 15 plus or minus 100 times 1%. The adjusted durations are 16 and 14. In this illustrative example, for every 100 basis point (bp) decrease in interest rates, the AAL increases by 15% plus 100 times 1% squared, or a total of 16%. For every 100 basis point (bp) increase in interest rates, the AAL decreases by 15% minus 100 times 1% squared, or a total of 14%.

Duration for Rate Decreases = $$\frac{\left(\frac{L_{i-x}}{L_i}\right) - 1}{x}$$

Duration for Rate Decreases =

$$\frac{\left(\frac{L_{4\%}}{L_{5\%}}\right) - 1}{1\%} = \frac{\left(\frac{\$1.16}{\$1.00}\right) - 1}{1\%} = 16$$

Duration for Rate Increases = $$\frac{1 - \left(\frac{L_{i+x}}{L_i}\right)}{x}$$

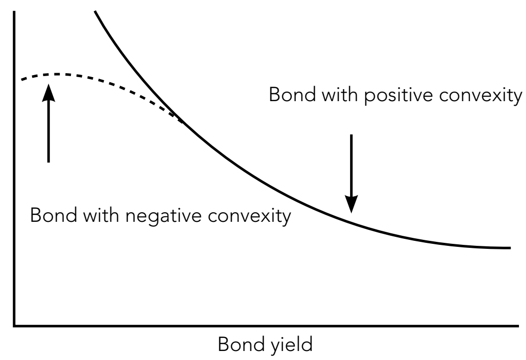
Duration for Rate Increases = $$\frac{1 - \left(\frac{L_{6\%}}{L_{5\%}}\right)}{1\%} = \frac{1 - \left(\frac{\$0.86}{\$1.00}\right)}{1\%} = 14$$

In the example, the increase of 16% in liabilities for the interest rate decrease from 5% to 4% is consistent with the increase from $1 million to $1.16 million in liabilities, and the decrease of 14% in liabilities for the interest rate increase from 5% to 6% is consistent with the decrease in liabilities from $1 million to $0.86 million. Using different durations for increases and decreases in rates improves the accuracy of estimates compared to using the same duration for increases and decreases.

NEGATIVE CONVEXITY

Figure 1: Graph of Negative Convexity

Negative convexity when interest rates fall



Note: This fugure is an Illustration only and is not intended to represent a specific mathematical relationship. Source: Vanguard.

The price/yield relationship for most bonds is convex. If the graph is concave, the relationship has negative convexity, as shown in Figure 2 above. Most callable bonds, mortgage backed securities (MBS), and asset backed securities have negative convexity at low rates due to the imbedded options. When rates decrease, the price will not increase as rapidly as non-callable bonds. At high interest rates, bonds with call options have positive convexity similar to bonds without call options.

KEY RATE DURATION

The duration calculations presented here are useful for parallel yield curve shifts and interest rate changes. Key rate duration considers the sensitivity of a liability's movement to different parts of the yield curve. When different rates move in different ways, key rate duration is more accurate. Key rate duration calculations require building a yield curve.

SUMMARY

Duration and convexity provide a risk metric for pension plan sponsors. The formula for Effective Duration can be used to estimate the value of pension liabilities at different interest rates. A convexity adjustment should be applied to reflect the fact that the pension liability increase for a decrease in interest rates is greater than the pension liability decrease for an increase in interest rates. There will be a lower duration for rate increases and a greater duration for rate decreases. ■

On the Research Front

NEW RESEARCH: EMBEDDED OPTIONS

Embedded options in pension plans play an increasing role in estimating pension liability values. With the credit interest rate floor of a cash balance plan as a model, this research, authored by Kailan Shang, Jen-Chieh Huang, and Hua Su, uses three valuation and risk analysis approaches to explore the existing techniques to value embedded options on an economic basis. In addition, several tools were built with comprehensive structures and documented implementation processes. The tools provide functions like economic assumption calibration, economic scenario generation (ESG), scenario validation and option value calculations using the three approaches.