Session 48 PD, Real World vs Risk Neutral: Practical Implications on Models

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[SOA Presentation Disclaimer](#)
ESGs – Case Studies

Josh Dobiac
Life and Annuity Symposium 2017 – Seattle, WA
ESG construction plays a role in many different frameworks from statutory capital to hedging.
Risk-Neutral vs Real-World

How do you decide? It depends on what you are trying to accomplish
Case Study – Hedging VAs with GLWBs

• Insurer has a block of Variable Annuities with Guaranteed Lifetime Withdrawal Benefits. The insurer wants to reduce the blocks sensitivity to market movements.
  • Institutes a hedging program that hedges both interest rate and equity movements using delta-one assets only (equity and rate futures, swaps, total return swaps, etc.)
  • How does the insurer determine how much of each contract to purchase? Market-consistent risk-neutral scenarios!
  • Insurer calculates market consistent value of the VAs, then shocks the base equity and rate levels and revalues the block. The difference between the base and shock valuations represents the block’s sensitivity to rate and equity movements.
  • With this information, derivatives can be purchased that match those sensitivities.
Case Study – Estimating Earnings Emergence

- Management is working on a financial plan for the next five years
  - Wants to understand how earnings emerge for all products over projection period under expected market behavior, with a 95% confidence interval.
- Confidence Interval ➔ Real-World Scenarios!!
- Construct a baseline line set of scenarios that represent a management best-estimate of how markets will behave over the next five years.
- Scenarios are stochastic, so they are allowed to vary over that time.
- At each time point, using the evolution of market information, projections of capital/reserves, cash flows, etc. can be made.
- Earnings emergence can then be calculated with confidence intervals, including different earnings metrics (IRR, NPV, accounting profit, or whatever other measures are of interest).
Case Study – Economic Capital

- Economic Capital often involves combined real-world and risk-neutral scenarios
  - Insurer wants to know what its 1-in-200 year risk exposure is on an economic basis.
  - 1-in-200 year = 99.5% Value-at-Risk (VaR).
  - VaR ➔ Real World
  - Economic Basis ➔ Risk Neutral
  - “Outer-Loop” Scenarios are used to estimate the VaR and are thus real-world. “Inner-Loop” Scenarios are generated for each outer-loop path and are used to calculate the economic capital on that path.
  - When economic capital has been estimated for all outer-loop paths, a 99.5% VaR can then be estimated.
  - This represents the EC for liabilities.
LAS 2017 Session 48

Real World vs Risk Neutral: Practical Implications on Models

Ricky Power, FSA, FIA, CERA
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Agenda

1. What is an ESG?
2. Real World and Risk Neutral
3. Real World Validation and Example Interest Rate model.
4. Risk Neutral Validation and Example Interest Rate model.
5. Summary
What is an ESG?
What is the Economic Scenario Generator?

» Generates scenario for economic variables and asset returns

Risk Factors (example)

» Real and nominal interest rates
» Credit spreads
» Equity volatility
» Exchange rates

Asset returns (example)

» Nominal & Index linked bonds
» Corporate Bond returns
» Equities
» Foreign assets

» Uses stochastic models to generate many paths for each risk factor and asset
Real World and Risk Neutral
Types of ESG Calibrations

Risk Neutral

For the *valuation* of complex liabilities

- All assets earn *risk-free rate*
- Monte-Carlo simulation *replicates market-price*
  - Distribution and statistics are market-implied

Real-World

Realistic *projection* of assets/liabilities

- Risky assets earn *risk-free rate* PLUS *risk-premium*
- Distribution and statistics are meaningful. One can set own assumptions about volatility and distributions of simulated rates.
Real World vs Risk Neutral: Practical Implications on Models

Calibration framework

- Regular target and methodology reviews
- Model R&D

Data

Market Data
- Initial Yield Curve
- Swaption Implied Volatilities

Historic Data
- Historic Standard Deviation
- Historic Average Returns
- Historic Correlations

Expert Judgement
- Long Term Assumptions
- Speed of Convergence

Review and update

Expert judgment

Target setting
- Long term (unconditional) targets
- Short term targets

Modeling

- Observe stylized facts
- Build and structure models accordingly

Symbols:
- $\alpha$
- $\beta$
- $\gamma$
- $r_t$
- $\sigma_t$
- $\pi_0$
- $\rho$
Real World
RW Calibration and Validation

1 Real-World Interest Rate Calibrations

1.1 Nominal Rates - Extended ZIBK Govt. Fit - Time-Varying Term Premium

Exhibit 1.1.1: Parameters in Extended ZIBK

- Parameters in the extended ZIBK (Exhibit 1.1.1) are calibrated to unconditional means of different interest rates. Details of our calibration method are available on our website. 

- The 30-year forward rate is a direct input in the unrestricted ZIBK model, which is defined by rolling regression spikes in the market yield (Exhibit 1.1.2) and corresponding to a 30-year forward rate. The extrapolation of current long-term rates is assumed.

- The 10-year forward rate is a direct input in the unrestricted ZIBK model, which is defined by rolling regression spikes in the market yield (Exhibit 1.1.2) and corresponding to a 10-year forward rate. The extrapolation of current long-term rates is assumed.

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Exhibit 1.1.2: Market Bond Yield vs Spindle Modal Fit

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Exhibit 1.1.3: Percentiles Nominal Short Rate

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Exhibit 1.1.4: Percentiles 10 Year Spot Rate

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Exhibit 1.1.5: Percentiles 5 Year Spot Rate

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Exhibit 1.1.6: Percentiles 1 Year Spot Rate

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Exhibit 1.1.7: Realised Back-testing

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Exhibit 1.1.8: Eurostoxx 50 Constant Vol

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Exhibit 1.1.9: Eurostoxx 50 SVJD
Real World Nominal Interest Rate Modelling

Extended 2-Factor Black-Karasinski
Key Features:
» Extended: input initial curve
» Displacement: absolute volatility gives more realistic distributions
» 2-Factors: de-correlated rates, realistic dynamics
» Easy to calibrate (good analytics)
» Easy to understand (moderate complexity)

Other ESG options:
LMM+, LMM, 3-Factor Cox-Ingersol-Ross, 2-Factor Hull-White, 2-Factor Vasicek.

\[
\begin{align*}
\text{d}ln(r'(t)) &= \alpha_1[ln(m(t)) - ln(r'(t))]dt + \sigma_1(dW_1 + \gamma_1(t)dt) \\
\text{d}ln(m(t)) &= \alpha_2[\mu'(t) - ln(m(t))]dt + \sigma_2(dW_2 + \gamma_2(t)dt)
\end{align*}
\]
Risk Neutral
**RN Calibration and Validation**

» **Returns**

- **ZCB Martingale**

- **Asset Martingale**

- **Implied Volatility or Price Test**

» **Volatility**

- **Swaption Tenor**

- **Swaption Maturity**

- **Implied Volatility or Price Test**

-Moody’s Analytics

Real World vs Risk Neutral: Practical Implications on Models
Nominal Rates - LMM+

1. Initial forward yield curve is a direct input to the model and is fit exactly.

2. Model of displaced forward rates where the magnitude of displacement can control the distribution of rates.

3. Stochastic volatility process allows for an excellent fit to market data both ATM and OTM.

\[
\frac{df_k(t)}{f_k(t) + \delta} = drift + \sqrt{V(t)} g_k(t) \cdot dW_t
\]

\[
dV(t) = \kappa (\theta - V(t))dt + \varepsilon \sqrt{V(t)}dZ_t
\]
Summary
Summary

Which model is best? Depends on what you are trying to achieve:

**Risk Neutral:***
- No risk premium
- Replicate the prices (or implied volatility) of market instruments

**Real World:***
- Risk premia (and control over them)
- Realistic forward looking distributions
ABOUT US
Moody’s Analytics helps capital markets and risk management professionals worldwide respond to an evolving marketplace with confidence. The company offers unique tools and best practices for measuring and managing risk through expertise and experience in credit analysis, economic research and financial risk management. By providing leading-edge software, advisory services, and research, including the proprietary analysis of Moody’s Investors Service, Moody’s Analytics integrates and customizes its offerings to address specific business challenges.
Section 1: Mathematical background

Melanie Dunn, ASA, MAAA
Overview

Mathematical background: real world vs risk neutral

Define

- Mathematical definition

Demonstrate

- Using a two scenario example, cover:
  - Pricing an asset
  - Transforming from real world to risk neutral
  - Pricing an option with a replicating portfolio

Compare

- Key conceptual differences
- Practical examples
Definitions
Risk neutral and real world

Risk neutral and real world are probability measures.

Risk neutral
- Probability measure such that the asset price is equal to the expected value of the cash flows discounted at the risk-free rate
- Risk neutral probabilities

Real world
- Probability measure such that the asset price is equal to the expected value of the cash flows discounted at the (risk-free rate + investor risk premium)
- Real world probabilities
Pricing an asset with two future scenarios

Real world

Scenarios

<table>
<thead>
<tr>
<th>p=0.5</th>
<th>1-p=0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50 payoff</td>
<td>$150 payoff</td>
</tr>
</tbody>
</table>

Risk free rate = 5%
Investor risk premium = 5%

Asset Value

\[
\frac{0.5 \times 50 + 0.5 \times 150}{1.00 + 0.05 + 0.05} = $90.91
\]
Pricing an asset with two future scenarios
Transform to risk neutral

Scenarios

<table>
<thead>
<tr>
<th>q=?</th>
<th>$50 payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-q=?</td>
<td>$150 payoff</td>
</tr>
</tbody>
</table>

Risk free rate = 5%
Investor risk premium = 0%

Asset Value

\[
\frac{q \times 50 + (1 - q) \times 150}{1.00 + 0.05 + 0} = 90.91
\]

\[
q = 0.5455
\]
Pricing an option with two future scenarios
Define the option and replicating portfolio

Risk free rate = 5%

Define an option

Pays (asset value - $100), if greater than zero.

\[
\begin{align*}
\text{asset} & : q \rightarrow \text{50 payoff} \\
& 1-q \rightarrow \text{150 payoff} \\
\text{option} & : q \rightarrow \text{0 payoff} \\
& 1-q \rightarrow \text{50 payoff}
\end{align*}
\]

Define a replicating portfolio

Purchasing ½ share of the asset and borrowing the PV of $25 is a replicating portfolio

\[
\begin{align*}
\text{replicating portfolio} & : q \rightarrow \text{asset payoff} = \frac{1}{2} \times 50 = 25 \\
& 1-q \rightarrow \text{borrowed CF} = -25 \\
& \quad \rightarrow \text{0} \\
& \quad \rightarrow \text{50}
\end{align*}
\]

\[
\text{replicating portfolio cost} = \frac{1}{2} \times 90.91 - \frac{25}{1.05} = 21.65
\]
### Pricing an option with two future scenarios

#### Price the option

**Real world**

<table>
<thead>
<tr>
<th>p=0.5</th>
<th>$0 payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-p=0.5</td>
<td>$50 payoff</td>
</tr>
</tbody>
</table>

**Option value**

\[
\frac{0.5 \times 0 + 0.5 \times 50}{1.00 + 0.05 + rp} = \text{unknown}
\]

**Risk premium unknown**

**Risk neutral**

<table>
<thead>
<tr>
<th>q=0.5455</th>
<th>$0 payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-q=0.4545</td>
<td>$50 payoff</td>
</tr>
</tbody>
</table>

**Option value**

\[
\frac{q \times 0 + (1 - q) \times 50}{1.00 + 0.05} = \$21.65
\]

**No risk premium needed!**

**Replicating portfolio cost**

\[
\frac{1}{2} \times \$90.91 - \frac{25}{1.05} = \$21.65
\]

---

**Risk free rate = 5%**

**Investor risk premium (on the option) = rp**

---

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### Real World
- Probabilities are those associated with each scenario occurring
- Discount rate is risk-free + risk premium
- Need to know the risk premium to price the asset (or know the market price to calculate the risk premium)
- Appropriate for assessing the range of likely outcomes

### Examples
- Pricing assets that cannot be hedged (e.g., earthquake insurance)
- Projecting balance sheets and income statements
- Value at risk
- Probability of losses exceeding a specified threshold

### Risk neutral
- Probabilities are solved for based on a known asset price
- Discount rate is risk-free
- No risk premium needed
- Appropriate when it is possible to replicate the asset using market securities

### Examples
- Pricing assets that can be hedged (e.g., swaptions)
- Market consistent valuation
- Embedded guarantees on VA products