The formulas presented in this paper are intended to solve two problems encountered in defined contribution plans sponsored by unincorporated sole proprietorships. One problem is determining the sole proprietor’s earned income. The other problem is determining an allocation formula that provides a specific addition to the sole proprietor’s account and at the same time meets the requirements defining earned income. The formulas are developed for both money purchase plans and target benefit plans.

The formulas also solve an analogous problem for a solely owned corporation, that is, distributing profits between compensation to the owner and contributions to the plan. Furthermore, the formulas may sometimes be adopted to solve the problem of distributing profits from a partnership.

Although iterative methods, such as successive bisection, can be used to solve these problems, iteration can be quite time-consuming. Therefore, the formulas have great practical value both in routine plan administration and in proposal work. They are ideally suited for an electronic spreadsheet.

FORMULAS FOR "EARNED INCOME"

The compensation used for a sole proprietor in a tax-qualified plan of an unincorporated business must meet certain requirements imposed by the Internal Revenue Code of the United States.1 In general, the sole proprietor’s compensation must be earned income. Essentially, earned income is defined in the code as the pretax profit realized by the sole proprietor from business operations, net of all expenses including the expense for deductible contributions made to the plan on behalf of both the proprietor and common law employees.2

1See Sections 401(c), 404(a)(8), 415(b)(3), and 415(c)(3) of the Internal Revenue Code of the United States for background.

2The net business profit reported on income tax form Schedule C (attachment to the U.S. individual income tax form 1040) already includes an expense for plan contributions on behalf of
If \( PR \) represents the preliminary profit of the business, net of all expenses except the expense of total deductible plan contributions, the sole proprietor’s earned income, \( EI \), can be symbolically defined as follows:

\[
EI = PR - (AS + AE - Z).
\]  

(1)

In Equation (1), the total expense for deductible plan contributions is \( AS + AE - Z \), where \( AS \) and \( AE \) represent total additions (that is, contributions plus forfeitures) allocated to the accounts of the sole proprietor and common law employees, respectively, and \( Z \) represents forfeitures, which are assumed to reduce the required contribution to the plan. (See Glossary and Tables 1 and 2 for definitions.)

The above equation for \( EI \) can be rewritten as follows:

\[
EI - PR + (AS + AE - Z) = 0.
\]

Now \( PR \) and \( Z \) are constants, and \( AE \) will be a constant if the plan’s allocation formula is fixed. So, because \( AS \) is a linear function of \( EI \) as defined in Table 1, the left-hand side of the last equation is a linear function \( f \) of the variable \( EI \). Thus, the sole proprietor’s earned income has a unique value because there is only one \( EI \) for which

\[
f(EI) = 0.
\]

**Formula for Earned Income**

The formula for the sole proprietor’s earned income is derived by assuming the business has an integrated money purchase pension plan; the earned income formula is similarly derived when the business has a target benefit plan. The earned income formula for both types of plans is shown in Table 1. (Refer to the Glossary for definitions of other variables in the formulas.)

common law employees. Thus, earned income will generally be the net business profit reported on income tax form Schedule C further reduced by deductible contributions to the plan on behalf of the sole proprietor as reported on income tax form 1040.

Ignoring any potential coverage and discrimination issues under code Sections 401(a)(4) and 410(b), assume that the plan requires a participant be credited with 1,000 hours of service to receive an allocation based on the parameters \( p \) and \( q \); and assume that—even though employee B is actively employed at all times during the plan year and is a plan participant—B is credited with only 501 hours of service. In this case, B will not receive an allocation of $900 (15% \times $6,000). But, because the plan is top-heavy, B must receive (under regulation 1.416-1/M-10) a top-heavy minimum contribution of $180 (3% \times $6,000).
As shown in Table 1, assuming $EI > IL$, the definition of $AS$ is

$$AS = p \times EI + q \times (EI - IL).$$

Substituting the above expression for $AS$ into the equation $EI = PR - (AS + AE - Z)$ and assuming that $EI > IL$ results in the formula:

$$EI = \frac{PR + Z - AE + q \times IL}{1 + (p + q)}.$$  \hfill (2)

An application of the formula follows. To use the above formula, the allocation parameters $p$, $q$, and $IL$ must already be known. The special cases in which a plan is not integrated or the assumption $EI > IL$ is not satisfied are discussed later (see Sections on Non-integrated Plan and Special Problems).

**Example 1**

Suppose an unincorporated sole proprietorship sponsors a top-heavy money purchase pension plan with allocation formula parameters $p = 15\%$, $q = 5.7\%$, and $IL = $48,000. Also assume that preliminary profits, $PR$, for
the year are $190,000 and that there are two common law employees as follows:

<table>
<thead>
<tr>
<th>Common Law Employee</th>
<th>Compensation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$50,000</td>
</tr>
<tr>
<td>B*</td>
<td>$6,000*</td>
</tr>
</tbody>
</table>

*Eligible only for 3% top-heavy minimum contribution of $180. (Ignoring any potential coverage and discrimination issues under code Sections 401(a)(4) and 410(b), assume that the plan requires a participant be credited with 1,000 hours of service to receive an allocation based on the parameters p and q; and assume that—even though employee B is actively employed at all times during the plan year and is a plan participant—B is credited with only 501 hours of service. In this case, B will not receive an allocation of $900 (15% × $6,000). But, because the plan is top-heavy, B must receive (under regulation 1.416-1/M-10) a top-heavy minimum contribution of $180 (3% × $6,000).

If there are no forfeitures to reduce the plan’s required contribution, What is the sole proprietor’s earned income? and What is the addition allocated to the proprietor’s account?

To solve for $E_1$ by using the above formula, first determine $A_E$, total additions for common law employees. Because employee B receives only a top-heavy minimum addition of $180, $A_E = $7,794. Using this value of $A_E$ in the formula for $E_1$, we obtain earned income of $E_1 = $153,224.52. Thus, the addition to the sole proprietor’s account is $A_S = $28,981.48.

As a consistency check, we validate that Equation (1) is satisfied, namely, that $E_1 = PR - (A_S + A_E - Z)$. Because there are no forfeitures to reduce the plan’s contribution requirement, $Z = 0$, so that

$$153,224.52 = 190,000 - (28,981.48 + 7,794 - 0).$$

**Forfeitures**

The formulas in Table 1 assume that forfeitures reduce the required contribution to the plan, as is the case with traditional money purchase plans. In this regard, $A_S$ and $A_E$ represent the total amount of contributions plus forfeitures that are allocated to accounts (not just contributions).
By setting $Z$ equal to zero, the formulas in Table 1 can be used when forfeitures do not reduce the required contribution to the plan, but are allocated in addition to contributions, as with traditional profit-sharing plans. In such cases, $AS$ and $AE$ represent only contributions allocated to accounts, not contributions plus forfeitures.

The following example illustrates the effect of forfeitures in not only reducing the required plan contribution, but also increasing the sole proprietor’s earned income.

**Example 2**

An unincorporated sole proprietorship has preliminary profits, $PR$, of $190,000$ and two common law employees as in Example 1. Suppose the business sponsors a money purchase pension plan as in Example 1 but that there are $3,000$ of forfeitures that reduce the plan’s contribution requirement. What is the sole proprietor’s earned income? and What is the net required contribution to the plan?

Because the total contribution to the plan in Example 1 was $36,775.48$, it may first appear that the net required plan contribution would be $33,775.48$ ($36,775.48 - $3,000). However, because $EI$ and $AS$ are not independent, forfeitures do not reduce the contribution requirement dollar-for-dollar. To determine the net contribution to the plan, apply the formulas from Table 1 as follows.

Proceeding as in Example 1, first calculate $AE$ of $7,794$, the same amount as in Example 1. Then use Equation (2) to calculate the sole proprietor’s earned income, $EI$, as $155,710.02$. Note that the sole proprietor’s earned income here is slightly greater than that in Example 1, because forfeitures reduce the amount of business profits needed to fund the plan. Based on this earned income, the addition to the sole proprietor’s account is $AS$ of $29,495.98$, so that the net required plan contribution is $34,289.98$ ($29,495.98 + $7,794 - $3,000).

Finally, as a consistency check, we validate that Equation (1) is satisfied, namely:

$$155,710.02 = 190,000 - (29,495.98 + 7,794 - 3,000).$$

**FORMULAS FOR ALLOCATION PARAMETER $p$**

In the previous two examples, the allocation formula was already established, so it was only necessary to find a solution for $EI$, after which all
plan computations could be made. However, the allocation formula may be undetermined as, for example, when a plan is being established.

In determining a plan allocation formula for businesses of this type, the objective is usually to provide the maximum allowable addition to the sole proprietor's account and, at the same time, provide the least possible amount—within the requirements of the law—to the accounts of common law employees. In the typical case, these objectives are achieved by (1) minimizing $p$ and (2) maximizing $q$ in the allocation formula for $AS$. Because the maximum value allowed for $q$ is usually fixed by the "integration rules" found in Section 401(l) of the Internal Revenue Code of the United States, we need only determine the minimum value of $p$ that will provide the desired addition to the owner's account.

**Formula for Minimum $p$**

The formula for the minimum allocation percentage, $p$, that will provide a $30,000 addition to the sole proprietor's account will be derived for an integrated money purchase plan of an unincorporated sole proprietorship. The derivation of $p$ for the other cases shown in Table 2 is similar.

### Table 2

**Quadratic Coefficients $a$, $b$, and $c$**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Money Purchase Plans</th>
<th>Target Benefit Plans</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$K$</td>
<td>$K$</td>
</tr>
<tr>
<td>$b$</td>
<td>$-0.25 \times K - (PR + Z - FA - q \times K)$</td>
<td>$(-0.25/f_s) \times K - (PR + Z - FA - q_s \times K)$</td>
</tr>
<tr>
<td>$c$</td>
<td>$0.25 \times (PR + Z - FA + q \times IL)$</td>
<td>$(0.25/f_s) \times (PR + Z - FA + f_s \times q_s \times IL_s)$</td>
</tr>
</tbody>
</table>

$AS = 25\% \times E/\$

$AS = 30,000$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Money Purchase Plans</th>
<th>Target Benefit Plans</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$30,000 - (PR + Z - FA - q \times K)$</td>
<td>$30,000 - (PR + Z - FA - q_s \times K)$</td>
</tr>
<tr>
<td>$b$</td>
<td>$30,000 \times (1 + q) - q$</td>
<td>$30,000 \times (1/f_s) + q_s - q_s$</td>
</tr>
<tr>
<td>$c$</td>
<td>$30,000 \times (PR + Z - FA - IL)$</td>
<td>$30,000 \times (PR + Z - FA - IL_s)$</td>
</tr>
</tbody>
</table>

Notes:
1. See Glossary for definition of terms.
2. Subscript $s$ in formulas indicates value for sole proprietor/owner.
3. Money purchase plan formulas result from target benefit plan formulas by assigning $f_s = 1$.

$^3$The Tax Reform Act of 1986 (TRA 86) limits the maximum value of $q$ to the lesser of $p$, or (at the current time) 5.7 percent. It is assumed here that $p$ is greater than 5.7% so that $q$ is not dependent on $p$.  

It is important to note that even though the formula results in an addition of $30,000 for the sole proprietor, the "25 percent limitation" under Section 415(c) of the Internal Revenue Code of the United States is not necessarily satisfied. This is an interesting point and is discussed later (see Special Problems).

First, because the addition to the sole proprietor will be $AS = $30,000, the definition of $AS$ from Table 1 (assuming $EI > IL$) results in the following requirement for $p$:

$$\$30,000 = p \times EI + q \times (EI - IL).$$

The above equation can be rewritten as:

$$EI = \frac{(30,000 + q \times IL)}{(p + q)} \quad (3)$$

Again, because we have assumed that $AS = $30,000, Equation (1), defining the sole proprietor's earned income, becomes:

$$EI = PR - (\$30,000 + AE - Z).$$

In this last equation, both $EI$ and $AE$ are functions of $p$. The right-hand side of the equation can be expressed solely in terms of $p$ if $AE$ can be broken down into two components, one that involves $p$ and the other that does not. By defining the terms $K$ and $FA$, which are constants (see Glossary), we assume $AE$ can be expressed as

$$AE = p \times K + FA.$$

Thus, the above equation for $EI$ can be written as

$$EI = PR - (30,000 + [p \times K + FA] - Z). \quad (4)$$

Both Equations (3) and (4) must be satisfied by the solution for $p$. So, by equating the right-hand side of both equations and thus eliminating the variable $EI$, the following quadratic equation involving $p$ results:

$$ap^2 + bp + c = 0$$

where $a$, $b$, and $c$ are defined in Table 2. The quadratic equation will have two solutions for $p$ given by:

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$
(Note that an electronic spreadsheet or calculator may not be able to compute a numerical value for the quadratic expression because of the magnitude of the $b^2$ term. A "transformation" that reduces the magnitude of the numbers can be used to avoid this problem. One such transformation for the radical is as follows:

$$\sqrt{b^2 - 4ac} = 10^n \times \sqrt{(b/10^n)^2 - (4ac/10^{2n})}$$

A transformation factor of 1,000 ($n = 3$) is sufficient for most cases.)

Thus, the minimum value of $p$ that satisfies both Equations (3) and (4) is the smallest non-negative solution using the above formula. Example 3 below gives an application of the formula.

Several observations can be made about the above formula for $p$. Note that the graph of Equation (3), when expressed in the form $EI = f_1(p)$, is a hyperbola with a vertical asymptote of $p = -q$ and a horizontal asymptote of $EI = 0$ (see Figure 1). The graph of Equation (4), when expressed in the form $EI = f_2(p)$, is a straight line with a downward slope.

**FIGURE 1**

**EQUATIONS (3) AND (4) FOR "$30,000 FORMULA"**

(BASED ON DATA FROM EXAMPLE 1)

\[
EI = \frac{30,000 + q \times IL}{p + q}
\]

\[
EI = PR - (30,000 + [p \times K + FA] - Z)
\]

$p = -5.7\%$
A solution for \( p \) exists only where the hyperbola and the straight line intersect. When \( b^2 - 4ac \) is zero, there is the single solution \( p = -b/2a \) and the straight line is tangent to the hyperbola. When \( b^2 - 4ac \) is positive, there are two different values for \( p \) given by the quadratic equation and the straight line intersects the hyperbola at two points. When \( b^2 - 4ac \) is negative, the quadratic formula yields imaginary solutions and the straight line and hyperbola do not intersect.

As a final observation, note that it is not possible to use linear interpolation to find the minimum solution for \( p \) because Equation (3) is a hyperbola. Without using the quadratic formula, the solution for \( p \) could be obtained by iterative methods such as successive bisections, but this might take several trials.

Finally, it would have been just as easy to derive a formula for \( E_1 \) from Equations (3) and (4) rather than a formula for \( p \). If a formula for \( E_1 \) had been derived, the solution for \( p \) could then be determined from the definition of \( AS \) found in Table 1.

**Example 3**

Suppose an integrated money purchase plan will be established by an unincorporated sole proprietorship. Assume that preliminary profits \( PR \) are $190,000, that the same employee data as in Example 1 apply, and that there are no forfeitures. What allocation formula will provide a $30,000 addition to the sole proprietor's account at the least possible employee funding cost?

To determine the allocation formula, first assign the integrated allocation parameters \( q = 5.7\% \) and \( IL = $48,000 \). The remaining undetermined variable in the allocation formula is \( p \), the non-integrated allocation percentage. To derive the minimum value of \( p \) that will provide a $30,000 addition to the sole proprietor's account, first calculate quadratic coefficients \( a, b, \) and \( c \) (see formulas in Table 2):

\[
\begin{align*}
    a &= 50,000.00 \\
    b &= (156,856.00) \\
    c &= 23,632.76.
\end{align*}
\]

In deriving the values of \( a, b, \) and \( c \) above, note that \( K = 50,000 \) and \( FA = 294 = (3\% \times $6,000) + 5.7\% \times ($50,000 - $4,800) \). Thus, compensation for employee B is not included as part of \( K \) because the addition for employee
B is a "fixed" addition that does not vary as a function of $p$. An employee's compensation is included in the term $K$ only if the employee's addition varies with the allocation percentage $p$.

Now solve the quadratic equation for $p$ by using the above values for $a$, $b$, and $c$ to obtain the two solutions $p = \{2.98; 0.158693\}$. Thus, the minimum solution for $p$ is $p = 15.8693\%$. To check the solution for $p$, we apply it to the basic employee data and calculate $AE = $8,228.64; and using Equation (2) we calculate $EI = $151,771.36. Thus,

$$AS = 15.8693\% \times EI + 5.7\% \times (EI - 48,000) = $30,000.$$  

Finally, as a consistency check, we validate that Equation (1) is satisfied, that is, that $EI = PR - (AS + AE - Z)$. Because there are no forfeitures that reduce the contribution requirement to the plan, $Z = 0$ and

$$151,771.36 = $190,000 - ($30,000 + $8,228.64 - 0).$$

**Generalized Formulas for $p$**

Some cases may require an allocation formula that provides an addition to the sole proprietor's account other than $30,000 or 25 percent of earned income. For example, an allocation formula providing an addition equal to 15 percent of earned income may be desired. A formula for this could be derived in the same way as the formulas in Tables 1 and 2 were derived above.

However, the expressions for $a$, $b$, and $c$ in Table 2 all have the same general form. The forms differ depending on $AS$, which is either a percentage of earned income or a fixed dollar amount. The equations have been written in such a way that the parameters "25 percent" and "$30,000" are isolated. Based on this observation, a generalized set of formulas can be written for the quadratic coefficients $a$, $b$, and $c$. If the addition to the sole proprietor's account is $r$ percent of earned income, the quadratic coefficients for a money purchase plan will be

$$a = K \quad (5a)$$

$$b = - \frac{r}{100} \times K - (PR + Z - FA - q \times K) \quad (5b)$$

$$c = \frac{r}{100} \times (PR + Z - FA + q \times IL) - q \times (PR + Z - FA - IL). \quad (5c)$$
If the addition to the sole proprietor’s account is a fixed dollar amount \( D \), the quadratic coefficients for a money purchase plan will be

\[
a = K \quad \text{(6a)}
\]

\[
b = D - (PR + Z - FA - q \times K) \quad \text{(6b)}
\]

\[
c = D \times (1 + q) - q \times (PR + Z - FA - IL). \quad \text{(6c)}
\]

Equations for target benefit plans can be similarly generalized.

**Non-Integrated Plans**

If the plan is not integrated or if \( EI < IL \), the formulas for \( EI \) and \( p \) collapse to the desired non-integrated expression by setting \( q \) equal to zero. When \( q \) equals zero and the 25 percent formulas are used, the formula for \( p \) becomes trivial. The trivial formula is \( p = 25\% \) for a non-integrated money purchase plan and \( p = 25\% \div f_x \) for a non-integrated target benefit plan (see Glossary for definition of \( f_x \)).

**Target Benefit Plans**

The formulas for target benefit plans found in Tables 1 and 2 are derived and applied in essentially the same way as the formulas for money purchase plans. The target benefit plan expressions have a form similar to those for money purchase plans, but differ by the appearance of a weighting factor, \( f_x \). The factor \( f_x \) is the funding factor for participant \( x \) (see definition in Glossary).

If the factor \( f_x \) is set equal to 1, the target benefit plan formulas collapse into the money purchase plan formulas. Thus, the money purchase plan formulas are a special case of the targeted benefit plan formulas.

**Modification for Funding Method**

If the Individual Level Premium (ILP) cost method is used to fund the target benefit plan, the sole proprietor’s addition in the second and later plan years is defined as

\[
A_S_y = A_S_{y-1} + \Delta A_S,
\]

where \( y \geq 2 \), and where

\[
\Delta A_S = f_y \times (TB_y - TB_{y-1}).
\]
In other words, the sole proprietor’s addition in year \( y \geq 2 \) is defined as the addition in the prior year plus an incremental funding amount, \( \Delta AS \). The incremental funding is the amount necessary to fund the change in the target benefit \( TB_y - TB_{y-1} \).

Clearly this definition of \( AS_y \) required by the ILP cost method for \( y \geq 2 \) is not equivalent to the definition of \( AS \) in Table 1. Therefore, the formula for earned income in Table 1 is invalid if the ILP cost method is used and if the target benefit plan is in its second or later year. However, a formula for earned income can be derived that satisfies this definition of \( AS_y \). The formula for \( EI \) for \( y \geq 2 \) for a target benefit plan using the ILP cost method is

\[
EI = \frac{PR + Z - AE + f_{s,y} \times q_s \times IL_s + \Delta f_s \times TB_{s,y-1}}{1 + f_{s,y} \times (p + q)}
\]

This expression for earned income has the same form as the expression for \( EI \) in Table 1 except for the adjustment term \( \Delta f_s \times TB_{s,y-1} \) in the numerator. The term \( \Delta f_s = f_{s,y} - f_{s,y-1} \) is the increment in the funding factor between the two years; and \( TB_{s,y-1} = p \times EI_{y-1} + q_s \times (EI_{y-1} - IL_s) \) is the sole proprietor’s target benefit from the prior year’s allocation. The derivation of the above formula assumes that \( p_s, q_s, \) and \( IL_s \) are the same in both plan years and that earned income exceeds the integration level in both years. If any of these assumptions are not met in actuality, the formula is invalid.

**SPECIAL PROBLEMS**

The formulas cannot always provide the desired solution for a given problem. And even where a solution for \( EI \) or \( p \) is possible, it may be invalidated for other reasons. Some of the problems that can arise in using the formulas are illustrated below. In each case, examples show how erroneous or inconsistent results can be avoided.

**Acid Test**

The most fundamental and important test of a solution for \( EI \) or \( p \) is the acid test. This test verifies that Equation (1), the basic definition of earned income, is satisfied, namely, that

\[
EI = PR - (AS + AE - Z).
\]

The test is made by actually performing the allocation (using the solutions for \( EI \) and \( p \)) and determining the values of \( AS \) and \( AE \). If the values of \( EI \), \( AS \), and \( AE \) do not satisfy Equation (1), then some inconsistency exists.
Conversely, however, passing the acid test does not guarantee that the solutions are free from other potential problems (see below).

In each of the previous examples, the solutions for $EI$ or $p$ passed the acid test. The examples below illustrate situations in which the acid test can fail and situations in which, even though the acid test is passed, the solutions cannot be used for other reasons.

**Integrated Allocation Formulas**

In certain instances, it may not be possible to derive a solution for $p$ when the allocation formula is integrated. The formulas in Tables 1 and 2 were derived by assuming the sole proprietor’s earned income was greater than the integration level, $IL$. However, the sole proprietor’s actual earned income may be less than $IL$. In such cases, the actual data are inconsistent with the assumptions implicit in the formulas themselves.

Actual earned income may be less than $IL$ when (1) preliminary profits are small or (2) when total additions for common law employees are quite large. In these cases, either imaginary solutions will be derived for $p$ or $p$ will be larger than 25 percent.

**Example 4**

To illustrate how the formulas may fail to provide a solution for $p$ for an integrated top-heavy money purchase plan when preliminary profits are small, assume that $PR = $70,000, $Z = 0$, $q = 5.7\%$, $IL = $48,000, and the same employee data apply as in Example 1.

If we try to solve for $p$ by using these parameters in the $30,000$ formula, the quadratic equation yields solutions that are imaginary numbers. On the other hand, if we try to solve for $p$ by using the 25 percent formula, the acid test is not satisfied, and furthermore, the solution for $p$ is greater than 25 percent. In this particular case, there is no “excess compensation,” so the only possible solution is a non-integrated plan (set $q$ equal to zero) using the 25 percent formula.

\[a = 50,000; b = (36,856); c = 30,472.76; \text{ and } b^2 - 4ac = (4,736,186,864).\]

\[p = 25.9838\%; AE = 12,940.89; EI = 45,647.29; AS = 11,545.93.\]
Fixed Dollar Addition of $30,000

A solution for \( p \) is sometimes impossible to obtain when the $30,000 formulas are being used—regardless of whether the plan is integrated. For instance, the problem presented in the last example cannot be solved with a non-integrated plan providing $30,000 to the sole proprietor. If an attempt is made to use the $30,000 formula, the solutions will be imaginary.\(^6\)

The result of obtaining imaginary solutions can be seen by referring to Figure 1. As a function of \( p \), the relative placement of the hyperbola in the Cartesian plane does not depend on preliminary profits, \( P_R \). However, the relative placement of the straight line of Equation (4) does depend on \( P_R \). As \( P_R \) increases, the straight line of Equation (4) moves up (without change of slope); as \( P_R \) decreases, the line moves down. If \( P_R \) decreases beyond a certain point, the straight line moves outside the concave region enclosed by the hyperbola. When the straight line moves outside this region, it fails to intersect the hyperbola, thus indicating that a solution for \( p \) is non-existent. A similar illustration can be made to show the effect of employee funding costs on obtaining a solution.

Top-Heavy Minimums

The formulas may fail to provide a solution for \( p \) if the apportionment of \( A_E \) into its component terms \( K \) and \( F_A \) is inconsistent with the actual results of the final allocation. This can happen, for example, in a target benefit plan in which the actual addition for an employee is a top-heavy minimum, but the initial assumption was that the employee would receive a formula addition dependent on the term \( K \). In such cases, the acid test will fail, indicating the inconsistency between that which was assumed and that which is actual.

It may also be impossible to obtain a solution for earned income in the unusual case in which \( p \) and \( q \) are very small and the allocation formula is integrated. The formulas cannot be used in such cases because the requirement for top-heavy contributions is not "fixed."

Example 5

To illustrate how the formulas may fail to provide a solution for earned income in a top-heavy money purchase plan when \( p \) and \( q \) are small and the

\[ a = 50,000; \quad b = (39,706); \quad c = 30,000; \quad \text{and} \quad b^2 - 4ac \approx (4,423,433,564). \]
plan is integrated, assume the same data as in Example 1 apply, except that \( p = q = 1 \) percent. If \( r \) represents the addition to the sole proprietor’s account as a percentage of earned income, then \( r \) must be greater than 1 percent but less than 2 percent. Thus, the top-heavy minimum addition for employee B must also be \( r \) percent of B’s compensation because \( r \) is less than 3 percent.7

Unless we already know \( r \), the earned income formula in Table 1 cannot be used because the formula uses \( AE \) as one of its components and \( AE \) depends on \( r \). If we accidentally overlook the fact that employee B’s addition is less than 3 percent of compensation and determine \( E_1 \) by assuming employee B gets a 3 percent addition, the acid test will fail, if the allocation is then independently performed, indicating the erroneous and inconsistent assumption.

**IRS Code Section 415 Limits**

Even though the formulas for \( p \) result in additions of $30,000 or 25 percent of earned income for the sole proprietor, the limitations under Section 415(c) of the code may not be satisfied. Furthermore, the limitation of code Section 415(e) may apply and the formulas would require an adjustment.

**Example 6**

To illustrate how the solution for \( p \) might violate the 25 percent limitation under code Section 415(c) in a money purchase plan, assume that \( PR = $160,000, Z = 0, q = 5.7\% \), and the same employee data apply as in the examples above. At first blush, \( PR \) appears sufficiently large to suggest using the $30,000 formula. Applying this formula, we obtain \( p = 21.8613\% \), and this yields a $30,000 addition to the sole proprietor’s account. However, earned income, \( EI \), is then $118,775.36, so the sole proprietor’s addition is 25.2578% of earned income, and this exceeds the 25 percent limitation under code Section 415(c). Thus, despite the fact the acid test is satisfied, this extraneous limitation is violated. To avoid violating the 415(c) limit, the solution for \( p \) must be re-determined by using the 25 percent formula.

**Example 7**

The dollar limitation under code Section 415(c) might similarly be violated. This can be illustrated with a money purchase plan assuming that \( PR = $170,000, Z = 0, q = 5.7\% \), and the same employee data apply as in the

7See Section 416 of the Internal Revenue Code of the United States for background.
examples above. Using the 25 percent formula under these assumptions provides the sole proprietor an addition of $AS = $31,796.08, which is 25 percent of earned income. But this addition violates the $30,000 limitation of code Section 415(c) even though the acid test is satisfied.

**Example 8**

A special problem arises when limitations under code Section 415(e) apply. The resolution of the problem depends on whether the allocation percentage $p$ is fixed or can be changed. In each of the two cases below, assume that code Section 415(e) limits the sole proprietor’s total annual addition to $22,000.

**Case 1.** This case illustrates how to determine $E1$ when the allocation percentage $p$ is fixed. Let us revisit Example 1, which shows that the sole proprietor would, without the 415(e) limitation, receive an annual addition of $AS = $28,981.48. Because only $22,000 may be allocated to the sole proprietor’s account, earned income simply becomes $160,206 = $190,000 − ($22,000 − $7,794).

In this case, Equation (2), which is normally used to determine $E1$, is no longer valid because the usual allocation formula defining $AS$ in terms of $p$, $q$, and $IL$ is no longer applicable. The limitation under code Section 415(e) superimposes an additional condition and Equation (2) becomes invalid. Therefore, $E1$ must be obtained directly from the fundamental equation, Equation (1). Note that additions to accounts of common law employees are not affected.

**Case 2.** This case illustrates how to determine $p$ and $E1$ when $p$ is not fixed. Assume now that the plan in Case 1 is a profit-sharing rather than a money purchase plan and that $p$ can be varied on a discretionary basis (equivalently, that the sponsor has discretion over the given dollar amount contributed to the plan). To determine the value of $p$ that will provide a $22,000 addition to the sole proprietor’s account, it is only necessary to invoke the use of the generalized quadratic coefficients in Equations (6a), (6b), and (6c). By setting $D = $22,000 in these equations, we obtain $p = 9.4786\%$ and $E1 = $162,966.72. In this case, because $p$ has been reduced, additions to the accounts of common law employees are also reduced to $AE = $5,033.28.
$200,000 Compensation Limitation

Another extraneous limitation that applies to plan calculations but that is not inherently reflected in the formulas is the $200,000 limitation on compensation found in Section 401(a)(17). This extraneous limit must also be scrutinized in the final allocation.

Example 9

Resolution of the special problems that arise when earned income exceeds $200,000 is illustrated in the two cases below depending on whether the allocation percentage $p$ is fixed or can be changed.

Case 1. This case illustrates how to determine $E_1$ when the allocation percentage $p$ is fixed. Assume for the illustration that there is a top-heavy money purchase plan with $p = 10\%$, $q = 5.7\%$, $IL = $48,000, $Z = 0$; that the same employee data as in Example 1 apply; and that $PR = $240,000. We initially calculate $E_1 = $205,222.13, $AS = $29,483.87, and $AE = $5,294. However, the value of $E_1$ exceeds the maximum amount of earned income allowed to be recognized in the allocation. So when the allocation is actually performed, the amount of earned income recognized must be limited to $200,000 and this reduces the sole proprietor’s addition to $AS = $28,664.

Thus, based on Equation (1), earned income is actually $E_1 = $240,000 - ($28,664 + $5,294 - 0) = $206,042.

As with Case 1 of Example 8, Equation (2), which is normally used to determine $E_1$, is no longer valid because the usual relationship between $E_1$ and $AS$ in terms of $p$, $q$, and $IL$ is no longer applicable. The $200,000 limitation superimposes an additional condition that makes Equation (2) invalid. Therefore, $E_1$ must be obtained directly from the fundamental equation, Equation (1). Note that additions to accounts of common law employees are not affected.

Case 2. This case illustrates how to determine $p$ and $E_1$ when $p$ is not fixed. Assume now that the plan in Case 1 is a profit-sharing rather than a money purchase plan and that $p$ can be varied on a discretionary basis (equivalently, that the sponsor has discretion over the given dollar amount that is contributed to the plan). Suppose, furthermore, that the desired addition to the sole proprietor’s account is $30,000.

We first use the $30,000 formulas in Table 2 to determine that applying $p_1 = 10.3036\%$ to earned income, $E_1$, of $204,554.21$ will result in a $30,000 addition for the sole proprietor. However, this amount of earned income exceeds the maximum amount that can be recognized in the allocation. So
when the allocation is actually performed, $E_I$ would be limited to $200,000$ and the sole proprietor's addition would be less than $30,000$ when based on $p_1$. Therefore, it is necessary to determine $p$ as follows:

$$p = 10.668\% = \left[\frac{$30,000 - 5.7\% \times ($200,000 - $48,000)}{$200,000}\right].$$

To verify this value of $p$ will produce the desired result, first use it to calculate $AE = $5,628. Then verify that earned income will be at least $200,000:

$$EI = $240,000 - ($30,000 + $5,628 - 0) = $204,372.$$

Because this value of $p$ results in earned income of at least $200,000$, it is exactly the value of $p$ we are seeking. If we select a larger value of $p$, we have not minimized $AE$, additions to the accounts of common law employees. And if any smaller value of $p$ is used, then the sole proprietor's addition will be less than $30,000$ (because even though decreasing $p$ would increase earned income, $E_I$ would still be limited to $200,000$, and the smaller value of $p$ applied to $200,000$ would result in an addition of less than $30,000$).

**Summary**

The above examples show that solutions obtained by rote application of the formulas sometimes need to be ruled out. As a practical matter though, solutions for all the formulas can be easily generated if an electronic spreadsheet is used. Then, by examination of the results, solutions that do not satisfy the various limitations or that are inconsistent can be eliminated or the input parameters adjusted.

**SOLELY OWNED CORPORATIONS**

A problem similar to that of determining earned income exists for businesses that are solely owned corporations rather than unincorporated sole proprietorships. Suppose a solely owned corporation that sponsors a defined contribution pension plan has residual profits at the end of a fiscal year and that the corporation's tax accountant advises the owner to "zero out" the residual profits by taking additional compensation in the form of a bonus. By paying a bonus equal to the entire residual profits, the corporation will be put into a loss position because the bonus will also increase the required plan contribution, assuming it is recognized as compensation in the plan's allocation formula. Thus, to have a bottom-line profit of exactly zero, the marginal increase in the plan's contribution must also be taken into account in determining the bonus.
The problem of exactly achieving a zero bottom-line profit can be stated symbolically as follows. First, let $PR$ represent the business's preliminary profit at the end of the fiscal year net of all expenses except (1) the expense of the owner's total compensation for the fiscal year, $C$, and (2) the expense of total net plan funding, $AS + AE - Z$. The net profit or loss for the corporation can be symbolized as a function $g$ whose independent variable is $C$:

$$g(C) = PR - (AS + AE - Z) - C.$$  

The variables $AS$, $AE$, and $Z$ in the above equation have the same meaning as previously defined. Thus, the objective is to find $C$ such that:

$$g(C) = 0.$$  

Recognizing that the functions $f$ and $g$ are essentially the same solves both problems with a single stroke. $C$ is determined in exactly the same way as $EI$. The following example illustrates how this is done.

**Example 10**

An incorporated professional practice, whose sole owner is $Q$, has two non-owner employees, $A$ and $B$, as in Example 1. A preliminary accounting just prior to the close of the corporate fiscal year shows that $Q$ has already been paid $120,000 in wages throughout the fiscal year and that the corporation has a profit of $70,000 net of all expenses except income taxes and the required contribution to the corporation's top-heavy money purchase plan. The money purchase plan has the same allocation parameters, $p$, $q$, and $IL$, as in Example 1. What bonus should $Q$ receive so there is no corporate profit?

The solution is obtained by restating the problem in a slightly different form. First, let $X$ be the bonus $Q$ will receive. After paying the bonus $X$ to $Q$ and making the required contribution of $AS + AE - Z$ to the money purchase plan, the net corporate profit will be $70,000 - X - (AS + AE - Z)$. Because this profit is to be zero, we have

$$X = 70,000 - (AS + AE - Z).$$

Now let $EI$ represent the total compensation paid to $Q$ for the fiscal year. Then $EI = X + $120,000, and by substituting the above expression for $X$ into this expression for $EI$, we have the equation

$$EI = 190,000 - (AS + AE - Z).$$
This last equation is the same as in Example 1. Hence, $EI = $153,224.52 and so Q's bonus is $33,224.52 ($153,224.52 − $120,000). Furthermore, we verify the corporate profit is zero by noting that the sum of the total plan contribution and the bonus is $70,000 = $36,775.48 + $33,224.52.

PARTNERSHIPS

The formulas can sometimes, with appropriate modifications, be applied to the problem of distributing profits from a partnership. This problem can be similar to determining "earned income" for a sole proprietor because contributions made to a plan reduce profits distributable to the partners; but distributable profits are, in turn, the basis for making the plan contributions.

Example 11

As a simple illustration, let Q1 and Q2 be partners in a business venture. The business has two employees, A and B, as in Example 1 and the partnership agreement provides that Q1 bears 60 percent and Q2 bears 40 percent of the business's expenses relating to employees A and B. Before making any plan contributions, a preliminary accounting for the business shows the following:

<table>
<thead>
<tr>
<th>Partner</th>
<th>Profit before Plan Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>$190,000</td>
</tr>
<tr>
<td>Q2</td>
<td>$120,000</td>
</tr>
<tr>
<td>Total</td>
<td>$310,000</td>
</tr>
</tbody>
</table>

If the business sponsors a top-heavy profit sharing plan with $q = 5.7\%$, $IL = $48,000, and $Z = 0$ and if we assume Q1 is to receive a $30,000 addition in the plan, what is the net profit distributable to Q1 and Q2?

The problem is solved by first determining the allocation percentage $p$ that will provide Q1 with a $30,000 addition, and then determining earned income (that is, distributable profit) for Q1 and Q2 by using Equation (2).

Because Q1 bears only 60 percent of the cost of additions for employees A and B, the $30,000 formula is applied by uniformly substituting 60 percent of both $K$ and $FA$ in each of the expressions for $a$, $b$, and $c$ in the quadratic formula. Thus, using $K1 = 60\% \times K = $30,000 and $FA1 = 60\% \times FA = $176.40, we determine $a = 30,000$, $b = <158,113.60>$, $c = 23,626.05$, and $p = 15.3920\%$. 


Based on this value of $p$, total additions to the plan for employees A and B are $AE = $7,990. The portions of $AE$ born by Q1 and Q2 are $AE1 = $4,794 and $AE2 = $3,196, respectively; and using these values in Equation (2), we have distributable profits $EI1 = $155,206 and $EI2 = $98,718.37.

To verify the results, note that the total contribution to the business’s plan is $56,075.64 ($30,000 + $18,085.64 + $7,990) and that total plan contributions plus total partnership distributions are $EI1 + EI2 + $56,075.64 = $310,000. This verification shows the acid test is satisfied.

GLOSSARY

$AE = \text{Total additions (that is, contributions and forfeitures) allocated to accounts of common law employees, assuming forfeitures reduce the required plan contribution (see note under variable Z below regarding forfeitures). It is assumed that } AE \text{ can be expressed in terms of the variables } K \text{ and } FA \text{ (see Table 1 for the algebraic expression for } AE \text{ in terms of } K \text{ and } FA). \text{ If it is not possible to express } AE \text{ in these terms, the formulas for } p \text{ in Tables 1 and 2 cannot be used.}$

$AS = \text{Total additions (that is, contributions and forfeitures) allocated to the sole proprietor’s account, assuming that forfeitures reduce the required plan contribution (see note under variable Z below regarding forfeitures). See formulas in Table 1 for the algebraic expression of } AS.$

$C_i = \text{Compensation for common law employee } i.$

$EI = \text{Earned income of the sole proprietor of an unincorporated business (see Sections 401(c), 404(a)(8), 415(b)(3), and 415(c)(3) of the Internal Revenue Code of the United States for background). If the business is incorporated, } EI \text{ represents the total compensation of the owner.}$

$FA = \text{The portion of total additions for common law employees that is fixed and does not with } p. \text{ } FA \text{ would include, for example, additions due to compensation in excess of the integration level, } IL, \text{ and any extra additions not provided by the plan’s allocation formula that are needed to satisfy the top-heavy minimum rules. Algebraically,}$

\[
FA = \sum q_x (C_i - IL > 0) + \sum \text{extra top-heavy minimums}
\]

\[
\text{Target Benefit Plans:}
FA = \sum f_i x q_i x (C_i - IL_i > 0) + \sum \text{extra top-heavy minimums}
\]
$f_i$ = Funding factor in target benefit plan for participant $i$. For example, in the first plan year when the Individual Level Premium cost method is used, the addition for participant $i$ equals his/her target benefit times $f_i$, where $f_i = \bar{a}_n + s_n$, and where $ra$ is the participant's retirement age, and $n$ the number of years to fund the target benefit.

$IL$ = Integration level in the allocation formula. Use of a subscript, for example, $IL_i$, indicates the integration level varies by participant as might covered compensation.

$K$ = Proportionality constant for common law employees based on compensation associated with allocation percentage $p$. Algebraically,

Money Purchase Plans: $K = \sum C_i$

Target Benefit Plans: $K = \sum f_i \times C_i$.

Compensation, $C_i$, for common law employee $i$ is included in the term $K$ only if his/her addition is a function of $p$. Thus, for example, compensation is not included in $K$ for common law employees who receive only top-heavy minimum additions that do not depend on $p$.

$p$ = The non-integrated allocation percentage of the allocation formula that is applied to a participant’s total compensation under the plan. See formulas in Table 1.

$PR$ = Preliminary profits of the business. For a sole proprietorship, $PR$ is generally the preliminary profit reported on Schedule C (attachment to individual income tax form 1040), net of all expenses except the expense for plan contributions on behalf of common law employees. For a solely owned corporation, $PR$ represents preliminary profit of the business, but not the expense of the owner’s total compensation for the fiscal year or the expense of plan funding.

$q$ = The integrated allocation percentage in the allocation formula that is applied only to a participant’s plan compensation in excess of the plan’s integration level, $IL$. Use of a subscript, for example, $q_i$, indicates the integration percentage varies by participant, as it might in a target benefit plan.

$Z$ = Total forfeitures to be allocated to accounts, assuming forfeitures reduce the required contribution to the plan. Note: If forfeitures do not reduce the required contribution but are allocated in addition to contributions to the plan, the formulas may be used by assigning $Z = 0$. When this is done, $AE$ and $AS$ represent only contributions allocated to the account (not contributions plus forfeitures).