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MARKET VALUE AND DURATION ESTIMATES OF INTEREST SENSITIVE LIFE CONTRACTS

Abstract

This article analyzes a fixed premium interest sensitive life contract with the objectives of: 1) developing an approach to estimating its market value and 2) assessing various notions of its duration. The results of these analyses have implications for market value accounting of life insurance liabilities and for life company portfolio management.

Thomas J. Merfeld
Vice President Asset/Liability Management
CUNA Mutual Group
Madison, WI 53701
(608) 231-8460*

*This article was written while the author was Chief Financial Officer of Savers Life Insurance Company of America. The author acknowledges useful imput from Mr. Dale R. Oldham, FSA.

Life insurance companies have not traditionally assessed the market value of their liabilities. Life actuaries compute statutory reserves under strictly defined rules. Neither state regulatory bodies nor the FASB have required or allowed market value liability reporting.

Nor have these companies needed to assess the market risk of their liabilities. Market values are sensitive to exogenous factors such as changing interest rates. The distribution of their sensitivities is a measure of risk. For much of this century, financial markets enjoyed enough stability that risk assessments were not necessary. Nor did many investment officers have the requisite liability understanding to raise the right questions. Finally, life products were relatively simple and margins relatively wide; insolvencies did not generally occur for lack of liability market risk measurements.

But needs have changed. Congressional bodies are applying pressure on state regulators to tighten statutory reporting requirements. Financial reporting under GAAP appears to be moving in a market value direction. And complex new products, funding complex new securities, are accentuating the effects of the heightened financial market volatility of the last 15 years. The market value of life company surplus has become volatile enough that effective insolvencies can result. These insolvencies can be masked by traditional accounting standards but revealed by market value methods.

Life company financial management generally has not yet installed the technology necessary to measure the risk of surplus volatility. Although statutorily-defined cash flow testing procedures adequately reveal certain cases of financial ruin, they can also provide false indications of problems. Most valuation actuary certifications are designed to protect contract holders without regard to stockholders; as a consequence, they constitute another measure of solvency rather than of surplus volatility. And even internal portfolio managers remain too far removed from liabilities to apply established market value assessment and risk management procedures to liabilities.

Securities-type valuation and risk management methodology can be applied to a particularly complex type of life product -- the interest sensitive whole life contract ("ISWL"). Life companies have added these and similar universal life contracts with an aggregate face amount of approximately $1.5 trillion over the past decade, as reported in 1991 Life Insurance Fact Book. As a result, interest sensitive products represent a significant portion of life company obligations. It is not clear that either the literature or the industry has assessed the valuation or risk of this particular liability in a thorough way.

This article analyzes a fixed premium interest sensitive life contract with the objectives of: 1) developing an approach to estimating its market value and 2) assessing various notions of its duration. The results of these analyses have implications for market value accounting of life insurance liabilities and for life company portfolio management.

THE CONTRACT

Provisions

ISWL contracts have similarities to traditional whole life contracts. For a large pool of insureds, whole life products are structured to produce a level premium at any issue age. This premium can be regarded as the periodic contribution to a fund that, when accumulated at an implicit rate of interest, produces periodic death benefits and is exhausted at the end of the pool of lives. This condition holds after giving effect to loadings for coverage of company issuance and maintenance expenses and a competitive return to capital. An ISWL contract is initially structured with a level premium.

ISWL contracts differ from traditional whole life contracts in two relevant ways. First, although the contracted premium must be paid in order to avoid nonforfeiture status, ISWL premiums can have a more complex structure. Much of this complexity can be considered as discrete contract owner options to adjust the mix of premium paid and coverage amount within constraints imposed by the issuer. Obviously, the contract owner uses private information in exercising these options. The issuer can then respond by adjusting the cost of insurance assessment within contractual and legal constraints. In a yet more general type of permanent life contract, universal life (i.e. flexible premium interest sensitive life), the contract owner effectively has continuous options to adjust the premium/coverage mix. Application of game theory modeling in this regard could provide rich insights.

Second, in an ISWL contract, the accumulation interest rate is explicit and adjustable. Whatever rate structure the issuer may originally have used to price the premium structure, in fact the contract fund value is credited with a rate that resets periodically at the option of the issuer, subject to a floor.

ISWL contracts also provide for the opportunity for the contract owner to remove the accumulated fund value either by: 1) taking a loan (which continues to commit the policyowner to the premium schedule and can expose the loaned portion of the fund value to a lower crediting rate), 2) making a partial withdrawal or 3) canceling the entire contract (which typically exposes the policyowner to a schedule of surrender assessments). ISWL contracts also provide the standard array of nonforfeiture options, ignored in this analysis. See Appendix A for a summary of terms and in force assumptions of the ISWL contracts analyzed.

Cash Flow Components

Contract cash flows belong to four classes:

- Premium Inflow

- Surrender Benefit
- Death Benefit
- Servicing Expense and Commission

Projecting surrender benefit cash flows entails certain complexities. First, the industry comparable product crediting rate must be modeled. See Appendix B for a model of industry interest sensitive life crediting rates as a function of U.S. Treasury interest rates. Second, the company crediting strategy must be established. Third, a function of contract holder surrender behavior with respect to these differential rates must be integrated into the overall model.

Contract holder behavior is affected fundamentally by the spread between the rate earned by a particular contract and the rate provided by a similar contract issued by an industry competitor. It will also be relatively higher during the early stages of the contract. It can be volatile at critical points in the life of the contract, such as premium redetermination periods. Furthermore, it will be affected by critical ages, such as retirement, of the contract holder and by critical stages of the contract, such as the reduction or termination of surrender charges. The crediting rate structure that a company establishes is the result of myriad marketing, investment, legal and game theory considerations. It is not, as yet, well understood from a theoretical perspective. See Appendix A for a description of the surrender functions used in this analysis.

Surrender cash flows include new policy loans. Policy loans typically do not pay cash interest and, in any case, are usually not repaid. Instead, interest is capitalized, increasing the loan amount. New policy loans are modeled as a premature cash outflow from the fund. When the contract is extinguished by surrender, the surrender benefit is reduced by the amount of the loan; when it is extinguished by ultimate death of the insured, the death benefit is reduced by the amount of the loan. We assume future loan balances to be a constant proportion of contract cash values. As a consequence, in high interest rate scenarios, cash values increase quickly, loan balances capitalize quickly and cash paid upon death of the insured is reduced. So in high interest rate scenarios our modeling shifts cash flows to earlier periods and from the death benefit component to the surrender benefit component.

Cash Flow Generation

In estimating market values and durations, various interest rate scenarios are generated. Each scenario is associated with a yield curve. Under each scenario, for each projected month, premium cash flows into and benefit and expense cash flows out of the fund are generated for each contract remaining in force. In addition, the fund accumulates by capitalizing interest at the credited rate and is reduced by the cost of insurance and expense charges deducted for each policy in force. As described in Appendix B, industry and company crediting rates can be regarded as functions of medium term forward rates implied by the scenario yield curve. As described in Appendix A, contract surrenders are triggered, generating surrender cash flows. Finally, mortality cash flows are generated, further reducing the number of policies in force. The process is repeated for each month and each yield curve.

MARKET VALUE ESTIMATE

Let $cf_{c,t}$ equal the cash flow of component c at projection month t where c takes a value from 1 to 4 (premium, surrender benefit, death benefit, and expense, respectively). Let z_t equal the risk-free zero coupon rate at projection month t. Let Δz_k equal the shift in the zero coupon rate over "key" regions one through nine, stipulated to refer to the terms of on-the-run Treasury instruments and at all t proximately bounding the k^{th} Treasury instrument. See the discussion of analogous key rates in Ho [1990].

Now consider an analogy whereby the accumulating ISWL fund represents the proceeds from company debt issuance in the same way as an industrial firm might issue debt as part of the financing structure for a capital project. By way of similarity, in either case, the issuer is accumulating assets with the promise to repay the debt, with interest, at a future date. The issuing firm in the case of debt issuance and the life company in the case of contract issuance are charged a basis point risk premium by entities providing the funds. Issuers in both cases incur issuance costs and enjoy related tax advantages. By way of distinction, since the ISWL contract provides risk-reducing services to the provider of funds (the contract holders), some or all of such cost of funds is offset. That is, the insurance company is rewarded for providing risk intermediation services. Consider the aggregate of these costs and their offsets to be a cost of funds basis point spread to Treasury.

The spread can be estimated for a line of business or a block of policies at any time in its life. To accomplish this, solve for the spread that, when added to the respective risk-free zero coupon rates, discounts all future expected contract cash flows to the market value of funds that insureds willingly provide to the insurer. Immediately prior to issuance, such assets equal zero; after the initial premium, such assets equal the premium reduced by the commission and other acquisition costs incurred in issuance; at any time, such assets equal the sum of all net cash flows and their investment earnings at risk adjusted rates. Let s equal the spread; in this analysis, the resultant presumed spread is set to 20 basis points.

Finally, using the semi-annual coupon convention of the investment literature, let d equal the market price discounting vector such that:

A simple procedure then provides the market value of ISWL (mv).

$$d_t = 1 + \left(\frac{z_t + \Delta z_k + s}{2}\right)^{\frac{-t}{6}} \quad (1)$$

$$mv = \sum_{c=1}^{4} \sum_{t=1}^{\infty} (cf_{c,t})(d_t) \quad (2)$$

This analysis uses 480 months as a practical analog for infinity. Applying the methodology produces Table 1. By convention, since this market value measure is that of a liability, cash flows of the premium component have a negative sign.

Table 1
Base Market Value ISWL

Component	Market Value
Premium	(71,687,488)
Surrender Benefit	53,262,613
Death Benefit	34,190,301
Expense	7,852,738
Aggregate	23,618,164

A few points are worth noting about these values relative to various Appendix 1 values:

- the aggregate market value is only about 50 percent of the statutory reserve, reflecting the conservative nature of statutory accounting principles.
- the surrender benefit is even higher than the fund or cash

values due primarily to the inclusion of future premiums in this component.

- the death benefit is only about five percent of face amount, indicating the impact of withdrawals and the time value of money.

NOTIONS OF DURATION

In general, duration is a measure of the sensitivity of a vector of a cash flow to an interest rate change. Consider the following four concepts of duration.

- Spread: the percent sensitivity of market value to a shift in *s*. This is similar to Macaulay duration and modified duration which are measures of sensitivities to changes in yield for fixed and certain cash flows. It is useful as an average life index and as a measure of the capitalized value of a unit of higher cost of funds. Mathematically,

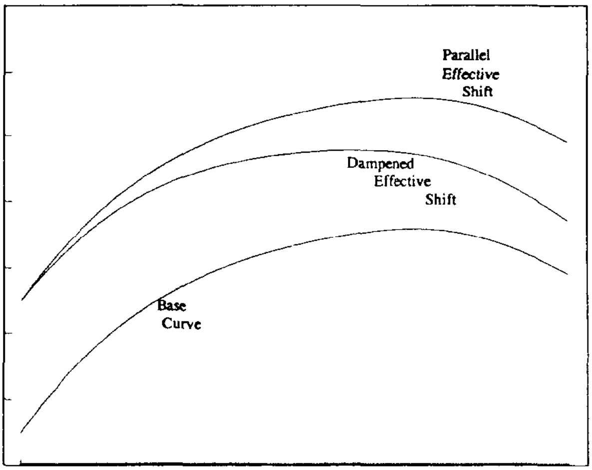
$$\frac{dmv}{ds}. \quad (3)$$

- Parallel Effective: the percent sensitivity of market value to a parallel shift in *z* simultaneously over all *k*. That is, Δz at all *k* are equal. This is the type of duration customarily used in asset and liability matching. Mathematically,

$$\frac{dmv}{dz}. \quad (4)$$

Graphically, see Graph 1.

GRAPH 1
Base and Shifted
Yield Curves



• Dampened the percent sensitivity of market value to a function that

Effective: non-parallel shift vector in z over all k: $\Delta z_k > \Delta z_{k+1}$.

Rates for months within a region are also shifted according to the non-parallel function. This is similar to effective duration, but recognizes that short rate volatility typically exceeds long rate volatility. In this analysis, the average shift over the region $k = 9$ is about

60 percent of the average shift over the region $k = 1$.

Mathematically,

$$\frac{dmv}{d\vec{z}}. \quad (5)$$

where: the vector z is produced according to the function described above.

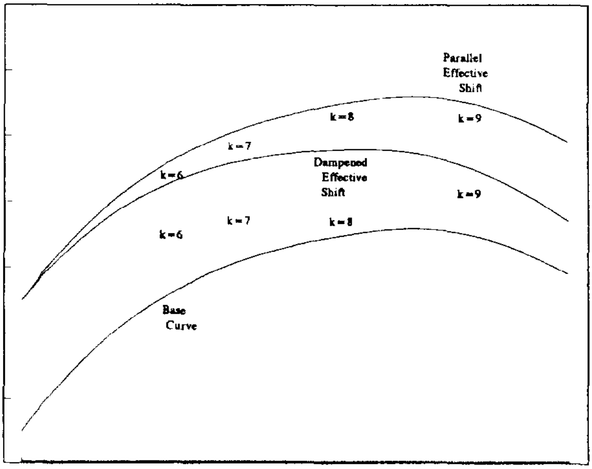
Graphically, see Graph 1.

- Key Region Effective: the percent sensitivity of market value to a set of shifts in z sequentially over each k. Measures under either of the following two subsets of this notion can be considered partial durations:
  - Parallel: similar to effective duration, but relates to the effect of parallel shifts by region of the curve. Mathematically,

$$\frac{dmv}{dz_k}. \quad (6)$$

Graphically, see Graph 2.

GRAPH 2
Base and Shifted
Yield Curves



•Dampened: similar to dampened effective duration, but relates to the effect of dampened shifts by region of the curve. Mathematically,

$$\frac{dmv}{d\vec{z}_k}. \quad (7)$$

Graphically, see Graph 2.

DURATION CALCULATIONS

Consider point estimates of these measures shown in Table 2.

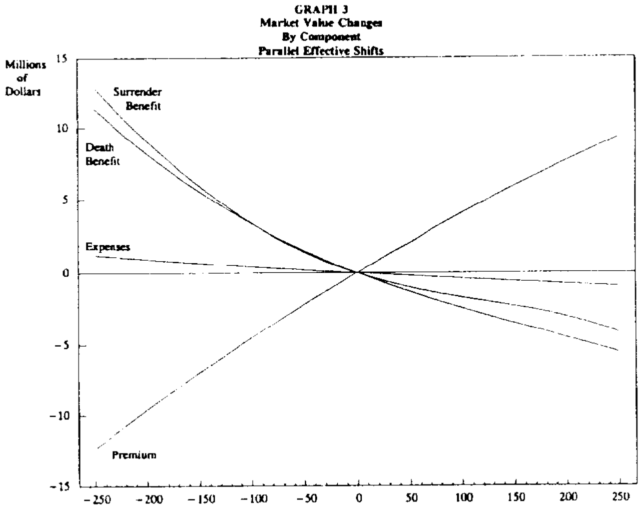
Table 2
Point Estimate Durations

Component	Spread	Parallel Effective	Dampened Effective	Dampened Key Region Effective over k = 1	2	3	4	5	6	7	8	9	Σ
Premium	(5.94)	(5.94)	(4.94)	(0.01)	(0.03)	(0.11)	(0.23)	(0.44)	(1.11)	(1.85)	(0.99)	(0.16)	(4.94)
Surrender Benefit	10.86	3.78	3.55	0.01	0.05	0.12	0.35	0.50	1.92	1.32	0.02	0.45	4.74
Death Benefit	14.15	7.93	6.51	0.00	0.02	0.06	0.17	0.29	1.37	3.45	2.13	0.21	7.71
Expense	5.49	5.49	4.59	0.01	0.04	0.12	0.24	0.44	1.08	1.58	0.88	0.19	4.59
Aggregate	28.79	3.82	3.95	0.00	0.05	0.05	0.42	0.37	3.31	2.89	0.34	1.75	9.18

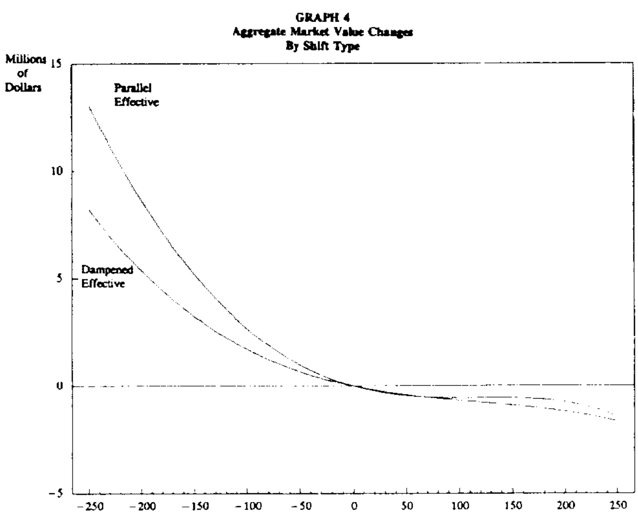
As is customary, these figures are scaled to a 100 basis point shift. Under dampened measures, figures are scaled to reflect an approximately 100 basis point shift over the region $k=1$. We show only the dampened category of key region measurements.

Note that premium duration figures are provided with respect to their effect on the value of the liability. For example, at a higher discount spread of 100 basis points, the market value of premium will become less negative, producing a higher overall liability. In this sense, premium has *a negative duration as well as a negative market* value.

Consider also the more intuitive market value change by component shown in Graph 3.



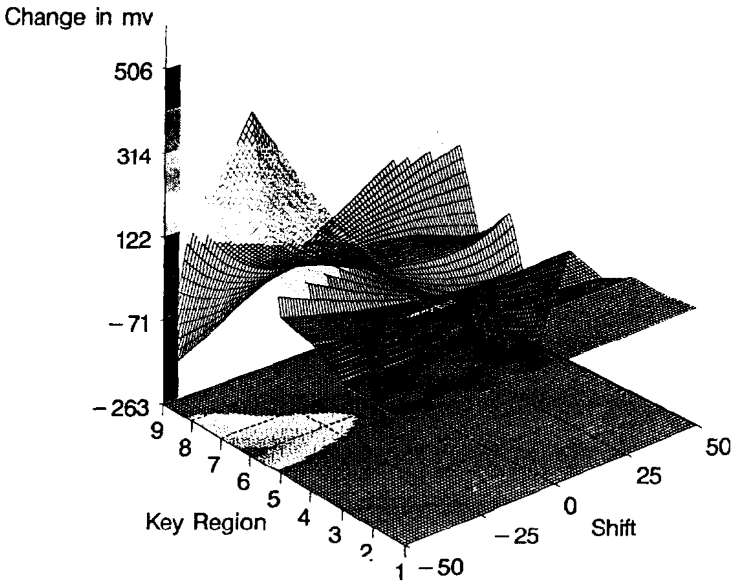
See also the aggregate market value changes by effective and dampened effective-type shifts shown in Graph 4.



Finally, see the surface in Graph 5.

ISWL Risk Surface

GRAPH 5



Note that each of the graphs shows a market value change from the base case resulting from the indicated shift.

DISCUSSION POINTS

- Estimating the cost of funds is a complex process. The rate credited to the fund accumulations explains less of such cost than analogous rates in depository institution or corporate bond environments. A simplistic comparison between a life company's asset earning rate and its crediting rate provides inadequate information with respect to its net spread. Such comparison, in a depository context, however, provides much more significant information.

- Clearly, however, market value approximations for complex insurance liabilities can be made. Using conventions to structure characteristics of the foregoing analysis, valuation actuaries could provide market value estimates.
- The death benefit component is highly convex due to the mere length of the cash flows and to the effect of policy loans. Our modeling treatment of policy loans causes the death benefit component to have greater absolute cash flows in lower interest rate environments, a combination sure to produce convexity. Obviously, liability convexity is not desirable.
- The surrender benefit component is also highly convex due primarily to the presence of the guaranteed rate in the contracts, which acts as an interest rate floor.
- The effective duration of the surrender benefit component, primarily a floating rate set of cash flows, is as long as it is due to the lagged nature of adjustments to crediting rates (see Appendix B).
- The surrender benefit component has a curious dampened effective profile over the regions $k = 7\text{-}9$. In these cases the market value of the component increases regardless of which direction rates are shocked. In using medium term forward

curves to project crediting rates, we affect cash flow during periods that are not affected by a change in discounting rates. Obviously, if we held cash flow fixed during the entire projection horizon, and increase z_k, then we can be certain that the present value of cash flow would fall. However, if we increase cash flow as a result of such higher rates, then we can predict the direction of the change in present value only by means of simulation. In this case, when z_k was increased, the cash flow effect dominated; when z_k was lowered, the discounting effect dominated. So the market value of the surrender benefit component increases with either shift.

- Financial managers need to decide whether to use the dampening notion of effective duration or whether to use the strictly parallel approach. The alternative methods can give significantly different results. Long-tailed cash benefits and the customary "front-loaded" premium structure of whole life combine to produce, in effect, a net asset at the beginning of contract time and a net liability at the end of such time. Obviously, the present value of this configuration is subject to yield curve twists such as are emphasized in the difference between effective and dampened effective durations.
- Spread duration, indicating average lives of cash flow components, are long with respect to the following

components:

- Surrender Benefits, since interest is capitalized, rather than paid currently, on the surviving contracts.
- Death Benefits, since, as the insureds of surviving contracts grow older, cash flow increases.
- Aggregate, since it is leveraged, in a sense, by negative premium flows in earlier periods. The leveraged characteristic of this component renders its spread duration a less reliable measure of cash flow life. It remains a measure of the value of a basis point change in discounting spread.

- Spread and effective durations are equal for fixed cash flow components, such as premium and expense, and different for interest rate sensitive cash flows such as surrender and death benefits.
- Dampened effective durations and aggregate dampened effective partial durations are equal for fixed cash flow components and different for interest sensitive cash flow components.
- Premium cash flows are fixed over different scenarios even though surrender cash flows change. We have fixed the incidence of surrender in the analysis by leaving the crediting strategy unchanged -- hence the constant premium cash flows. On the other hand, the amount of surrender cash flows can

change dramatically under different forward rate scenarios.

- In the context of Reitano [1990], since the partial durations with respect to interest sensitive cash flows are relatively high with respect to an equivalent dampened effective duration, it appears that the interest sensitive nature of the cash flows has the effect of adding "durational leverage" to the component. Other elements able to cause the effect include negative partial durations, portfolios of securities and short positions.
- Option-adjusted spread methodology would provide yet additional texture to the duration results. Additionally, analogous shocks can be made in mortality and surrender behavior space to reveal additional risk measures.
- A further consideration would be the duration of cash flows distributable to the stockholders. Under life insurance accounting conventions in most states, dividends can generally be paid without restriction to the extent of the prior year statutory earnings. So, in this case, statutorily-defined accounting and legal conventions actually drive a cash flow stream. And, in most cases, these conventions have a significant historical cost component, serving to stabilize earnings. As a result, intra-company durations and the durations of returns to the ownership rights to such company may diverge substantially. Management and stockholders rarely

synthesize an optimal duration position.

CONCLUSION

Life insurance companies can adapt pricing methodologies that are customarily used in the financial area to the cash flow provisions of their particular contracts. Doing so can provide rich insight into the market value of liabilities. These modeling methodologies can guide investment strategies and provide management with measures of the market value of life company surplus.

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APPENDIX A

Hypothetical ISWL Contract Terms and Inforce Assumptions

Following is a sample cell of the ISWL model used in the analysis.

Representative Cell Information

Field	Value
Gender	Male
Weighted Average Issue Age	32
Smoking Classification	Nonsmoker
Premium Mode	Monthly
Underwriting Class	Nonmedical
Weighted Average Policy Age	10 Years
Number of Policies	70
Basic Annual Premium	$5.60/$1000
Total Face Amount (Policy yrs 1-10)	$7,605,500
Total Face Amount (Policy yrs 11-)	6,844,950
Total Fund Value	266,633
Total Cash Value	226,639
Total Statutory Reserve	250,740
Total Policy Loan	22,664

Common to all 102 cells of the analysis are: 1) a guaranteed crediting rate of 5.5 percent, 2) a 15 year declining surrender charge schedule, as a percentage of existing fund value, 3) a provision whereby interest crediting on loaned balances can be less than on unloaned balances, 4) a specified agent commission schedule and 5) a standard servicing expense structure.

Following are descriptive statistics for the insured groups, weighted by face amount.

Issue Age: 41.5
Months Seasoned: 55

Other beginning characteristics of the model contract: 1) face amount: $709,585,000, 2) cash value: $36,348,600, 3) fund value: $50,426,020, 4) statutory reserve: $47,150,500, 5) annualized premium in projection month one: $5,886,800 and 6) number of policies: 6,951.

The analysis assumes a simple mortality experience equal to 80 percent of the 75-80 Basic select and ultimate mortality table.

The analysis assumes baseline surrenders of 7.5 percent annually with adjustments for: 1) early policy years due to buyers' remorse, 2) later policy years, when surrender charges are lower, 3) higher insured ages, when contract owners typically have a post-retirement need for cash and when they find surrender to be tax-efficient and 4) policy crediting rates that are different from rates on policies of competitor companies. Over the projection horizon, the crediting rate is set at the projected industry rate less 125 basis points. This low crediting rate produces moderately higher surrenders. Little empirical analysis, however, has been conducted in this area. This analysis uses a quadratic function in which the surrender rate is slightly lower when crediting is high relative to the industry and significantly higher when contract crediting is low relative to the industry.

APPENDIX B

Industry Interest Sensitive Life Crediting Rate Modeling

Industry interest sensitive life crediting rates track fixed income market rates with lags for several institutional reasons: 1) company investment committees often establish crediting rates as a spread to the asset book value earnings rate. Once a particular asset is on the books at a given rate of return for purposes of accruing interest income, its rate of return will be assumed to remain constant until those particular assets are rolled into a new security with a new rate of return. Obviously this practice delays the recognition that earning rates have, in fact, changed. 2) the institution then needs to declare the new rate. 3) companies are reluctant to change rates too often for marketing purposes. 4) the effective crediting rate can be further delayed due to certain contractual provisions allowing new rates to be credited only at the policy's anniversary; hence, contracts containing this provision have a built-in expected delay of six months.

The crediting rate series used was monthly data from January 1985 through December 1992 from the TULAS industry survey published by the actuarial firm Tillinghast, A Towers Perrin Company (calculations made by use of these rates are provided with the permission of Tillinghast). The period covers a substantial fall in rates and changes in yield curve shape.

Consider the following regression coefficients to explain the interest sensitive life crediting rate_t:

Explanatory Variable	Coefficient	t-Statistic
Constant	3.780	7.146
$T3M_{t-6}$	.119	3.451
$T5Y_t$	.342	6.447
$T5Y_{t-18}$	.146	3.168

Note that $T3M_{t-6}$ refers to the 3 month risk free bill rate six months prior to the interest sensitive life crediting rate (subsequent subscripts are treated consistently); T5Y refers to the 5 year risk free note rate.

This regression would yield the following equations:

$$\text{Industry Crediting Rate}_t = 3.78 + (.119 \times T3M_{t-6}) + (.342 \times T5Y_t) + (.146 \times T5Y_{t-18}).$$

The regression $r^2 = .72$.

Although this model appears to have extensive explanatory power to describe crediting rates, it is dominated by the constant term. The model is not useful in describing the response of the crediting rate to a change in market rate since the non-constant term coefficients sum to significantly less than one. For example, the model indicates that the crediting rate would never fully respond to a permanent parallel increase in rates of 100 basis points.

Consequently, although it is an interesting model, it is not useful for our purposes. Following is a similar model, fit without the constant term. The following model also weighs recent observations more heavily.

Explanatory Variable	Coefficient	t-Statistic
$T5Y_t$	.424	8.882
$T5Y_{t-6}$	.297	5.560
$T5Y_{t-18}$	.328	10.193

This regression would yield the following equation:

$$\text{Industry Crediting Rate}_t = (.424 \times T5Y_t) + (.297 \times T5Y_{t-6}) + (.328 \times T5Y_{t-18}).$$

The regression $r^2 = .99$.

In this case, we compromise no model fit; however, since the coefficients sum nearly to one, the model merely describes the timing by which the crediting rate will fully reflect changed market rates.