Model To Develop A Provision For Adverse Deviation (PAD) For The Longevity Risk for Impaired Lives

Sudath Ranasinghe
University of Connecticut

41st Actuarial Research Conference - August 2006
Recent Mortality Trend

• Recent trends in life expectancies over the past century show dramatic improvement mostly at later ages.
• Mortality is improving due to recent medical advances, improvement in healthcare and health education, genetic research, therapeutic advances etc.,
• Recent trends in mortality improvement call for updated survival models when pricing and reserving life annuities and other LTC benefits.

Source: National Vital Statistics
Projecting Mortality Improvements

• Mortality Improvement Projection procedures or models
  ➢ Should reduce inconsistencies that may emerge as a result of the extrapolation.
  ➢ Should recognize the current mortality trend.
  ➢ Should be able to minimize Random mortality fluctuations and systematic deviations.
Mortality Risk

There are two types of risks:

1. **Risk of Random Fluctuation (Statistical Volatility)**

- Well-known type of risk in the insurance business, in both life and non-life areas.
- Fundamental results in risk theory state that the severity of this risk decreases as the portfolio size increases.

![Graph showing future mortality experience vs projected mortality over calendar years.]

- Future mortality experience
- Projected mortality

$q_r(y)$

calendar year $y$
2. Longevity Risk (Systematic Risk)

- Future mortality experience
- Projected mortality

- The risk exists as the result of an actual mortality trend different from the forecasted one.
- The systematic deviations can be thought of as a “model risk” or “parameter risk”, referring to the model used for projecting mortality and the relevant parameters.
- The risk cannot be hedged by increasing the portfolio size. On the contrary, its financial impact increases as the portfolio size increases.
Analyzing Mortality Risk in Deterministic and Stochastic Framework

- The survival function \( S(x) \) (\( S(x) = P\{T_0 > x\} \) where \( T_0 \) is a random lifetime of a newborn) can be obtained from the past data or projections.

- The random present value of benefits, \( Y \) is given by
  \[
  Y = \sum_{k=1}^{K_x} R_k v^k
  \]
  \( K_x \) = the curtate residual lifetime of the insured age \( x \)
  \( R_k \) = payment made by the insurer in the \( k^{\text{th}} \) year

- The random present value of benefits for the portfolio, \( Y_{\text{Tot}} \)
  \[
  Y_{\text{Tot}} = \sum_{i=1}^{N} Y_i
  \]
  \( Y_i \) = random present value of the \( i^{\text{th}} \) insured

- Under the hypothesis of homogenous and independent risks, we can obtain the followings for fixed \( S \):
  \[
  E[Y_{\text{Tot}}] = N \cdot E[Y_i]
  \]
  \[
  \text{Var}(Y_{\text{Tot}}) = N \cdot \text{Var}(Y_i)
  \]

- The mortality risk can be measured by the coefficient of variation of \( Y_{\text{Tot}} \) (Risk Index)
  \[
  \text{Risk Index} = r = \frac{\sqrt{\text{Var}(Y_{\text{Tot}})}}{E[Y_{\text{Tot}}]} = \frac{1}{\sqrt{N}} \cdot \frac{\sqrt{\text{Var}(Y_i)}}{E[Y_i]}
  \]
Analyzing mortality risk in deterministic and Stochastic framework (Cont’d)

- In a stochastic framework, a finite set of survival functions, S will be adopted and assigned to them probability distribution P, where $P = \{p_1, p_2, \ldots, p_k\}$ with $\sum p_i = 1$

- The probability distribution, P is assigned according to the degree of confidence in corresponding projection.

- The risk index of the portfolio can be obtained as follows:

$$E[Y_{Tot}] = E_p[E[Y_{Tot} | S]] = N E_p[E[Y | S]]$$

$$Var(Y_{Tot}) = E_p[Var(Y_{Tot} | S)] + Var_p(E[Y_{Tot} | S]) = N E_p[Var(Y | S)] + N^2 Var_p(E[Y | S])$$

$$r = \frac{\sqrt{Var(Y_{Tot} | S)}}{E[Y_{Tot} | S]} = \sqrt{\frac{1}{N} \cdot \frac{E_p[Var(Y | S)]}{E^2[Y | S]} + \frac{Var_p(E[Y | S])}{E^2 [Y | S]}}$$

- The 1st term of $r$ shows the random fluctuation risk as in the deterministic case. The 2nd term is the longevity risk which is independent of $N$.

- The deterministic approach can only address the random fluctuation risk.
The Mortality risk can be addressed by:

- Establishing an adequate solvency margin
- Reinsuring
- Investing in Longevity Bonds
- Developing a model to calculate a Provision for Adverse Deviation (PAD)

The current study only focuses on developing a PAD model.
PAD Model

- Two components of the longevity:
  - Slope risk – Risk of **under pricing** given benefit because of failure to capture the correct impaired mortality slope (impaired mortality pattern) of the policyholder;
  - Misstatement risk – Risk that the underwriter will **understate** true life expectancy (LE) based on medical information obtained at underwriting;

- Statistical volatility risk – Risk that **actual** future years lived will exceed “true” LE;

Statistical volatility and slope risks exist even if there is **no** misstatement risk.
Methodology

1. Obtain impaired mortality table (deterministically or Stochastically) using u/w LE and impairment type
2. Obtain Misstatement Risk and Statistical Volatility PADs using impaired mortality table
3. Final PAD = Misstatement Risk PAD + Statistical Volatility PAD

Relevant Medical information

Obtain impaired mortality table (deterministically or Stochastically) using u/w LE and impairment type

Population data, Life insurance Data, etc.

Obtain Misstatement Risk and Statistical Volatility PADs using impaired mortality table

Final PAD = Misstatement Risk PAD + Statistical Volatility PAD

Estimated LE of Impaired Life

Slope Risk Adjustment

Misstatement Risk & Statistical Volatility Risk Adjustment

Estimate Life Expectancy

Final PADs
Slope Risk Adjustment

- Hardest risk to quantify because different impaired mortality slopes can have the same LE, but different annuity costs.

- Impaired mortality table can be constructed after obtaining initial estimated LE.

- There are 2 methods will be used to estimate the future mortality, \( q'_{x+t} \) for impaired annuitants.
  - For acute or degenerative chronic conditions \( q'_{x+t} \) can be represented as a generic model: 
    \[
    q'_{x+t} = A_t q_{x+t} + b_t
    \]
    where
    - \( q_{x+t} = \) Mortality after \( t \) years of healthy life who purchased at age \( x \)
    - \( b_t = \) Substandard flat extra
    - \( A_t = \) Substandard mortality multiple

Time parameter \( t \) is needed because many chronic condition’s extra mortality would be expected to tail off with advancing years.
Slope Risk Adjustment (Cont’d)

For static chronic medical conditions, \( q'_{x+t} \) can be represented by a combination of Generic model and Log-Linear Declining Relative Rate (LDR) method.

Log-Linear Declining Relative Rate (LDR) methods

LDR method: 
\[
\ln\left(\frac{q'_{x+t}}{q_{x+t}}\right) = \beta(\alpha - x),
\]
where,

\[
q_{x+t} = \text{Mortality after } t \text{ years of healthy life who purchased at age } x
\]
\[
\alpha = \text{Parity age (i.e. estimated mortality rate is equal to } q_{x+t} \text{ at } x = \alpha
\]
\[
q'_{x+t} = q_{x+t}\text{ for } x + t > \alpha
\]
\[
\beta = \text{Declining rate of log relative risk (depends on the level of impairment)}
\]

The parameters \( \alpha \) and \( \beta \) are estimated from the observed data.

Example: Consider a Spinal cord injury situation

- The period shortly after spinal cord injury is one of especially high risk.
  \( q'_{x+} = Aq_{x+t} \) is appropriate for \( q'_{x+t} \)
- After that he has fairly low risk over his life span
  LDR method gives better estimates for \( q'_{x+t} \)
Slope Risk (Cont’d)

• Finally, $q^{'}_{x+t}$ will be estimated by using one of the above methods depending on the following disability scenarios: Policyholder has

  ➢ Temporary high risk to period $n$ and normal health after
  ➢ Permanent impairment
  ➢ Permanent impairment with temporary high risk for period $n$
PAD For Misstatement Risk

- Requires 2 initial inputs:
  - Level of confidence / reliability of underwriter;
  - Level of tolerance of the cost of the misstatement risk.

- Underwriter reliability at level \((1-\alpha)\) is translated into:
  \[
  \Pr \left[ \text{true} \ \text{LE} \leq \text{underwriting} \ \text{LE} \right] = 1-\alpha
  \]

- Probability distribution assumed on LE understatement satisfies two constraints:
  - Sum of probabilities for each year of LE understatement must equal \(\alpha\)
  - Probability decreases as level of LE understatement increases.

- Probabilities are assigned exponentially for each LE understatement under three degrees of difficulty in estimating LE:
  - Low
  - Medium
  - High
PAD For Misstatement Risk (Cont’d)

• Cost of misstatement risk is normalized to equal \((A - B)/B\) where:
  \(A = \text{annuity cost or loss function value at issue}\) when LE equal to ‘true’ LE
  \(B = \text{annuity cost or loss function value at issue}\) when LE equal to u/w LE

• PAD is chosen such that expected normalized misstatement cost with PAD is within tolerance level.

\[
\begin{align*}
K_x \\
\text{Life annuity cost} &= \sum_{k=1}^{K_x} R_k k p_x v^k \\
\text{Loss function at issue} &= \sum_{k=1}^{K_x} R_k v^k - \sum_{k=1}^{K_x} p_k v^k
\end{align*}
\]

*Expected Normalized Misstatement Cost* = \(\sum (A - B)/B \times \Pr(\text{LE understatement})\)

**Example:** An impaired policyholder needs a lifetime annuity and gives $1M premium to the insurance company.

<table>
<thead>
<tr>
<th>If Estimated LE</th>
<th>5 years</th>
<th>Annual benefit</th>
<th>$200K (approximately)</th>
</tr>
</thead>
<tbody>
<tr>
<td>If he lives 1 year longer than expected, The company has paid out 20% more in benefits.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>If Estimated LE</th>
<th>10 years</th>
<th>Annual benefits</th>
<th>$100 K (approximately)</th>
</tr>
</thead>
<tbody>
<tr>
<td>If he lives 1 year longer than expected, The company has paid 10% more in benefits.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Simplified Example of Misstatement Risk PAD

- Assume the following:
  - Impaired u/w LE = 5 years
  - LE of corresponding healthy lives at same issue age = 10 years
  - \( i = 0 \)
  - Underwriter reliability = 85%
  - Level of tolerance = 5%

- If ‘true’ LE is 6 years, then normalized cost is \((6-5)/5 = 1/5\)

Assume that the underwriter can recognize the level of difficulty in estimating in LE is “Medium” then the probability distribution of the normalized cost is given by:

<table>
<thead>
<tr>
<th>TRUE LE</th>
<th>Normalized Cost</th>
<th>Probability</th>
<th>Normalized Cost * Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤ 5</td>
<td>-</td>
<td>0.850</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.2</td>
<td>0.077</td>
<td>0.155</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>0.037</td>
<td>0.150</td>
</tr>
<tr>
<td>8</td>
<td>0.6</td>
<td>0.018</td>
<td>0.111</td>
</tr>
<tr>
<td>9</td>
<td>0.8</td>
<td>0.009</td>
<td>0.007</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
<td>0.004</td>
<td>0.004</td>
</tr>
</tbody>
</table>

TOTAL 0.053

\[ \text{E[Normalized Cost]} > 5\% \]
Simplified **Example** (cont’d)

- Suppose the PAD of 1 year increase in LE is used i.e. pricing LE = 6 years
- Then, normalized cost distribution is as follows:

<table>
<thead>
<tr>
<th>LE</th>
<th>NO PAD</th>
<th>PAD OF 1 YEAR INCREASE IN LE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TRUE</td>
<td>Normalized Cost</td>
</tr>
<tr>
<td></td>
<td>≥ 5</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.2</td>
<td>0.077</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>0.037</td>
</tr>
<tr>
<td>8</td>
<td>0.6</td>
<td>0.018</td>
</tr>
<tr>
<td>9</td>
<td>0.8</td>
<td>0.009</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>TOTAL</td>
<td>0.053</td>
</tr>
</tbody>
</table>

**Normalized Probability** = (6 – 5)/5

**Normalized Probability** = (7 – 6)/6

- Since E[Normalized Cost] < 5%, PAD for misstatement risk equals 1 year increase in LE.
PAD For Statistical Volatility Risk

- Exists because LE is the expected value of the future lifetime random variable.

- **Actual** future years lived have roughly a 50-50 chance of exceeding the underwriting LE, even if it is correct.

- PAD for statistical volatility risk takes the form:
  \[ \frac{C \times \sigma}{\sqrt{N}} \]
  where
  - \(C\) = level of confidence required for PAD
  - \(\sigma\) = standard deviation of the future lifetime random variable
  - \(N\) = average number of policies sold.
Measuring Riskiness of the Portfolio

- Assume the following:
  - A person age 30 is suffering from a spinal cord injury - Frankel Grade ABC (C1–C4)
  - Impaired u/w LE = 25 years
  - LE of corresponding healthy lives at same issue age = 49.75 years
  - i = 6%  Annual benefit = $100
  - Underwriter reliability = 85%,  Level of tolerance = 5%
  - The level of difficulty in estimating in LE is “Medium”

Risk Index of Annuity Portfolio

<table>
<thead>
<tr>
<th></th>
<th>N = 500</th>
<th></th>
<th>N = 1000</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NO PAD</td>
<td>PAD = 3</td>
<td>NO PAD</td>
<td>PAD = 2</td>
</tr>
<tr>
<td>E[Y_{Tot}</td>
<td>S]</td>
<td>477494.991</td>
<td>618040.577</td>
<td>904989.982</td>
</tr>
<tr>
<td>Var(Y_{Tot}</td>
<td>S)</td>
<td>43445745.989</td>
<td>39129265.057</td>
<td>86891491.978</td>
</tr>
<tr>
<td>r</td>
<td>0.01380</td>
<td><strong>0.01012</strong></td>
<td>0.01030</td>
<td><strong>0.00827</strong></td>
</tr>
</tbody>
</table>

- Riskiness of the portfolio is decreasing after applying the PAD to the initial Estimate.
- Risk index (r) is also decreasing with the size of the annuity portfolio.
Application of the Model

• Applied PAD model for a leading New England insurance company’s *Structured Settlements* business

• A financial or insurance arrangement, including periodic payments, that a claimant accepts to resolve a personal injury or to reflect a statutory period payment obligation
Actual To Expected Analysis

- Actual to expected in force deaths for calendar years 2001 through 2004 were compared using our PAD model vs company’s model.

<table>
<thead>
<tr>
<th>Calendar Year</th>
<th>Our Model</th>
<th>Company Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>1.44</td>
<td>1.10</td>
</tr>
<tr>
<td>2002</td>
<td>1.71</td>
<td>1.02</td>
</tr>
<tr>
<td>2003</td>
<td>1.25</td>
<td>0.95</td>
</tr>
<tr>
<td>2004</td>
<td>1.51</td>
<td>1.08</td>
</tr>
</tbody>
</table>

*Expected deaths based on 1983 IAM table (Adjusted table)*

- Actual to expected ratios of death are **higher** using our PADs compared to the company’s model.
Implications Of PAD Model : Life Settlements

• Model has generated interest by Life Settlements companies as a means to improve on the underwriting information provided by outside underwriting agencies.

• Model lends itself naturally for commercialization to be used for:
  – Impaired Annuity Products;
  – Structured Settlements pricing;
  – Life Settlements pricing;
References


- David Strauss, Robert Shavelle, Christopher Pflaum and Christopher Bruce (2001), *Discounting the Cost of Future Care for person with disabilities*, Life expectancy Project, CA


- Tom Ng Chu (2003), *Stochastic Simulation in Valuing Mortality and Investment risks in life annuity contracts*, University of Philippines, Dilmiman, Philippines

- Ronald Lee, Timothy Miller (2000), *Evaluating the performance of Lee-Carter Mortality forecasts*, University of California, CA