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A RATIO OF INTEREST-ADJUSTED COST INDEXES FOR THE COMPARISON OF DISSIMILAR LIFE INSURANCE CONTRACTS

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ABSTRACT

This paper describes a method of comparing net costs of dissimilar life insurance contracts which differ from others in several crucial respects:

1. It treats the policy as a single entity (neither calculation nor explanation of the ratio involves splitting out the amount at risk).
2. It does not require use of year-by-year cash values.
3. It does not require any assumption as to mortality rates applicable to the prospective policyholder.
4. It makes full use of the knowledge that agents are acquiring of the interest-adjusted method, without requiring either agents or their clients to master sophisticated techniques of policy analysis.

The paper includes a discussion of how the relative cost of early termination of the contract may best be shown to the buyer.

I. INTRODUCTION

This paper shows how net costs of virtually all life insurance contracts may be compared, using, for each contract, the ratio of two interest-adjusted cost indexes (denoted "IAC indexes" below). The ratio is that of the IAC index as defined by the Moorhead report[1] to a control IAC index. The control index differs from the Moorhead index only in that a control premium is used in place of the gross premium. This control premium is based on a "control price scale" for insurance protection and on the interest rate chosen for determining the Moorhead index. It is calculated to provide the policy benefits during the cost analysis period, the dividends (if any) payable during this period, and the cash value and terminal dividend payable at the end of the period.

Under the proposed method the policy is treated as a single entity. Since both numerator and denominator in the ratio are IAC indexes,

[1] Joint Special Committee on Life Insurance Costs, Report to American Life Convention, Institute of Life Insurance, Life Insurance Association of America, May 4, 1970. The interest-adjusted method is also described in *TSA*, XXIII, 290–91 (steps 1–5).

full advantage is taken of the knowledge of the interest-adjusted method that agents are acquiring. The only other concept needed is that of a premium on a standard basis to provide the policy benefits to the end of the analysis period.

Section II develops a theoretical analysis of the method. In Section III the choice of control price scale is considered, and use of the Commissioners 1958 Standard Ordinary Mortality Table for this purpose is recommended. Section IV considers the validity, for the prospect, of the basis of contract valuation implied by the suggested method, as against the use of cash values for this purpose. Section V shows how the method may be adapted to special underwriting classifications.

The method assumes that the contract will not be surrendered before the end of the analysis period. However, because this eventuality cannot be ruled out altogether, it is important that the relative cost of early surrender be shown. Section VI discusses the most effective way this can be done.

The Appendix shows, with illustrative calculations, how twenty-year ratios for standard policies can be calculated by a seven-factor formula on a desk calculator, using data available in the trade publications. Such ratios are generally correct to within ± 0.01 and take account of premium refunds in the year of death, post mortem dividends, and terminal dividends payable on death. So that twenty-year ratios of IAC indexes may be obtained readily, the requisite factors are given for quinquennial issues ages at interest rates of 4 and 5 per cent. These factors are so laid out that they can be used, with little or no briefing, by a clerk with no knowledge of the method.

II. THEORY

The relation between the distinctive functions of this paper, namely, that of the "control price scale" to the "control premium" and "prospect reserves" for a given benefit, will be shown to be the same as that of the mortality table to the net premium and net level premium reserves for a policy that has the same benefits during the analysis period and that matures at the end of the analysis period with a pure endowment benefit of the then cash value (and any terminal dividend). On account of this analogy, and to facilitate interpretation of the formulas, conventional actuarial symbols will be used, but with primes to serve as a reminder of their special connotation in the present context.

For simplicity, the theory will be developed for a contract with level premiums and a level death benefit through the cost analysis period. The adjustments required for contracts with nonlevel premiums or death benefits will then be delineated.

The annual premium, π, charged for a life insurance contract with unit death benefit may be regarded as being composed of a "control premium," or "prospect's valuation net premium," P', and a "prospect's loading," $\pi - P'$ (which may be negative). From the buyer's viewpoint, P' should be based on a rate of interest, i, deemed appropriate by him and a "control price scale" for insurance protection. This scale should be consistent with the prevailing market rates of competitively priced policies but is independent of the individual characteristics of the prospective policyholders within any single underwriting rate classification. It is specified by a set of values, q'_x, one for each age x, where $(1 + i)^{-1/2}q'_x$ is taken to be the control price for an individual aged x at issue for a one-year term insurance benefit of 1 unit.

We denote by ${}_tV'$ the value of a contract at the end of the tth policy year (after payment of any policy dividend for the year) to a prospect who intends to keep his contract in force to the end of the n-year analysis period. This value increases to ${}_tV' + P'$ on payment of the annual premium π, since P' has been defined as the prospect's valuation net premium.

The prospect's valuation, ${}_tV' + P'$, of the contract at the start of the $(t + 1)$st year must be just sufficient to provide (a) the discounted value, $v\ {}_{t+1}V'$, of his valuation of the contract at the end of the $(t + 1)$st year; (b) the discounted value, $v\ {}_{t+1}DIV$, of any policy dividend payable at the end of the $(t + 1)$st year; and (c) the amount needed to purchase, at the control price, one-year term insurance for the excess of the unit death benefit over $v^{1/2}\ {}_{t+1}V'$ and $v^{1/2}\ {}_{t+1}DIV$. (This assumes that no post mortem dividend is payable. When a post mortem dividend is payable pro rata, the latter term should be halved.)

It follows that

$$V' + P' = v\ {}_{t+1}V' + v\ {}_{t+1}DIV + v^{1/2}q'_{x+t}(1 - v^{1/2}\ {}_{t+1}V' - v^{1/2}\ {}_{t+1}DIV)\ . \tag{1}$$

Multiplying by $1 + i$ and rearranging terms, we have

$$P'(1 + i) - {}_{t+1}DIV - [{}_{t+1}V' - (1 + i)\ {}_tV'] = q'_{x+t}[(1 + i)^{1/2} - {}_{t+1}V' - {}_{t+1}DIV]\ .$$

Multiplying now by $(1 + i)^{n-t-1}$, and summing for values of t from 0 to $n - 1$, we obtain (since ${}_0V' = 0$)

$$P'\ddot{s}_{\overline{n}|} - \sum_{t=0}^{n-1} (1 + i)^{n-t-1}\ {}_{t+1}DIV - {}_nV' = \sum_{t=0}^{n-1} (1 + i)^{n-t-1}q'_{x+t}[(1 + i)^{1/2} - {}_{t+1}V' - {}_{t+1}DIV]\ .$$

Since ${}_nV' = {}_nCV + {}_nTD$, the cash value (plus any terminal dividend) payable n years hence, we obtain, on dividing by $\ddot{s}_{\overline{n}|}$, the control IAC index:

$$P' - \left[\sum_{t=1}^{n} (1+i)^{n-t}\, {}_tDIV + {}_nCV + {}_nTD\right] \Big/ \ddot{s}_{\overline{n}|}$$

$$= \left\{\sum_{t=1}^{n} (1+i)^{n-t} q'_{x+t-1}[(1+i)^{1/2} - {}_tV' - {}_tDIV]\right\} \Big/ \ddot{s}_{\overline{n}|}\,.$$

If to this control IAC index the prospect's loading, $\pi - P'$, is added, the Moorhead IAC index is obtained:

$$\pi - \left[\sum_{t=1}^{n} (1+i)^{n-t}\, {}_tDIV + {}_nCV + {}_nTD\right] \Big/ \ddot{s}_{\overline{n}|}\,.$$

The working form of the ratio of IAC indexes, for purposes both of explanation and of computation, is

$$\frac{\pi - \left[\sum_{t=1}^{n} (1+i)^{n-t}\, {}_tDIV + {}_nCV + {}_nTD\right] \Big/ \ddot{s}_{\overline{n}|}}{P' - \left[\sum_{t=1}^{n} (1+i)^{n-t}\, {}_tDIV + {}_nCV + {}_nTD\right] \Big/ \ddot{s}_{\overline{n}|}}\,.$$

Consideration need be given to the analytic form of the control IAC index,

$$\left\{\sum_{t=1}^{n} (1+i)^{n-t} q'_{x+t-1}[(1+i)^{1/2} - {}_tV' - {}_tDIV]\right\} \Big/ \ddot{s}_{\overline{n}|}\,,$$

only when, as in Section III, the control price scale is being chosen.

The value of P' may be derived from relationship (1) above, which, with p'_{x+t} denoting $1 - q'_{x+t}$, may be restated as

$$(vp'_{x+t}\, {}_{t+1}V' - {}_tV') + vp'_{x+t}\, {}_{t+1}DIV = P' - v^{1/2} q'_{x+t}\,. \tag{2}$$

While p'_{x+t} here bears the same algebraic relation to q'_{x+t} as p_{x+t} customarily bears to q_{x+t}, it cannot be regarded as a probability of survival, because of the special interpretation given to q'_{x+t}. Similarly, while the primed actuarial functions used below bear the same relation to the values of q'_x as the corresponding unprimed functions bear to the values of q_x, they cannot be given the usual probabilistic interpretations.

To obtain P' from relationship (2), we multiply by $v^t\, {}_tp'_x$ and sum for values of t from 0 to $n-1$. Since ${}_0V' = 0$, we obtain

$$v^n\, {}_np'_x\, {}_nV'_x + \sum_{t=1}^{n} v^t\, {}_tp'_x\, {}_tDIV = P'\ddot{a}'_{x:\overline{n}|} - (1+i)^{1/2} A'^{\,1}_{x:\overline{n}|}\,.$$

As ${}_nV'_x = {}_nCV + {}_nTD$, it follows that

$$P' = (1+i)^{1/2} P'^{\,1}_{x:\overline{n}|} + \left(\sum_{t=1}^{n} v^t \, {}_tp'_x \, {}_tDIV\right) \Big/ \ddot{a}'_{x:\overline{n}|} + P'_{x:\overline{n}|}{}^{\!\!\!\!\!\!1}\;({}_nCV + {}_nTD).$$

Thus the value of P is determined as the net annual premium to give the benefits (including any policy dividends) payable during the n-year analysis period, together with a pure endowment at the end of the period of the cash value and any terminal dividend then payable, using, for the "mortality table," the control price scale.

While the above formal development assumes a level death benefit, the formulas may readily be generalized to show that the description just given of how P' is determined applies also when the death benefit is not level. The treatment of any pure endowment benefits payable within the analysis period is identical with that of policy dividends.

For a policy already in force, the formulas apply, with the present cash-surrender value treated as a negative pure endowment (or dividend) immediately payable. Thus the method can be used to analyze replacement proposals.

Where the gross premiums are not level, the series of net level premiums that bear a constant ratio to the corresponding gross premiums and are equivalent in value to the benefits payable through the analysis period (including ${}_nCV$ and ${}_nTD$ as pure endowments) are first derived. P' is then calculated as the level equivalent (with allowance for interest only) to these net premiums, while π is the corresponding level equivalent to the gross premiums.

The formulas developed above ignore both apportionable premium and terminal dividend death benefits. When payable, their average amount in the $(t+1)$st policy year should be added to the bracketed term in relationship (1). Thus the value of these benefits (and of post mortem dividends, previously mentioned), through the analysis period, should be included in the calculation of P'.

The assumption that ${}_nV' = {}_nCV + {}_nTD$ should not be regarded as relevant only when the policy is to be surrendered at the end of n years. The difference between the values, for two participating policies, of ${}_nV'$ so calculated will approximate the difference between the companies' nth-year asset shares for the policies. Hence, after n years, the net annual charge (premium less dividend) of the company with the higher value of ${}_nV'$ will, for a given plan of insurance, tend to be lower than that of the company with the lower value of ${}_nV'$. Thus, while a method that takes account only of premiums and dividends payable in the n-year analysis period can give a fair measure of relative cost for the policyholder who dies near the end of the period, it usually will not be as valid as the method here suggested for those whose contracts remain in force much longer than n years.

III. CHOICE OF CONTROL PRICE SCALE

The analytic form of the control IAC index developed above, namely,

$$\left\{\sum_{t=1}^{n} (1+i)^{n-t} q'_{x+t-1}[(1+i)^{1/2} - {}_tV' - {}_tDI\,V]\right\} \Big/ \ddot{s}_{\overline{n}|}\,, \qquad (3)$$

shows that the control IAC index may be regarded as the weighted total of the values of q'_{x+t-1}, the weights being dependent on the amounts at risk. Thus the control IAC index for term insurance is higher than for permanent plans.

On account of the relative amount at risk at shorter and longer policy durations, if the control price scale is increased, with the proportionate increase greater at the higher ages, then the control IAC index for term insurance will be raised more, proportionately, than the control IAC index for a permanent plan. Conversely, if an increase in control price scale is proportionately greater at the younger ages, then the control IAC index for term insurance will be raised less, proportionately, than the control IAC index for a permanent plan. It is this latter situation which prevails if the control price scale is increased from the 1958 CSO Basic Table of Mortality to the 1958 CSO Table, for the latter table has margins over the former which decrease from 107.5 per cent at age 25 to 15.0 per cent at age 65. However, even this substantial change in the pattern of the control price scale generally has only a slight effect on comparisons among costs of different plans. For example, with a twenty-year analysis period, use of the 1958 CSO Table (instead of the 1958 CSO Basic Table) as the control price scale decreases the quotient

$$\frac{\text{Ratio of 4 per cent IAC indexes for ordinary life insurance}}{\text{Ratio of 4 per cent IAC indexes for twenty-year term insurance}}$$

by less than 2 per cent at all issue ages.

Since small variations in the control price scale have a negligible effect on comparisons among policyholder costs, development of a special scale of values for this purpose is not justified. Instead, a well-known existing table may be employed. We show below that an experience table is not suitable for use as the control price scale and that, rather, the 1958 CSO Table is satisfactory for this purpose.

A policyholder cost comparison method is intended to help the prospect who is already in the market for life insurance choose among alternative contracts. The typical prospect can determine within only broad limits the amount of insurance that he will need ten or twenty years hence. His appraisal of the relative costs of differing types of contracts that fall within the range of his anticipated needs depends on how the cost of each type of contract compares with the cost of contracts on the market with similar benefits. Specifically, the relative economic value to him, at his different future attained ages, of a given net amount at risk depends on the relative market price of insurance at these ages. The ratio of this price to the corresponding experience mortality rate is, generally, not constant but decreases with increasing age. This decrease arises from companies' proper analysis of their expenses and hence should not be regarded as inequitable.

As a consequence of the incidence of company expenses, the expense charge for a lower-premium plan with larger amounts at risk at later durations would normally be a somewhat smaller fraction of the experience mortality cost than would be the case for a higher-premium plan. To do justice to higher-premium plans (and also decreasing term insurance), in relation to lower-premium level death benefit plans, the control price scale should reflect the higher ratio, at the lower attained ages, of the market price of protection (from which the buyer cannot escape) to experience mortality.

The foregoing suggests that the 1958 CSO Table may be acceptable as the control price scale. To test the suitability of this basis, the table has been used to calculate ratios of twenty-year 4 per cent IAC indexes for twenty participating and fifteen nonparticipating ordinary life policies issued by thirty-five large companies at ages 25, 35, 45, and 55. Since it is convenient to use the same control price scale for all policy sizes, and since the policy fees are fixed independently of the value of the control IAC indexes, these fees may be considered part of the prospect's loading, $\pi - P'$. Thus the ratios of IAC indexes have here been determined with the policy fees excluded from the gross premiums.

If the control price scale is consistent with market rates, then the ratios of IAC indexes (with the policy fee excluded) for competitively priced contracts will not vary greatly with issue age. Furthermore, because the control IAC index is dependent on the amounts at risk in successive policy years, and because these amounts in turn are a function of the control price scale, the described ratios ideally should be close to unity. However, the reserves underlying the amounts at risk build to the same value (namely, ${}_nCV + {}_nTD$) at the end of the n-year analysis period, and changes in the control price scale will generally have a somewhat similar effect on the amounts at risk associated with the policies compared. Thus the change in the difference between the ratios of IAC indexes of two contracts, arising from the effect of a change in the control price scale on the amounts at risk, is very slight.

With the 1958 CSO Table as control price scale, the ratios of the twenty-year 4 per cent IAC indexes for the twenty participating ordinary life policies average 1.07, 1.01, 0.94, and 0.88 for issue ages 25, 35, 45, and 55, respectively, using 1972 rates and values. For each issue age the lowest ten of these ratios average 0.92, 0.93, 0.88, and 0.83, respectively, and the lowest ratios are 0.77, 0.81, 0.81, and 0.72, respectively.

Ratios corresponding to the above for the fifteen nonparticipating ordinary life policies average 1.51, 1.36, 1.19, and 1.07 for issue ages 25, 35, 45, and 55, respectively. For each issue age the lowest ratios are 0.97, 1.01, 1.04, and 1.02, respectively.

While the yearly renewable term insurance plan is of much less importance than the ordinary life plan, it is of interest to compute for it the ratio of the one-year 4 per cent IAC indexes, that is, the ratio of the gross premium (for the present purpose, net of policy fee) for (x) to the value of $v^{1/2}q_x$ on the 1958 CSO Table. The ratios of the one-year 4 per cent IAC indexes for the nonparticipating yearly renewable term policies of twelve companies average 1.35, 1.25, 1.12, and 1.09, for ages 25, 35, 45, and 55, respectively. For each age the lowest ratios among these companies are 1.06, 1.06, 1.03, and 1.00, respectively.

As has already been noted, even a quite drastic change in the control price scale has very little effect on comparisons among the plans that might suit any one prospect's circumstances. It appears that, for competitively priced policies, ratios of IAC indexes based on the 1958 CSO Table as control price scale do not vary greatly from age to age. The acceptability of the 1958 CSO Table for this purpose is thus confirmed.

The policy fee was omitted from the numerators of the ratios of IAC indexes quoted above (in the second to fourth preceding paragraphs), solely for the purpose of testing the suitability of the 1958 CSO Table as control price scale. Normally it would always be taken into account. The resulting increase in the ratios of IAC indexes is greater at the lower ages, where the indexes are smaller.

IV. PROSPECT APPRAISAL OF CONTRACT VALUATION

The ratio of IAC indexes would always be explained to a prospect in terms of the relationship between the Moorhead IAC index and a control IAC index based on a standardized premium applicable to the policy. The concept is thus expounded without reference to any valuation of the policy. However, as the theoretical development in Section II shows, a valuation is implicit in the method. The relation between the implied "prospect reserves" (based on the 1958 CSO Table as control price scale) and the cash values of the policy, and the reason why the former are a better measure of the prospect's future interest, are discussed below. First, however, we consider one category of life insurance buyer for whom the method of this paper is not applicable.

This paper contends that "mortality" per se is irrelevant to the comparison of policyholder costs, and advances a method that requires no assumption as to mortality rates applicable to the prospect. The great majority of those in the market for insurance are concerned with price, not probabilities. However, probabilities that the contingency insured against will occur must be weighed by those who purchase insurance with intent to make a speculative gain. A buyer in this category would, typically, place a substantially higher value on his newly issued policy than the gross premium paid. The proposed method cannot be applied to further his nefarious interests.

The prospect reserves will be greater than the reserves held by the company for a participating policy, if the illustrative dividends are based on an interest rate higher than the rate used to determine the IAC indexes, and if the loading and mortality factors, taken together, tend not to decrease with increasing policy duration. At the shorter durations, especially, the prospect reserves generally will be appreciably greater than the cash values. This also will be true of nonparticipating policies (except those on forms featuring high early cash values). However, if, as generally will be the case under the present unrealistic valuation laws, the interest rate underlying the IAC indexes is higher than the company valuation rate, then the prospect reserves of nonparticipating policies will be lower than the cash values when these approach and equal company net level premium reserves within the analysis period.

The prospect reserve is a concept that is used in this paper for purposes of analysis only. It need not, of course, be the same as the value a policyholder would put on a contract after it had been in force some time. This generally would be much larger than the prospect reserve if the policyholder's health had deteriorated since issue. Even if a policyholder had no reason to reappraise the prospect reserves, and even if he knew their values and their relation to the cash values, this intelligence would be of no use to him. If he wanted to evaluate a replacement proposal, for example, he would proceed as in the fourth from the last paragraph of Section II, without using any prospect reserve values (except, of course, the cash-surrender values at the beginning and end of the analysis period).

It is sometimes suggested that the buyer's "net amount at risk" is the excess of the face amount over the cash value.[2] If this were true, then the prospect would have to regard either (*a*) his net insurance benefit as the excess of the amount payable on death over the cash value, even though the cash value is different from his valuation of his interest in the policy, or (*b*) the cash value as the valuation of his interest in the policy. Both alternatives will be shown to be untenable.

No one who takes out a magazine subscription for several years, with the proviso that the unexpired portion of the subscription will be refunded only in the event of the subscriber's death, regards this as an insurance contract. Yet, under alternative *a*, the refund would be a "net insurance benefit"! Clearly, a prospect can regard a contract as incorporating an insurance benefit only to the extent that it pays on death more than his anticipated interest in the contract at the time of death, even though this interest may not be realizable if he lives.

The proposition that the cash values represent the prospect's future interest in the policy accords with an analysis that, in effect, treats every life insurance contract as a series of one-year contracts, each giving term insurance for the face amount together with a pure endowment of the cash value payable at the end of the year, and with a premium equal to the sum of the previous year's cash value and the level premium charged. If these were isolated, strictly nonrenewable contracts, then the interest at the end of the year of the buyer in any one of them would, indeed, be the cash value then payable. However, they are neither isolated nor nonrenewable.

The extent to which the renewability benefit is of value to the prospect depends on the sizes of the successive cash values. If, as will generally be the case, the early cash values are appreciably less than net level premium reserves, then the net cost of the initial contract will be much higher than that of subsequent ones. Obviously, the only reason why the buyer would accept the high initial net cost is that he has the right to renew at a lower net cost in subsequent years. In these circumstances, he could be expected, in the early policy years, to consider the renewability benefit as worth a large proportion of the difference between first- and renewal-year net costs. Thus his valuation of the policy is better represented on a net level premium reserve basis, as implied by the method of this paper, than by the lower early cash values.

The fallacy of comparing policyholder costs by a method that takes the net insurance benefit to be the excess of the death benefit payable over the cash value is shown when costs are compared under two policies

[2] For example, *TSA*, XXIII, 305–6.

with the same rates and values, except for certain cash values payable prior to the end of the analysis period. On the assumption that no terminations occur before the end of the period, costs under the policies are obviously equal. While these policies do have the same ratio of IAC indexes, methods that postulate the net insurance benefit as the excess of the amount payable on death over the cash value show, incongruously, a lower cost for the policy with the lower cash values. (Sec. VI below shows how the relative cost of early surrender may be effectively demonstrated.)

Any implication that the buyer should regard the cash values, and in particular the low early cash values, as the measure of his interest in the policy is pernicious. It can only weaken his resolve to keep his contract in force.

V. SPECIAL UNDERWRITING CLASSIFICATIONS

Adjustments to the control price scale for differences in sex and underwriting classification are suggested below. Also considered are the treatment of policies with age determined on a last-birthday basis and limitations on the use of the method when the insurance element in the contract is slight.

On quotations to female prospects, the commonly used three-year rate-down in age for premiums and values should be applied also to the control price scale.

For impaired prospects the control price scale should be given an appropriate age rating, except when the impairment is generally covered by a temporary extra premium. In this case the values of the control price scale should be appropriately increased at those attained ages for which the extra premium would typically be payable. No precision is called for in determining the age rating or other adjustment, since, as shown in the second paragraph of Section III above, comparisons of ratios of IAC indexes are quite insensitive to changes in the control price scale. However, it is important that the control IAC indexes of all policies compared be based on the same control price scale, even when the underwriting or rating practices of the carriers differ.

In comparing the competitive position of a company that uses the "age last birthday" classification with that of one that uses the "age nearest birthday" basis, it is satisfactory, when no quotation is to be made to a prospect, to use the control price scale on an "age last birthday" basis for the company with rates so classified. However, when a quotation is to be given to a specific prospect, exactly the same values for the price control scale should be used by both companies. (These would, presumably, be on the more commonly used "age nearest birthday" basis.)

Otherwise, the relatively better competitive position of the "age nearest birthday" company in the first half of the year of age, and vice versa in the last half, would be concealed.

The analytic form of the control IAC index, formula (3) above, shows that, as the amounts at risk over the prospect reserves approach zero, so does the control IAC index. Thus, for the ratio of IAC indexes to be of significance, the average death benefit during the analysis period should exceed the average reserve (or, say, for convenience, the average cash value) by some appreciable proportion, such as 10 per cent. Policies sold as insurance contracts will almost always meet this requirement. An exception might arise in the case of an analysis of a proposal to replace a retirement income policy (having an initial level death benefit) a number of years after issue.

VI. ELUCIDATION OF THE EFFECT OF EARLY TERMINATION

The ratio of IAC indexes takes no account of the possibility of early termination. Some would consider this a weakness and argue that a measure of policyholder cost to be used for purposes of contract comparison should incorporate a scale of lapse rates.

When any comparison of contract costs is made, the prospect should certainly be shown, in as cogent a manner as possible, the consequences of early termination. The question is: "How can this be done effectively, so that the prospect can judge for himself, in the light of his knowledge of his own circumstances, the relative weight he should attach to a lower short-term cost as against a lower long-term cost?"

There are serious problems in including lapse rates in the cost calculation. It is to be doubted whether the prospective policyholder can make a meaningful choice from among arrays of such rates. He should rather be faced with choices expressed in terms that he can readily comprehend.

A consideration that militates against the use of lapse rates is that the probability of termination is liable to be influenced by the scale of cash values. Inclusion of lapse rates is most likely to affect the result of a cost comparison when surrendered net costs in the early policy years (when lapse rates on any scale are highest) are very different. However, if, after a sizable proportion of lapses, another policy is purchased, an analysis of the cost of replacement is more likely to favor termination of the original policy when it has low surrendered net cost. Thus for a prospect comparing two policies it would be quite logical to use not the same lapse rates but, rather, higher rates for the policy with the lower surrendered net cost at short policy durations.

While it is not practical to incorporate lapse rates into cost calculations, it is important that the prospect be apprised of the relative effect of early termination on cost. It is not sufficient to quote only the dividends and cash values of the first five years. These figures have to be related to one another (and to the premium payable) to make a meaningful cost comparison. It should not be left to the unsophisticated buyer of ordinary insurance to perceive the relationships and perform the necessary calculations.

To ascertain an appropriate simple indicator of the cost of early termination, average annual net costs were determined on the assumption that the contract remains in force for the first t years, with $t = 1, 2, 3, 4,$ and 5, and is then surrendered. These costs were calculated from 1972 data for participating policies issued at age 35 with face amount $25,000 on the whole life or other long-term plans of twenty large companies, chosen on the basis of the amount of insurance they issued on participating life and endowment plans in 1970. For each company the five costs were then averaged. The over-all averages thus obtained, being derived from the average annual net costs rather than the total net costs, throw more weight on the costs for the shortest durations, at which the probability of lapse is greatest.

While the above over-all average values, which show the effect of termination in the years when lapse rates are highest, correlated poorly or not at all with the corresponding ten- and twenty-year IAC indexes, they correlated very well with the two-year average annual surrendered net costs. In 97 per cent of the 190 comparisons that could be made among the companies, the two-year cost ranking agreed with the ranking of the averages of the five costs. (To preserve this correlation, legislation might be needed requiring companies to justify any apparent inconsistency among early cash values.)

The two-year surrendered net cost is easy to calculate, the disregard of interest being inconsequential for very short durations. (The one-year cost does not correlate so well with subsequent ones, because the first-year cash value may be zero, the increase in cash value in the second year sometimes being significantly less than the increases thereafter.) Given this two-year cost (along with the ten- and twenty-year ratios of IAC indexes), the prospect can better appraise the relative cost of the policies to him. In making this appraisal, the prospect would (perhaps subconsciously) weigh the likelihood of his withdrawing, but he would not have to go through the baffling experience of trying to translate this likelihood into a set of numerical probabilities.

VII. CONCLUSION

Life insurance policies are complex, many-faceted instruments. The agent should have adequate time to present these to the prospect and relate them to his needs. Thus it is important that any cost comparisons made should be explainable as simply as possible.

Once the customary application of the interest-adjusted method has been understood, the notion of a second application of the method, differing from the first only in that a standardized premium for the benefits of the analysis period is used, is readily comprehensible. No attempt to make the policyholder visualize a year-by-year breakdown of the policy into constituent parts is needed.

If the prospect is given the two-year surrendered net costs as a guide to the relative cost of early termination, he can readily perceive, in the light of his knowledge of his present and probable future circumstances, the weight he should attach to the differences among them. On the other hand, the imposition by some other party of a scale of lapse rates, so that both short-term and long-term surrender costs are promiscuously subsumed in a single index, effectively conceals the significance of either from the prospect. He is thus precluded from being his own man.

APPENDIX

SHORT FORMULA FOR TWENTY-YEAR RATIOS OF IAC INDEXES

Using only seven or eight factors, twenty-year ratios of IAC indexes may be calculated for a participating policy that has, throughout the twenty-year period, a level death benefit and level premiums. For such a policy issued to (x), the ratio may be expressed as

$$\left[\pi - \left(\sum_{t=1}^{20} v^t\ {}_tDIV\right)\Big/\ \ddot{a}_{\overline{20|}} - ({}_{20}CV + {}_{20}TD)/\ddot{s}_{\overline{20|}}\right]\Big/$$

$$\left[(1+i)^{1/2}P^{\prime\,1}_{x:\overline{20|}} + \left(\sum_{t=1}^{20} v^t\ {}_tP'_x\ {}_tDIV\right)\Big/\ \ddot{a}'_{x:\overline{20|}} - \left(\sum_{t=1}^{20} v^t\ {}_tDIV\right)\Big/\ \ddot{a}_{\overline{20|}}\right.$$

$$\left. - \left(\frac{1}{\ddot{s}_{\overline{20|}}} - \frac{1}{\ddot{s}'_{x:\overline{20|}}}\right)({}_{20}CV + {}_{20}TD)\right].$$

The "equivalent level dividends" in numerator and denominator can be derived quite accurately from ten- and twenty-year dividend totals.[3] Errors in costs based on such approximate equivalent level dividends have been investigated.[4] The relation between these errors and the sizes of the corresponding control IAC indexes is such that, at every issue age, ratios of IAC indexes calculated using ten- and twenty-year dividend

[3] Peter L. J. Ryall, "A Fast, More Meaningful Twenty-Year Net Cost Formula," *TSA*, XXI, 101–18.

[4] *TSA*, XXII, D707–D708.

totals are correct to within ± 0.01 for more than 90 per cent of policies written. Errors greater than 0.02 occur only for policies issued by those few small companies that pay extra dividends at certain policy durations. (However, errors fall within the above bounds where an extra dividend is payable for the fifth policy year only.)

In the working formulas below, "Prem," "Tot 10 Div," "Tot 20 Div," "${}_{20}$CV," and "${}_{20}$TD" denote, per \$1,000 sum insured, the gross annual premium, the totals of the first ten and twenty years' dividends, the twentieth-year cash value, and the twentieth-year terminal dividend, respectively. The coefficients in the numerator are drawn, on the 4 per cent basis, from the values (multiplied by 10^5) in Table 2 of *TSA*, XXII, D707, and, on the 5 per cent basis, from similarly derived unpublished values.

With respect to the denominator, the first column, as indicated, is the issue age (x, say). The second column is the value of $10^5(1+i)^{1/2}P'^{1}_{x:\overline{20}|}$. The next five columns are (for issue ages 15–60) the differences between the appropriate values in Table A of *TSA*, XXI, 118 and (for the 4 per cent basis) corresponding values (multiplied by 10^5) from Table 2 of *TSA*, XXII, D707. The values on the 5 per cent basis (and for issue ages 0, 5, 10, and 65) in these columns are the differences between similarly derived but, in part, unpublished values.

The coefficients of ${}_{20}$TD are those of ${}_{20}$CV rated down ten years of issue age. This rating is to allow for the customary payment of the terminal dividend on death. More precisely, the rate-down should "range from five years when terminal dividends are first payable [at the higher issue ages] after fifteen policy years on a steeply sloping scale to twenty-five years when terminal dividends are payable [at these ages] after ten policy years as a constant proportion of the cash value."[5]

ILLUSTRATIVE CALCULATIONS (1958 CSO 4 PER CENT BASIS)

Example 1

Data:

First-year dividend payable	
Issue age	45
Premium charged per \$1,000	\$ 31.79
Total dividends first ten years	\$ 55.50
Total dividends first twenty years	\$180.71
Twentieth-year cash value	\$474.00

No premium refund past month of death, no post mortem dividend, and no terminal dividend

[5] *TSA*, XXI, 111–12.

Calculation:

$$\text{Ratio} = \frac{10^5(31.79) - 2{,}484(55.50) - 3{,}566(180.71) - 3{,}229(474.00)}{1{,}000(1{,}194) + 775(55.50) - 446(180.71) - 610(474.00)}$$

$$= \frac{8.66}{8.67}$$

$$= 1.00$$

Example 2

Data:

Year of payment of first dividend, issue age, premium charged per \$1,000, and total dividends first ten and first twenty years as for example 1 above

Twentieth-year cash value	\$450.00
Twentieth-year terminal dividend	\$ 24.00

Premium refunded past month of death and post mortem dividend payable

Calculation:

Ratio =

$$\frac{10^5(31.79) - 2{,}484(55.50) - 3{,}566(180.71) - 3{,}229(474.00)}{1.015(1{,}194) + 760(55.50) - 410(180.71) - 610(450.00) - 263(24.00)}$$

$$= \frac{8.66}{8.99}$$

$$= 0.96$$

It may be noted that the difference of 0.04 between the ratios illustrated in examples 1 and 2 arises solely from the inclusion of ancillary death benefits in example 2.

WORKING FORMULAS FOR TWENTY-YEAR RATIOS OF IAC INDEXES

1958 CSO 4 Per Cent

1. Use data per $1,000 sum insured.
2. If premiums are apportionable (refund past death month), add half the gross annual premium (per $1,000 sum insured), rounded to the lower dollar, to "1,000" in the denominator.
3. If post mortem dividends are payable, use the values shown in parentheses.

FIRST-YEAR DIVIDEND PAYABLE

$$\frac{10^5\,(\text{Prem}) - 2{,}484\,(\text{Tot 10 Div}) - 3{,}566\,(\text{Tot 20 Div}) - 3{,}229\,({}_{20}CV + {}_{20}TD)}{1{,}000(\quad) + (\quad)(\text{Tot 10 Div}) - (\quad)(\text{Tot 20 Div}) - (\quad)\,{}_{20}CV - (\quad)\,{}_{20}TD}$$

Issue Age	1,000( )	( )(Tot 10 Div)		( )(Tot 20 Div)		( ) $_{20}CV$	( ) $_{20}TD$
0	179	77	(83)	48	(46)	55	55
5	142	86	(86)	50	(47)	61	55
10	159	100	(100)	58	(54)	69	55
15	184	113	(114)	66	(62)	77	61
20	211	128	(129)	75	(70)	91	69
25	258	157	(157)	92	(85)	120	77
30	348	217	(214)	126	(116)	174	91
35	509	326	(319)	188	(172)	263	120
40	777	502	(492)	289	(265)	402	174
45	1,194	775	(760)	446	(410)	610	263
50	1,828	1,189	(1,172)	684	(631)	912	402
55	2,755	1,800	(1,785)	1,035	(961)	1,315	610
60	4,040	2,636	(2,641)	1,516	(1,420)	1,789	912
65	5,791	3,706	(3,764)	2,137	(2,024)	2,310	1,315

NO FIRST-YEAR DIVIDEND PAYABLE

$$\frac{10^6\,(\text{Prem}) - 2{,}315\,(\text{Tot 10 Div}) - 3{,}606\,(\text{Tot 20 Div}) - 3{,}229\,({}_{20}CV + {}_{20}TD)}{1{,}000(\quad) + (\quad)(\text{Tot 10 Div}) - (\quad)(\text{Tot 20 Div}) - (\quad)\,{}_{20}CV - (\quad)\,{}_{20}TD}$$

Issue Age	1,000( )	( )(Tot 10 Div)		( )(Tot 20 Div)		( ) $_{20}CV$	( ) $_{20}TD$
0	179	71	(73)	46	(44)	55	55
5	142	81	(81)	49	(46)	61	55
10	159	94	(95)	57	(53)	69	55
15	184	106	(107)	64	(60)	77	61
20	211	119	(120)	72	(68)	91	69
25	258	148	(147)	89	(83)	120	77
30	348	207	(204)	124	(113)	174	91
35	509	312	(305)	185	(169)	263	120
40	777	479	(470)	284	(260)	402	174
45	1,194	738	(725)	437	(401)	610	263
50	1,828	1,129	(1,112)	670	(617)	912	402
55	2,755	1,699	(1,686)	1,011	(937)	1,315	610
60	4,040	2,462	(2,470)	1,475	(1,379)	1,789	912
65	5,791	3,407	(3,468)	2,067	(1,954)	2,310	1,315

1958 CSO 5 Per Cent

1. Use data per $1,000 sum insured.
2. If premiums are apportionable (refund past death month), add half the gross annual premium (per $1,000 sum insured), rounded to the lower dollar, to "1,000" in the denominator.
3. If post mortem dividends are payable, use the values shown in parentheses.

FIRST-YEAR DIVIDEND PAYABLE

$$\frac{10^6\,(\text{Prem}) - 3{,}042\,(\text{Tot 10 Div}) - 3{,}241\,(\text{Tot 20 Div}) - 2{,}880\,({}_{20}\text{CV} + {}_{20}\text{TD})}{1{,}000(\quad) + (\quad)(\text{Tot 10 Div}) - (\quad)(\text{Tot 20 Div}) - (\quad)\,{}_{20}\text{CV} - (\quad)\,{}_{20}\text{TD}}$$

Issue Age	1,000( )	( )(Tot 10 Div)		( )(Tot 20 Div)		( ) $_{20}$CV	( ) $_{20}$TD
0	181	76	(82)	47	(45)	50	50
5	141	84	(85)	49	(46)	55	50
10	157	97	(98)	56	(53)	62	50
15	182	110	(112)	64	(61)	70	55
20	208	124	(126)	72	(68)	83	62
25	253	152	(152)	88	(82)	109	70
30	339	209	(207)	121	(112)	158	83
35	494	312	(308)	180	(166)	239	109
40	753	480	(472)	277	(255)	364	158
45	1,156	739	(729)	426	(393)	553	239
50	1,770	1,133	(1,121)	653	(604)	825	364
55	2,670	1,712	(1,705)	986	(918)	1,187	553
60	3,925	2,501	(2,518)	1,442	(1,355)	1,612	825
65	5,641	3,507	(3,579)	2,029	(1,927)	2,075	1,187

NO FIRST-YEAR DIVIDEND PAYABLE

$$\frac{10^6\,(\text{Prem}) - 2{,}821\,(\text{Tot 10 Div}) - 3{,}294\,(\text{Tot 20 Div}) - 2{,}880\,({}_{20}\text{CV} + {}_{20}\text{TD})}{1{,}000(\quad) + (\quad)(\text{Tot 10 Div}) - (\quad)(\text{Tot 20 Div}) - (\quad)\,{}_{20}\text{CV} - (\quad)\,{}_{20}\text{TD}}$$

Issue Age	1,000( )	( )(Tot 10 Div)		( )(Tot 20 Div)		( ) $_{20}$CV	( ) $_{20}$TD
0	181	67	(69)	45	(43)	50	50
5	141	76	(77)	48	(45)	55	50
10	157	89	(90)	55	(52)	62	50
15	182	100	(102)	62	(59)	70	55
20	208	113	(114)	70	(66)	83	62
25	253	140	(140)	86	(80)	109	70
30	339	195	(193)	119	(109)	158	83
35	494	294	(290)	177	(162)	239	109
40	753	452	(445)	271	(249)	364	158
45	1,156	696	(687)	417	(383)	553	239
50	1,770	1,063	(1,053)	637	(588)	825	364
55	2,670	1,597	(1,592)	960	(892)	1,187	553
60	3,925	2,310	(2,328)	1,398	(1,310)	1,612	825
65	5,641	3,184	(3,257)	1,953	(1,852)	2,075	1,187

DISCUSSION OF PRECEDING PAPER

DAPHNE D. BARTLETT:

At the present time, when existing disclosure and comparison methods are receiving considerable industry, governmental, and public attention, it is satisfying to observe that the actuarial profession has not abandoned its efforts to determine the ultimate method. Professor Ryall is to be congratulated on continuing the research in this area, and on presenting a new concept, that of the "standard," for our consideration. When the dust settles on Senator Hart's hearings, the Pennsylvania *Shoppers' Guide*, and the various current discussions on this subject, it is likely that there will emerge the need for two types of information:

1. A cost disclosure method. This would provide a basis for determining the actual cost of a specific policy and would probably be of interest only to an individual prospective purchaser.
2. A policy comparison method. This would provide a basis for a purchaser to compare one plan of insurance with another, in the same or a different company. It would also be used for published comparisons between plans and companies. (I have deliberately avoided the use of the expression "*cost* comparison method," since this carries the implication that use of such a method produces results which represent the actual costs of the policies being compared.)

Cost Disclosure Method

Because of the individual attitudes and characteristics of prospective buyers of life insurance, it will be very difficult to obtain a cost disclosure method which will provide true and satisfactory information to the purchaser in the form of a single number or index, or even a series of them. Perhaps cost is best disclosed in the form of a ledger illustration of the year-by-year premiums, dividends, death benefits, and cash values for the policy under consideration.

Policy Comparison Method

In competitive situations several cost disclosures of the type described above probably would tend to confuse rather than enlighten the prospect. Despite the many arguments for and against the comparison methods developed to date, there seems to be general agreement that a reasonably simple means for the prospect to compare plans and companies is essential. The necessary characteristics of the ideal method appear to reduce to the following: (1) the method should be simple to calculate and to understand; (2) the method should fairly and objectively allow comparisons between different plans and companies; (3) the method should guard against "actuarial manipulation"; and (4) the method should be "foolproof" against its use in a way that could misinform or be misinterpreted.

Against these criteria, Professor Ryall's method stands up fairly well. In the era of computers just about any method can be considered simple to calculate. (Also, for policy comparison purposes, I would suggest that it is not really necessary to have available a figure for the exact age of the insured and the exact amount of insurance being discussed. The nearest decennial or quinquennial age and the nearest $5,000 of amount might be considered adequate, thus reducing the burden of calculation and paperwork for companies and their agents.) Professor Ryall's method also satisfies the criterion that it be simple to understand. However, I do not think that is is necessary that either the agent or the prospect know that his index is based on the ratio of the two interest-adjusted indexes. Nor is it necessary that they understand how the indexes are calculated. Instead, an explanation that "the policy ranks *X* per cent above standard" would probably be sufficiently informative in most instances.

With respect to the criterion of fairness, Professor Ryall is silent on the yet-unsolved problem of reasonable comparison between participating and nonparticipating policies. It is difficult to judge as "fair" a method which grants equal credibility to guaranteed and unguaranteed payments, or even to "conservatively" and "generously" determined unguaranteed payments. Proper qualification of comparisons between participating and nonparticipating policies seems to be essential in the absence of further research on the subject. In practice, it is unlikely that proper qualification will occur.

The third criterion, protection against actuarial manipulation, is where I feel that Professor Ryall's method is most deficient. Although he recognizes that differences in early cash values may occur between policies which his index shows as identical, he attempts to reflect these differences by means of additional calculations and suggests legislative action and monitoring as a means of preserving a somewhat artificial correlation. This suggested solution does not cover extreme, but possibly existing, situations where, currently quite legally, low cash values could be provided in all policy years until the one at which the index is calculated.

At this point it is possibly worthwhile to mention that, while many of the existing comparison methods produce rankings which have high correlations with each other, this does not necessarily imply that each method is equally good. The correlations are high currently, because whatever actuarial manipulation has occurred to date has been to produce favorable results on the traditional or the interest-adjusted cost methods. If another method permitting manipulation were to become commonplace, the manipulation would occur to produce favorable results by that method. This could produce significantly different rankings. For the reasons discussed above, Professor Ryall's method could not be described as one which would never mislead or be misinterpreted. To be realistic, however, I doubt whether any method can satisfy that particular criterion!

Another "Standard" Policy Comparison Method

Professor Ryall points out in his paper that the choice of a standard makes very little difference in the resulting rankings of policies being compared. When rankings are all that are required (under the assumption that full disclosure of costs has already been made), the actual numerical value of the index used for comparison has no absolute significance.

I would like to suggest a method involving a standard, which also facilitates comparisons between different plans and companies, which produces a result which is reasonably simple to calculate and explain, and which does not lie open to actuarial manipulation.

The method uses a standard set of lapse and mortality rates. Objections as to the inapplicability of these to a particular prospect are many, but the objections generally assume that the comparison illustration using these assumptions is the only one provided. If a proper cost disclosure is also provided, however, the prospect becomes interested in comparison only in a general sense ("is this a good deal or isn't it?)" rather than with respect to his individual situation, and in this sense, average assumptions are appropriate.

Under the suggested concept, a "standard policy" is created which would be used in all policy comparison situations. A possible standard might be a whole life policy with a "gross premium," G_x^s, equal to the net level annual premium on 1958 CSO 3 per cent. The standard cash values, ${}_tCV_x^s$, in this policy might equal minimum cash values, also on 1958 CSO 3 per cent.

An index, ${}_nI_x$, would be calculated for the policy being compared which would be equal to the present value of the deviations of this policy from the standard. The formula would be as follows:

$$
\begin{aligned}
{}_nI_x = \sum_{t=1}^{n} v^t \; {}_{t|}q_x^{(w)}({}_tCV_x - {}_tCV_x^s) + \sum_{t=1}^{n} v^t \; {}_{t|}q_x^{(m)}({}_tF_x - {}_tF_x^s) \\
+ \sum_{t=1}^{n} v^t \; {}_tp_x^{(T)} \; {}_tD_x - \sum_{t=1}^{n} v^{t-1} \; {}_{t-1}p_x^{(T)}({}_tG_x - G_x^s) \,,
\end{aligned}
$$

where x is the issue age; n is the comparison period; t is the policy year; ${}_tCV_x$, ${}_tF_x$, ${}_tD_x$, and ${}_tG_x$ are the cash values, face amounts, illustrative dividends, and gross premiums in policy year for the policy being compared; ${}_tCV_x^s$, F_x^s, and G_x^s are the cash values, face amount, and gross premium for the standard policy; v^t is calculated on a standard interest rate; and ${}_{t|}q_x^{(w)}$, ${}_{t|}q_x^{(m)}$, and ${}_tp_x^{(T)}$ are probabilities of withdrawal, death, and survival, calculated according to standard persistency and mortality tables. Appropriate provision would be made for termination dividends.

A policy identical with the standard in all respects would thus have an index of zero. A policy, otherwise identical, but with a lower gross premium, would develop a positive (above-standard) index. Level term and decreasing term policies could be compared with each other or with permanent policies by means of comparison with the same standard. This would be accomplished over the entire comparison period. A plan which had a shorter benefit period than the comparison period, therefore, would have ${}_tG_x$, ${}_tCV_x$, and ${}_tD_x$ values of zero in the years beyond its expiry.

This method, I believe, satisfies the first three criteria described above, with the exceptions noted below.

SIMPLICITY OF CALCULATION

The method requires more calculation than Professor Ryall's. However, the basic formula can be expressed as follows, resulting in a need for calculation of the standard policy's present values only once for each age:

$$\begin{aligned} {}_nI_x = {} & \sum_{t=1}^{n} v^t \; {}_{t|}q_x^{(w)} \; {}_tCV_x + \sum_{t=1}^{n} v^t \; {}_{t|}q_x^{(m)} \; {}_tF_x + \sum_{t=1}^{n} v^t \; {}_tp_x^{(T)} \; {}_tD_x \\ & - \sum_{t=1}^{n} v^{t-1} \; {}_{t-1}p_x^{(T)} \; {}_tG_x - \sum_{t=1}^{n} v^t \; {}_{t|}q_x^{(w)} \; {}_tCV_x^s \\ & - \sum_{t=1}^{n} v^t \; {}_{t|}q_x^{(m)} \; F_x^s + \sum_{t=1}^{n} v^{t-1} \; {}_{t-1}p_x^{(T)} \; G_x^s \, . \end{aligned}$$

With computers, calculation of the present values for the policy under consideration are not too burdensome, particularly if, as I suggest, only a few ages are required. (The expression above suggests an alternative index:

$${}_nI'_x = \frac{\sum_{t=1}^{n} v^t \; {}_{t|}q_x^{(w)} \; {}_tCV_x + \sum_{t=1}^{n} v^t \; {}_{t|}q_x^{(m)} \; {}_tF_x + \sum_{t=1}^{n} v^t \; {}_tp_x^{(T)} \; {}_tD_x - \sum_{t=1}^{n} v^{t-1} \; {}_{t-1}p_x^{(T)} \; {}_tG_x}{\sum_{t=1}^{n} v^t \; {}_{t|}q_x^{(w)} \; {}_tCV_x^s + \sum_{t=1}^{n} v^t \; {}_{t|}q_x^{(m)} \; F_x^s - \sum_{t=1}^{n} v^{t-1} \; {}_{t-1}p_x^{(T)} \; G_x^s} \, .$$

The "identical with standard" policy described above would then have an index value of unity. The "above standard" policy would have an index value greater than unity.)

ILLUSTRATIVE DIVIDENDS

I can offer no solution to the problem that an index for a participating policy will be used in comparison with one for a nonparticipating policy, however careful the required qualifications may be. It has been suggested that participating policies be illustrated on both a guaranteed and a nonguaranteed basis in all instances, but this is somewhat unfair to policies with high premiums and high dividend scales. There has also been discussion of analysis of dividend histories as a basis for determining the credibility of future illustrations. This does not seem realistic in view of the trend of interest rates over the last twenty-five years, the currently increasingly competitive life insurance marketplace, and the fact that management philosophies could have changed significantly over the period being reviewed.

While the most obvious problem exists in the direct and unqualified comparison of a participating with a nonparticipating policy, similar situations can arise, as stated earlier, between "conservatively" and "generously" determined illustrative dividend scales. This is not exclusively a problem for stock companies, therefore. I have a few ideas on this subject, which may be worthy of further investigation:

1. Creation of a participating standard which is actuarially equivalent to the nonparticipating standard policy, as a means of reducing the absolute differences between the two types of policies.
2. Allowing credibility for dividend illustrations only to the extent that their "value" (with appropriate adjustment for premium and cash-value levels) does not exceed that of some generally acceptable standard scale of illustrations.
3. Analysis of the assumptions inherent in a participating and a nonparticipating premium and dividend illustration scale. Considering interest only, it is common for the actuary determining a projected dividend scale to assume a level rate of interest and for the actuary determining a nonparticipating premium scale to assume a decreasing rate of interest. Both are reasonable actuarial assumptions; neither may be realized. Is there a way to merge the two sets of assumptions to create a "most likely" scale which could be used for policy comparison purposes?

Again, I would like to congratulate Professor Ryall on his work. I offer my comments, criticisms, and suggestions only in the interest of furthering the debate on this challenging and timely topic.

J. STANLEY HILL:

In reviewing yet another modern cost comparison method, one hopes that the prospective users of such methods may be encouraged, rather than dismayed, by the increasing number of choices available to them. Having walked the same path as Professor Ryall in the search for a valid, understandable, and practical approach to comparing the costs of dissimilar policies, I find myself in agreement with him on major points:

1. Rejection of the concept of applying individual lapse rates.
2. Rejection of the need for a mortality table specifically suitable to the individual prospect.
3. Emphasis on the need for the individual to select an interest rate suitable to him.
4. The observation that younger policyholders pay higher costs as a percentage of the pure cost of death protection.
5. Applicability of our methods to replacement considerations.

There is a most striking similarity in the methods themselves: both his method and mine (*TSA*, XXIII, 289) are based on the ratio of the interest-adjusted cost (IAC) to a standardized IAC; the difference is in the method of calculating the standardized IAC. The author derives his standardized IAC without the use of intervening cash values, which does not disturb me seriously. But in his (to me) unconvincing arguments against the use of intervening cash values, I feel "he doth protest too much."

The concept of the net amount at risk as the excess of the face over the cash value is (1) traditional, (2) easily explained, (3) justifiable under utility theory, and (4) true in a real and practical sense. Statements 1 and 2 hopefully need no enlargement. I have dealt with statement 3 at length in my paper (*TSA*, XXIII, 305). As to statement 4, let us consider two whole life policies fully loaned throughout their lifetime, both taken with the intention of surrendering them at the end of twenty years for their identical twentieth-year cash values, identical in all other respects except that policy A has lower intervening cash values. Let anyone delivering settlement checks to the widow ten years after issue try to explain that policy B, even though the claim check is smaller, was really the "better buy"!

The fact that policy A would have cost more if surrendered after ten years is irrelevant in the calculation of the twenty-year cost factor but is fully and properly reflected in the calculation of the ten-year cost factor. And I agree readily with the author that cost factors for more than one period should be examined by the prospective buyer. To argue otherwise, one must go all the way with Professor Belth and espouse the inclusion of individual mortality and lapse rates in a complex, single calculation which, however theoretically satisfying, defies valid practical application—not so much for the difficulty of calculation and understanding as for the difficulty in selecting valid individual termination rates.

Although comparative evaluation might better be left to a more objective reviewer, I could not help comparing the author's method with mine on the criteria of validity, understandability, and ease of calculation. Both methods seem valid and subject to the same qualifications (e.g., the desirability of determining the values for several periods; the need to use a near-riskless, aftertax, individually valid interest rate; the nonguaranteed nature of dividends; the nonmathematical aspects of the contract; the value of the agent's services; and other considerations such as the three reservations expressed by the author in his discussion of my paper [*TSA*, XXIII, 312]). Although I would maintain some slight superiority for the standard mortality cost method (because it does take intervening cash values into account), I suspect that the differences are not sufficiently large to be of much practical importance.

Both methods seem understandable to a prospect in the same sense: he can readily understand their significance *if* he takes the standardized IAC on faith.

The relative ease of calculation could be debated, but it seems academic in the light of the following consideration: if a company chooses to include either cost measure in its computerized sales illustrations, the additional cost is negligible; if it does not, the use of such measures will be limited to brokers (or agents acting as brokers) who can buy such calculations through time-sharing networks much more economically than they can have them done on desk calculators.

CHARLES L. TROWBRIDGE:

Professor Ryall's paper is an addition to the already substantial literature on methods of price comparison for individual life insurance. Continued interest by consumer-oriented legislators and regulators in this fascinating but elusive subject is an indication that this newest contribution is particularly well timed.

Professor Ryall seems to accept the interest-adjusted method proposed by the Moorhead report as a satisfactory method for comparing essentially similar life insurance contracts but suggests the ratio of the results of two interest-adjusted calculations to compare dissimilar policies. The denominator of the ratio performs the role of standardizing the price illustrations for plan differences.

An implication of the initial point in Professor Ryall's abstract is that the interest-adjusted method is one that views the life policy as a single entity and avoids the fragmentation of the life insurance contract into savings and protection elements. It is true that splitting out of the amount at risk is not so obviously accomplished as under certain other methods, but two of the formulas in this paper rather neatly indicate that the interest-adjusted method really belongs in the category of methods that divide the premium into protection and savings elements.

First,

$$\text{IAC index} = \pi - X,$$

where

$$X = \left[\sum_{t=1}^{n}(1+i)^{n-t}\ {}_{t}DIV + {}_{n}CV + {}_{n}TD\right] \Big/ \ddot{s}_{\overline{n}|},$$

can be interpreted to mean that the IAC index is the remainder of the gross premium after the savings (X) element has been subtracted. Here the dividends are treated as additions to savings rather than as subtractions from premium.

Second,

$$\text{IAC index} = \pi - P' + Y,$$

where

$$Y = \left\{\sum_{t=1}^{n}(1+i)^{n-t}\ q'_{x+t-1}[(1+i)^{1/2} - {}_{t}V' - {}_{t}DIV]\right\} \Big/ \ddot{s}_{\overline{n}|},$$

can be interpreted to mean that the IAC index is a loading ($\pi - P'$) element plus a protection (Y) element. The protection element is a function of the values of q'_x (on the "standardizing" mortality table) and the amounts at risk, and P' is the net premium, on the same mortality table, to provide the benefits (including dividends) within the n-year analysis period and the cash value (and terminal dividend) at the end of such period.

The ratios that Professor Ryall is interested in then become $(\pi - X)/(P' - X)$ in one form and $(\pi - P' + Y)/Y$ in the other. It is obvious, in either form, that the ratio is greater than unity if $\pi > P'$—that is, the gross premium is greater than the net premium P'—calculated on the standardizing mortality assumptions and the rate of interest employed in the interest-adjusted cost calculation. The use of the 1958 CSO Table, with its built-in margins, and an interest rate of 4 per cent seems to produce ratios of less than unity rather frequently. The implication of a negative loading may hurt the creditability of this approach.

I wonder whether it has occurred to the author that he might have based his comparisons on π/P' rather than on $(\pi - X)/(P' - X)$. The interpretation would be easier, and one could truly say that this approach looks at the policy as a single entity. Of course all connection with the interest-adjusted method would have been lost.

Professor Ryall's attempt to compare dissimilar policies in a systematic way reminds me that another cost illustration method can easily be adapted to make such comparisons. The basic cost index, by which policies with similar death and endowment benefits are to be compared, is simply the present value of future premiums less the present value of future dividends. Discounts for mortality and interest only are contemplated, and cash values are to be ignored (on the assumption that the policyholder bought with the intention of continuing). Adjustment of such an index to reflect fairly differences in death or endowment features could be accomplished by dividing the basic index by the net single premium (on identical mortality and interest assumptions) for the death and endowment benefits provided. This method of adjustment has many features in common with the similarly intended adjustment proposed by Professor Ryall.

(AUTHOR'S REVIEW OF DISCUSSION)

PETER L. J. RYALL:

I appreciate the thoughtful comments of Mrs. Bartlett and Messrs. Hill and Trowbridge. Their discussions will be of interest to all those interested in the problem of cost comparison among dissimilar policies.

I cannot agree with Mrs. Bartlett that her method of cost comparison is fair, at least for comparisons among participating policies. Comparisons for an analysis period shorter than the full term of the policy are, under most methods, substantially affected by the cash values available at the end of the period. This is not the case with Mrs. Bartlett's method. Thus, if the probabilities of withdrawal are taken to be zero, her index is unaffected by the scale of cash values.

The over-all cost for a policyholder who continues his contract in force beyond the end of the analysis period is clearly affected by the size of the dividends then payable. However, the size of these dividends is related to the size of the cash value at the end of the analysis period. Thus, on a given plan, a participating policy with high cash values calculated on a low interest basis will generally have a much steeper dividend scale than a policy with values calculated on a comparatively high interest basis. Hence the size of the cash value at the end of the analysis period is of significance for the continuing as well as the then terminating policyholder. (For further discussion of this matter see the last paragraph of Sec. II of the paper.)