On improving pension product design

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On improving pension product design

- Focus on DC pension plans:
  - Quickly expanding,
  - Easier and cheaper to administer,
  - More transparent and flexible so they can capture individuals’ needs.

- However,
  - If too much flexibility (e.g. U.S.), the participants do not know how to manage their saving and investment decisions.
  - If too little flexibility (e.g. Denmark), the product is generic and does not capture the individuals’ needs.
On improving pension product design

- Asset allocation, payout profile and level of death benefit capture the individual’s personal and economical characteristics:
  - current wealth, expected lifetime salary progression, mandatory and voluntary pension contributions, expected state retirement pension, risk preferences, choice of assets, health condition and bequest motive.

- Combine two optimization approaches:
  - Multistage stochastic programming (MSP)
  - Stochastic optimal control (dynamic programming, DP).
On improving pension product design

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Optimization approaches

stochastic optimal control (DP) - explicit solutions

✓ ideal framework - produce an optimal policy that is easy to understand and implement
✓ explicit solution may not exist
✓ difficult to solve when dealing with details

stochastic programming (MSP) - optimization software

✓ general purpose decision model with an objective function that can take a wide variety of forms
✓ can address realistic considerations, such as transaction costs
✓ can deal with details
✓ difficult to understand the solution
✓ problem size grows quickly as a function of number of periods and scenarios
✓ challenge to select a representative set of scenarios for the model
Combined MSP and DP approach

<table>
<thead>
<tr>
<th>( n_0, x_0 = 550 )</th>
<th>Benefits</th>
<th>34.4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Purchases</strong></td>
<td><strong>Sales</strong></td>
<td><strong>Allocation</strong></td>
</tr>
<tr>
<td>Cash</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>300.6</td>
<td>0.58</td>
</tr>
<tr>
<td>Dom. Stocks</td>
<td>177.3</td>
<td>0.34</td>
</tr>
<tr>
<td>Int. Stocks</td>
<td>37.7</td>
<td>0.08</td>
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<table>
<thead>
<tr>
<th>( n_1 )</th>
<th>Benefits</th>
<th>31.6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Purchases</strong></td>
<td><strong>Sales</strong></td>
<td><strong>Allocation</strong></td>
</tr>
<tr>
<td>Cash</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>98.8</td>
<td>0.49</td>
</tr>
<tr>
<td>Dom. Stocks</td>
<td>8.3</td>
<td>0.44</td>
</tr>
<tr>
<td>Int. Stocks</td>
<td>4.4</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Agnieszka K. Konicz - Technical University of Denmark
Maximize the expected utility of total retirement benefits and bequest given uncertain lifetime,

\[
\max \sum_{s=\max(t_0, T_R)}^{T-1} \sum_{n \in \mathcal{N}_s} sp_x u(s, B_{s,n}^{tot}) \cdot \text{prob}_n
\]

\[
+ \sum_{s=t_0}^{T-1} \sum_{n \in \mathcal{N}_s} sp_x q_{x+s} Ku(s, I_{s,n}^{tot}) \cdot \text{prob}_n
\]

\[
+ Tp_x \sum_{n \in \mathcal{N}_T} V\left(T, \sum_i X_{i,T,n}\right) \cdot \text{prob}_n
\]

**Parameters:**
- \(T_R\) retirement time,
- \(T\) end of decision horizon and beginning of DP,
- \(tp_x\) probability of surviving to age \(x + t\) given alive at age \(x\),
- \(q_x\) mort. rate for an \(x\)-year old,
- \(\text{prob}_n\) probability of being in node \(n\),
- \(K\) weight on bequest motive.

**Variables:**
- \(B_{t,n}^{tot}\) total benefits at time \(t\), node \(n\),
- \(I_{t,n}^{tot}\) bequest at time \(t\), node \(n\),
- \(X_{i,t,n}\) amount allocated to asset \(i\), period \(t\), node \(n\).

Richard, S. F. (1975),
Optimal consumption, portfolio and life insurance rules for an uncertain lived individual in a continuous time model.
Conclusions I

- Equally fair payout profiles given CRRA utility:

\[ u(t, B_t) = \frac{1}{\gamma} w_t^{1-\gamma} B_t^\gamma, \ w_t = e^{-1/(1-\gamma)\rho t} \]

Sensitivity to risk aversion \(1 - \gamma\) and impatience (time preference) factor \(\rho\).

\[ \bar{a}^*_{y+t} = \int_t^{\bar{T}} e^{-\int_t^s (\bar{r} + \bar{\mu}_\tau) \, d\tau} \, ds, \]

\[ B_t^* = \frac{X_t}{\bar{a}^*_{y+t}}, \]

Subjective mortality rate \(\mu_t = 10\nu_t\):
30\% chances to survive until age 75, <1\% chance to survive until age 85.

Savings upon retirement \(X_{TR} = 550,000\) EUR, \(b_{TR}^{state} = 0\), risk-free investment, no insurance.
More aggressive investment strategy and higher benefits given state retirement pension $b^{state}_{TR}$

<table>
<thead>
<tr>
<th>Expected asset allocation \ Age</th>
<th>$b^{state}_{TR} = 0$</th>
<th>$b^{state}_{TR} = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>20%</td>
<td>4%</td>
</tr>
<tr>
<td>Bonds</td>
<td>44</td>
<td>53</td>
</tr>
<tr>
<td>Dom. Stocks</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>Int. Stocks</td>
<td>11</td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Benefits $B^*<em>t$, $b^{state}</em>{TR} = 0$</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefits $B^*<em>t$, $b^{state}</em>{TR} = 5$</td>
<td>32.7</td>
<td>34.8</td>
<td>36.9</td>
<td>39.1</td>
<td>41.5</td>
<td>44.1</td>
</tr>
</tbody>
</table>

Left plot: expected optimal benefits. Right plot: optimal benefits based on historical returns: 3-m U.S. T-Bills, Barclays Agg. Bond, S&P500, MSCI EAFE.
Conclusions III

- Possible to adjust the investment strategy such that $B_{t}^{\text{tot}*} \geq b_{t}^{\text{min}}$
- Possible to adjust the investment strategy such that $\sum_{i} X_{i,t,n} \geq x_{t}^{\text{min}}$

(a) immediate annuity, $age_0 = 65$, $x_0 = 550$, $b_{TR}^{\text{state}} = 5$

(b) deferred annuity, $age_0 = 45$, $x_0 = 130$, $l_0 = 50$, $p_{\text{fixed}}^{\text{fixed}} = 15\%$, $p_{\text{vol}}^{\text{vol}} = 10\%$ (right plot only), $b_{TR}^{\text{state}} = 5$, $ins_{0}^{\text{fixed}} = 150$
Conclusions IV

- Possible to include individual’s preferences on portfolio composition,
  
  \[ X_{i,t,n} \geq d_i \sum_i X_{i,t,n}, \quad X_{i,t,n} \leq u_i \sum_i X_{i,t,n} \]
  
  e.g. \( d_{\text{bonds}} = 50\% \) and \( u_{\text{bonds}} = 70\% \).

- Though any additional constraints lead to a suboptimal solution (\( \Longrightarrow \) lower of more volatile benefits).

- Optimal investment vs. optimal fixed-mix portfolio:

![Optimal asset allocation](image)

Deferred life annuity. 20% lower expected benefits given the same risk level.
Left: optimal investment, \( E[B_t^{\text{tot}*}] = 46,200 \) EUR. Right: fixed-mix portfolio, \( E[B_t^{\text{tot}*}] = 37,700 \) EUR.
Selected references

A Heuristic for Moment-Matching Scenario Generation.


A combined stochastic programming and optimal control approach to personal finance and pensions.
http://www.staff.dtu.dk/agko/Research/~/media/agko/konicz_combined.ashx.

Spending retirement on planet Vulcan: The impact of longevity risk aversion on optimal withdrawal rates.

Assisting defined-benefit pension plans.

Optimal consumption, portfolio and life insurance rules for an uncertain lived individual in a continuous time model.
Appendix
Constraints I

**Budget equation** while the person is alive, $t \in \{t_0, \ldots, T - 1\}$, $n \in \mathcal{N}_t$:

$$B_{t,n}1_{\{t \geq T_R\}} + \nu_t I_{t,n}^{tot} + \sum_i X_{i,t,n}^{buy} = P_{t,n}^{tot} 1_{\{t < T_R\}} + \sum_i X_{i,t,n}^{sell} + \nu_t \sum_i X_{i,t,n}$$

**Value of the savings** at the beginning of period $t$:
- **before** rebalancing in asset $i$, $t \in \{t_0, \ldots, T\}$, $n \in \mathcal{N}_t$, $i \in \mathcal{A}$,

$$X_{i,t,n}^{\rightarrow} = x_{i,0} 1_{\{t=t_0\}} + (1 + r_{i,t,n}) X_{i,t-,n-} 1_{\{t>t_0\}} ,$$

- **after** rebalancing in asset $i$, $t \in \{t_0, \ldots, T - 1\}$, $n \in \mathcal{N}_t$, $i \in \mathcal{A}$,

$$X_{i,t,n} = X_{i,t,n}^{\rightarrow} + X_{i,t,n}^{buy} - X_{i,t,n}^{sell} ,$$

**Purchases and sales**, $t \in \{t_0, \ldots, T - 1\}$, $n \in \mathcal{N}_t$, $i \in \mathcal{A}$,

$$X_{i,t,n}^{buy} \geq 0, \ X_{i,t,n}^{sell} \geq 0.$$
Constraints II

Premiums, \( t \in \{t_0, \ldots, T - 1\}, \ n \in \mathcal{N}_t, \)

\[
P^{\text{tot}}_{t,n} = P_{t,n} + p^{\text{fixed}}_l, \\
P_{t,n} \leq p^{\text{vol}}_l,
\]

Benefits, \( t \in \{t_0, \ldots, T - 1\}, \ n \in \mathcal{N}_t, \)

\[
B^{\text{tot}}_{t,n} = B_{t,n} + b^{\text{state}}_t, \\
B^{\text{tot}}_{t,n} \geq b^{\text{min}}_t,
\]

Insurance, \( t \in \{t_0, \ldots, T - 1\}, \ n \in \mathcal{N}_t, \)

\[
l^{\text{tot}}_{t,n} = l_{t,n} + \text{ins}^{\text{fixed}}_t, \\
l_{t,n} \geq \text{ins}^{\text{min}} \sum_{i} X_{i,t,n},
\]

Portfolio composition, \( t \in \{t_0, \ldots, T - 1\}, \ n \in \mathcal{N}_t, \ i \in \mathcal{A}, \)

\[
X_{i,t,n} \leq u_i \sum_{i} X_{i,t,n}, \ X_{i,t,n} \geq d_i \sum_{i} X_{i,t,n},
\]

Minimum savings, \( t \in \{t_1, \ldots, T\}, \ n \in \mathcal{N}_t, \)

\[
\sum_{i} X_{i,t,n} \geq x^{\text{min}}_t.
\]
**End effect**

- DP - very specific and simplified model: power utility, risk-free asset, risky assets following GBM, Gompertz-Makeham mortality rate model, deterministic labor income and state retirement pension, no constraints on portfolio composition and no constraints on the size of savings or benefits.

Utility:

\[ u(t, B_t) = \frac{1}{\gamma} w_t^{1-\gamma} B_t^\gamma, \quad w_t = e^{-1/(1-\gamma)\rho t} \]

Optimal value function (**end effect**):

\[ V(t, x) = \frac{1}{\gamma} f_t^{1-\gamma} (x + g_t)^\gamma \]

Optimal controls:

- benefits: \[ B_t^* = \frac{w_t}{f_t} (X_t + g_t) - b_{state}^t \]
- sum insured: \[ I_t^{tot*} = \left( K \frac{\mu_t}{\nu_t} \right)^{1/(1-\gamma)} \frac{w_t}{f_t} (X_t + g_t) \]
- proportion in risky assets: \[ \pi_t^* = \frac{\alpha-r}{\sigma^2(1-\gamma)} \frac{X_t + g_t}{X_t} \]

- \( g_t \) - present value of future cashflows (labor income, retirement state pension, insurance price)
- \( f_t \) - optimal life annuity