Investment Symposium
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I3 – Challenges in Index Replication

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Miguel Alvarez

Moderator
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Tracking Error and Taming the Culprits
Miguel Alvarez
Deutsche Bank

Outline
- Tracking error and its role in index replication
- Dissecting tracking error
  - Mechanics
  - Drivers
- What are the dominant culprits?
- How to tame the culprits?
Tracking Error

- Tracking error (TE) is used to measure the accuracy of the replication.

- Usually defined as the volatility of the difference in returns between the replicating instrument and the index:

\[ TE = \text{std} \left( r_{\text{replication}} - r_{\text{index}} \right) \]
What Drives Tracking Error?

- Not just volatility!

![Graph showing S&P 500 Volatility and SPY Tracking Error over time]

Mechanics of Index Replication

\[ r_{\text{index}} = \sum_{i \in \text{index}} w_i r_i \quad r_{\text{rep}} = \sum_{j \in \text{rep}} \hat{x}_j r_j \]

- The weights \( \hat{x}_j \)'s are computed to minimize TE:

\[ \hat{x} = \min_x \text{var} \left( r_{\text{index}} - \sum_{j \in \text{rep}} x_j r_j \right) \]

- The optimal \( \hat{x}_j \)'s are highly dependent on the variances and correlations of the assets (cov matrix)

* Constraints and other terms are usually added to the optimization
Mechanics of Tracking Error

- Model asset return as:
  
  \[ r_{\text{asset},t} = \beta_{\text{asset},t} \cdot r_{\text{index},t} + res_{\text{asset},t} \]

  Aggregate assets by optimal weights to form replicate

  \[ r_{\text{rep},t} = \beta_{\text{rep},t} \cdot r_{\text{index},t} + res_{\text{rep},t} \]

  \[ \sum \beta_{\text{wm},i,t} \]

  risk diversifies

  \[ r_{\text{rep},t} = r_{\text{index},t} + (\beta_{\text{rep},t} - 1) \cdot r_{\text{index},t} + res_{\text{rep},t} \]

  Tracking Error = Volatility of this term

Drivers of Tracking Error

- Tracking Error:

  \[ TE^2 = \text{var} \left[ (\beta_{\text{rep}} - 1) \cdot r_{\text{index}} \right] + \text{var} \left[ res_{\text{rep}} \right] \]

  \[ \text{systematic risk} \]

  \[ \text{specific risk} \]

- Systematic risk
  - Volatility of index coupled with
  - Dispersion (variability) of betas

- Specific risk
  - Levels and changes in specific risk of assets
Tracking Error vs. Specific Risk

Median Specific Risk*  
SPY Tracking Error (12-month std dev)

(TE/Volatility) vs. Beta Dispersion

TE / (S&P 500 Volatility)  
Beta* Dispersion

* Axioma specific risk forecasts for top 500 assets by mktcap

* Axioma beta forecasts for top 500 assets by mktcap
More Generally

- Systematic risk depends on
  - Volatility of systematic factors
  - Correlation between the assets
    - E.g., betas and their dispersion are dependent on correlation and factor volatility
- Specific risk depends on
  - Specific risk of the assets
  - Amount of overlap between the assets in the index and the assets in the replication instrument

Culprits: What Amplifies TE?

- Changes in index and asset relationships
  - Rapid or abrupt changes in volatility or correlation
- Estimation error in the inputs
  - Resulting from model misspecification or,
  - Too many inputs and not enough data
- Variance and covariance may be insufficient to properly track the index return distribution
  - May need higher moments and co-moments
Taming the Culprits

- Capturing changes in parameters:
  - Higher frequency data
    - More data may allow to identify changes more quickly
    - Limits: may capture more noise than signal
  - Use estimation techniques that are more responsive to changes in the parameters
    - E.g. Kalman filter (state space model)
    - Recursive estimates (sequential statistics)

Taming the Culprits (continued)

- Estimation error:
  - Use factor models to improve model specification and reduce dimensionality
  - Use Bayesian methods (shrinkage)
  - Higher frequency data
Future Research

- Use skewness and co-skewness in addition to variance and covariance
  - Difficult to measure with accuracy
  - Portfolio construction methodology not well defined and difficult to implement
Delivering Index Returns
SOA Investment Symposium

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How should I get my index exposure?
So Many Choices!

Instruments for Index Replication

- Cash Basket
- Listed Futures
- ETFs on Cash
- ETFs on Swap
- Index Swap
- Index-Linked Note
- Custom Indices
Cash Basket of Stocks

**Pro’s**
- Complete control over a basket of securities, including record ownership, voting rights
- No bells & whistles, no documentation required, no counterparty risk
- Relative ease of commencement – just trade a program

**Con’s**
- Unlevered (by default, unless posted at a Prime Broker)
- Minimal pass-through of lending value – names are traditionally General Collateral
- Cash collateral reinvestment risk associated with agency lending programs
- Account setup in smaller/emerging markets may not be easy (or possible)
- Operationally intensive portfolio rebalances
- No access to index data

**Best Application:** Short-term holdings in simple indices (Dow Jones 30, S&P 100)

Listed Futures

**Pro’s**
- Simple execution: one click to trade deep screen market
- Overnight trading capability (market open almost 24 hours a day)
- Embedded leverage
- No rebalancing required
- Listed instrument with central clearing – no counterparty risk
- Emerging markets futures sometimes roll at a discount to fair value (beneficial for long holders)

**Con’s**
- Quarterly rolls
- Interest rates – future decays at the onshore risk-free rate (can be expensive in some markets)
- Basis risk – futures can trade & roll at a premium if balance sheets are constrained
- Standalone marging versus exchange (no cross-margining vs other positions)
- Trading & clearing accounts required (including international clearing as appropriate)

**Best Application:** Short term holdings in highly liquid futures markets (S&P 500)
Pro's
- Ease of execution: screen market plus capital commitment from trading desks when necessary
- Creation/redemption at NAV of large clips (for a price)
- No rebalancing required
- US-hours liquidity for some foreign markets
- No counterparty risk if held by custodian

Con's
- Management fee (9.5bps for SPY, but 35bps for EFA, 56bps for EWJ, 72bps for EEM)
- Tracking risk against benchmarks – some ETFs do not replicate the index, especially for international underlyings; divergence can be substantial
- Premium/discount to NAV – common for funds traded outside the US
- Cash collateral reinvestment risk associated with agency lending programs
- Only levered if deposited at Prime Broker
- Typically GC borrow – minimal repo value passed through by custodian

Best Application: domestic & international indices in which you need to be able to source liquidity via capital commitments during US hours
ETF Dislocations vs NAV

-15.00% 
-10.00% 
-5.00% 
0.00% 
5.00% 
10.00% 
15.00% 
20.00% 
2823 Prem

2823 HK premium/discount versus NAV. Source: Bloomberg, Deutsche Bank, 2Mar10

ETF Tracking Risk vs Index in 2009

<table>
<thead>
<tr>
<th>Ticker</th>
<th>Underlying Index</th>
<th>Mgmt Fee</th>
<th>Index</th>
<th>Fund Return</th>
<th>NAV Perf</th>
<th>Index Return</th>
<th>Fund vs Index</th>
<th>NAV vs Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>EWZ</td>
<td>TR Net Brazil USD</td>
<td>0.65%</td>
<td>NDUBRAF</td>
<td>113.23%</td>
<td>115.65%</td>
<td>128.06%</td>
<td>-14.83%</td>
<td>-12.41%</td>
</tr>
<tr>
<td>EEM</td>
<td>Daily TR Net Emerging Mkts USD</td>
<td>0.72%</td>
<td>NDUEEGF</td>
<td>66.25%</td>
<td>69.00%</td>
<td>75.51%</td>
<td>-12.26%</td>
<td>-9.51%</td>
</tr>
<tr>
<td>IEA</td>
<td>EM South Africa USD</td>
<td>0.66%</td>
<td>NDUEUSA</td>
<td>45.26%</td>
<td>49.84%</td>
<td>57.82%</td>
<td>-12.55%</td>
<td>-7.98%</td>
</tr>
<tr>
<td>EPP</td>
<td>Pacific Free Ex Japan USD</td>
<td>0.50%</td>
<td>NDUDPFXJ</td>
<td>57.00%</td>
<td>65.04%</td>
<td>72.81%</td>
<td>-15.81%</td>
<td>-7.76%</td>
</tr>
<tr>
<td>ILF</td>
<td>S&amp;P LAC 40 Net TR</td>
<td>0.50%</td>
<td>SPTRARJ</td>
<td>87.63%</td>
<td>89.28%</td>
<td>95.96%</td>
<td>-9.33%</td>
<td>-7.68%</td>
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<tr>
<td>EWA</td>
<td>Australia USD</td>
<td>0.55%</td>
<td>NDUASA</td>
<td>63.03%</td>
<td>69.83%</td>
<td>76.43%</td>
<td>-13.41%</td>
<td>-6.60%</td>
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<tr>
<td>FUR</td>
<td>Turkey</td>
<td>0.65%</td>
<td>MMUTR</td>
<td>98.75%</td>
<td>103.50%</td>
<td>109.02%</td>
<td>-10.28%</td>
<td>-5.52%</td>
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<tr>
<td>EWF</td>
<td>MSCI Daily TR Net UK USD</td>
<td>0.55%</td>
<td>NDUKUK</td>
<td>32.24%</td>
<td>38.05%</td>
<td>43.30%</td>
<td>-11.06%</td>
<td>-5.26%</td>
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<tr>
<td>BKF</td>
<td>BRIC USD</td>
<td>0.72%</td>
<td>NDUBRIC</td>
<td>83.62%</td>
<td>87.96%</td>
<td>93.12%</td>
<td>-9.50%</td>
<td>-5.16%</td>
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<tr>
<td>EWH</td>
<td>Hong Kong USD</td>
<td>0.55%</td>
<td>NDUHK</td>
<td>51.01%</td>
<td>55.08%</td>
<td>60.15%</td>
<td>-9.14%</td>
<td>-5.07%</td>
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<tr>
<td>EFV</td>
<td>Value EAFE Free USD</td>
<td>0.40%</td>
<td>NDUEAFF</td>
<td>24.14%</td>
<td>29.39%</td>
<td>34.23%</td>
<td>-10.96%</td>
<td>-5.84%</td>
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<tr>
<td>EWJ</td>
<td>EM Net Taiwan USD</td>
<td>0.65%</td>
<td>NDUETW</td>
<td>70.88%</td>
<td>74.46%</td>
<td>79.21%</td>
<td>-8.33%</td>
<td>-4.76%</td>
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<tr>
<td>EMA</td>
<td>EMU</td>
<td>0.56%</td>
<td>NDUEMU</td>
<td>22.41%</td>
<td>26.85%</td>
<td>31.41%</td>
<td>-9.00%</td>
<td>-4.52%</td>
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<tr>
<td>EWM</td>
<td>Malaysia USD</td>
<td>0.56%</td>
<td>NDUMAP</td>
<td>45.68%</td>
<td>47.62%</td>
<td>52.06%</td>
<td>-6.38%</td>
<td>-4.44%</td>
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<td>IFA</td>
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<td>0.35%</td>
<td>NDUEAFE</td>
<td>23.23%</td>
<td>27.54%</td>
<td>31.78%</td>
<td>-8.55%</td>
<td>-4.24%</td>
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<tr>
<td>EWA</td>
<td>France USD</td>
<td>0.55%</td>
<td>NDUFRA</td>
<td>23.51%</td>
<td>27.82%</td>
<td>31.83%</td>
<td>-8.33%</td>
<td>-4.02%</td>
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<tr>
<td>ACWX</td>
<td>TR Net Ex USA USD</td>
<td>0.35%</td>
<td>NDUEAWX</td>
<td>33.17%</td>
<td>37.53%</td>
<td>41.45%</td>
<td>-8.28%</td>
<td>-3.92%</td>
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<td>EWG</td>
<td>Germany USD</td>
<td>0.55%</td>
<td>NDUEUG</td>
<td>16.75%</td>
<td>21.25%</td>
<td>25.15%</td>
<td>-8.40%</td>
<td>-4.91%</td>
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<tr>
<td>AAXJ</td>
<td>Asia Ex Japan USD</td>
<td>0.72%</td>
<td>NDUEAXJ</td>
<td>66.15%</td>
<td>68.54%</td>
<td>72.07%</td>
<td>-5.92%</td>
<td>-3.53%</td>
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<tr>
<td>EWW</td>
<td>EM Mexico USD</td>
<td>0.55%</td>
<td>NDUEWMX</td>
<td>51.44%</td>
<td>53.30%</td>
<td>56.63%</td>
<td>-5.19%</td>
<td>-3.33%</td>
</tr>
<tr>
<td>EFH</td>
<td>EAFE Free USD</td>
<td>0.40%</td>
<td>NDUEAFF</td>
<td>21.28%</td>
<td>26.09%</td>
<td>29.36%</td>
<td>-8.09%</td>
<td>-3.28%</td>
</tr>
<tr>
<td>IIOO</td>
<td>Global 100 Net</td>
<td>0.40%</td>
<td>SPTR100N</td>
<td>20.16%</td>
<td>22.57%</td>
<td>25.74%</td>
<td>-5.57%</td>
<td>-3.17%</td>
</tr>
<tr>
<td>EWC</td>
<td>Canada USD</td>
<td>0.55%</td>
<td>NDUCA</td>
<td>51.06%</td>
<td>53.70%</td>
<td>56.18%</td>
<td>-5.12%</td>
<td>-2.49%</td>
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<tr>
<td>EWI</td>
<td>Japan USD</td>
<td>0.56%</td>
<td>NDUJ</td>
<td>1.67%</td>
<td>3.63%</td>
<td>6.25%</td>
<td>-4.55%</td>
<td>-2.43%</td>
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<tr>
<td>EWL</td>
<td>Switzerland USD</td>
<td>0.56%</td>
<td>NDUSZ</td>
<td>20.96%</td>
<td>23.83%</td>
<td>25.31%</td>
<td>-5.25%</td>
<td>-1.48%</td>
</tr>
</tbody>
</table>

Source: Bloomberg, Deutsche Bank, 19 Feb 2010
ETF Tracking Risk vs Index: EFA

Comparative Performance

EFA vs MSCI EAFE TR Net USD
Source: Deutsche Bank, Bloomberg 19 Feb 2010

ETFs on Swap

Pro’s
- Many of the benefits of ETF trading
- Leverage via swap / synthetic PB facility; cross margining vs. other positions
- Transparent capture of repo (trade-specific financing rate) – generally easy for swap dealers to loan
- Control your cash collateral reinvestment risk associated with agency lending programs

Con’s
- Previously-mentioned drawbacks (management fee, tracking risk)
- Counterparty risk vs swap issuer on the MTM of the position since last reset
- While repo rate is generally better, most widely-held ETFs are unlikely to be “special”

Best Application: medium-term holdings in funds with good lending value (XRT, IYR, KRE)
Index Swaps

Pro's
- Precise index replication
- Rebalances handled by DB
- Efficient capture of constituents' lending value, especially on termed trades
- Levered; cross-margined against other swap/PB holdings
- Control your cash collateral reinvestment risk associated with agency lending programs
- Customized: your choice of tenor, reset frequency, interest benchmark
- Outperformance: Less liquid indices often fund below Libor

Con's
- Counterparty risk vs swap issuer on the MTM of the position since last reset
- ISDA & credit facility required
- Some term trades include breakfees
- Deltas tradable only during underlying market hours

Best Application: long term holdings in almost any index, especially MSCI's

Global Index Swap Levels

<table>
<thead>
<tr>
<th>Global</th>
<th>Bloomberg</th>
<th>Term</th>
<th>Size (MM)</th>
<th>Reference Rate</th>
<th>Bid</th>
<th>Offer</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI World</td>
<td>NDXIX</td>
<td>YR</td>
<td>100</td>
<td>USD 3M Libor</td>
<td>-35</td>
<td>-24</td>
<td>N</td>
</tr>
<tr>
<td>MSCI Europe</td>
<td>NDXII</td>
<td>YR</td>
<td>100</td>
<td>USD 3M Libor</td>
<td>-4</td>
<td>-28</td>
<td>N</td>
</tr>
<tr>
<td>MSCI EAFE</td>
<td>NDXIE</td>
<td>YR</td>
<td>100</td>
<td>USD 3M Libor</td>
<td>-43</td>
<td>-37</td>
<td>N</td>
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<tr>
<td>MSCI EM World</td>
<td>NDXIF</td>
<td>YR</td>
<td>100</td>
<td>USD 3M Libor</td>
<td>-48</td>
<td>-43</td>
<td>N</td>
</tr>
</tbody>
</table>

Source: Deutsche Bank 18 Feb 2010
Index-Linked Notes

Pro’s
- Precise index replication
- Rebalances handled by DB
- Outperformance payable on some less liquid indices
- Minimal documentation required; No ISDA
- Can be listed on an exchange to meet funds’ requirements
- No management fees

Con’s
- No leverage
- Counterparty risk versus issuer on full trade notional

Best Application: long term holdings in almost any index, especially MSCI’s

Custom Indices: When No ETF is a Perfect Match

Pro’s
- **Expensive borrow:** construct a basket designed to track the ETF, using easily-available constituents
  - Example: Cheap basket versions of XLF, XRT, IYR, others
- **Imperfect hedge:** start with a given ETF or index, and then add/subtract constituents to get the exposure you want
  - Example: Hedging AAPL with QQQQ’s while avoiding redundancy (14% of index)
- **Liquidity:** if your ideal ETF does not trade actively, or your index contains illiquid members – remove & reweight w/ optimization
  - Example: Russian GDR index (see November 2008 example on future slides)
- **Thematic mismatch:** index/ETF contains or overweights names you don’t want (or, no ETF exists)
  - Example: SMH Semis HOLDR. Intel (23%) + Texas Instruments (21%) + Applied Materials (14%) = 58% of the index (only 18 constituents in total)

Con’s
- Bespoke & non-portable; same risks as swap or P-Note

Best Application: Expensive (short side), Illiquid, or misweighted indices.
DAX & EuroStoxx

Volkswagen local shares price history. Source: Bloomberg, Deutsche Bank, 1May08 to 1Mar10

RDXUSD: Creating a liquid Russia hedge

RDXUSD: A capitalization-weighted index of the most liquid depository receipts on Russian ordinary shares.

<table>
<thead>
<tr>
<th>RDXUSD</th>
<th>Name</th>
<th>Wgt</th>
<th>TO $/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>OGZD LI Equity</td>
<td>Gazprom OAO</td>
<td>23.80%</td>
<td>446,969,364</td>
</tr>
<tr>
<td>LKOD LI Equity</td>
<td>LUKOIL</td>
<td>19.49%</td>
<td>185,913,068</td>
</tr>
<tr>
<td>ROSN LI Equity</td>
<td>Rosneft Oil Co</td>
<td>12.85%</td>
<td>74,460,257</td>
</tr>
<tr>
<td>MNOD LI Equity</td>
<td>MMC Norilsk Nickel</td>
<td>11.81%</td>
<td>80,086,037</td>
</tr>
<tr>
<td>SGGD LI Equity</td>
<td>Surgutneftegaz</td>
<td>13.23%</td>
<td>43,431,125</td>
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<tr>
<td>ATAD LI Equity</td>
<td>Tatneft</td>
<td>3.67%</td>
<td>8,862,986</td>
</tr>
<tr>
<td>NVTK LI Equity</td>
<td>NovaTek OAO</td>
<td>3.37%</td>
<td>24,103,133</td>
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<tr>
<td>VTBR LI Equity</td>
<td>VTB Bank OJSC</td>
<td>2.68%</td>
<td>18,110,726</td>
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<tr>
<td>PLZL LI Equity</td>
<td>Polymetals Gold Co</td>
<td>2.19%</td>
<td>3,063,030</td>
</tr>
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<td>URKA LI Equity</td>
<td>Uralkali</td>
<td>2.05%</td>
<td>13,641,677</td>
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<tr>
<td>EVR LI Equity</td>
<td>Evraz Group SA</td>
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<td>15,046,350</td>
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<td>NLMK LI Equity</td>
<td>Novolipetsk Steel OJSC</td>
<td>1.49%</td>
<td>5,588,700</td>
</tr>
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<td>SVST LI Equity</td>
<td>Severstal</td>
<td>1.00%</td>
<td>4,955,306</td>
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<tr>
<td>SSA LI Equity</td>
<td>Sistema JSFC</td>
<td>0.88%</td>
<td>7,038,996</td>
</tr>
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</table>

Max index notional when trading <25% of each name’s average daily volume: $35mm

Largest limiting factors in RED

Source: Bloomberg, Deutsche Bank.
RDXUSD: Creating a liquid Russia hedge

- Top 4 names represent 68% of the index by market cap
- Market-cap weightings preserved (pro-rata reweight)
- Correlation to index YTD: 99.15%

Max basket notional when trading <20% of each name’s average daily volume: $100mm

Source: Bloomberg, Deutsche Bank

Side-by-Side Comparison

<table>
<thead>
<tr>
<th>BENEFICIAL OWNER</th>
<th>CASH</th>
<th>FUTURES</th>
<th>ETF (CASH)</th>
<th>ETF (SWAP)</th>
<th>INDEX SWAP</th>
<th>CUSTOM SWAP</th>
<th>NOTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
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<td>DOCUMENTATION</td>
<td>ACCOUNT</td>
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<td>ISDA/CSA</td>
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<td>ISDA/CSA</td>
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<td>LEVERAGE</td>
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<td>REPO VALUE</td>
<td>SHARED</td>
<td>ON ROLL</td>
<td>SHARED</td>
<td>PASS THROUGH</td>
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<td>REINVESTMENT RISK</td>
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<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td></td>
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<td>MANUAL REBALANCE</td>
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<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td></td>
</tr>
<tr>
<td>MANAGEMENT FEE</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td></td>
</tr>
<tr>
<td>TRACKING RISK</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td></td>
</tr>
<tr>
<td>COUNTERPARTY RISK</td>
<td>NO (CUSTOM)</td>
<td>NO (CUSTOM)</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td></td>
</tr>
</tbody>
</table>

Source: Deutsche Bank
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- Source: Bloomberg closing prices, backtest uses the closing price of the day before for a country or exchange specific holiday. Please refer to earlier slides in the presentation for term sheets and structure. Back testing does not represent historical prices.
- The backtesting results enclosed herein do not represent historical prices. The model reflects historical payout and does not include related costs or tax implications. The back test does not delineate how prices have varied historically, and does not include volatility, or interest rate assumptions. The backtesting results discussed herein have inherent limitations. Unlike an actual performance record based on trading actual client portfolios, simulated results are achieved by means of the retroactive application of a back tested model itself designed with the benefit of hindsight. Taking into account historical events the back testing of performance also differs from actual account performance because an actual investment strategy may be adjusted at any time, for any reason, including a response to material, economic or market factors. The back tested performance includes hypothetical adjustments to reflect the application of the back tested strategy. No representation is made that any trading strategy or account will or is likely to achieve profits or losses similar to those shown. Alternative modeling techniques or assumptions might produce significantly different results and prove to be more appropriate. Past hypothetical backtest results are neither an indicator nor guarantee of future returns. Actual results will vary, perhaps materially, from the analysis. Results represent the performance of each basket on a back tested basis, tied to a structure whose economics are determined by current economic factors such as current volatilities and interest rates. There is no guarantee that a similar structure would have been available at any point in the past and that such results could have been achieved.

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Leveraged and Inverse Index Exposure

Solomon Teller, CFA, Head of Investment Analytics
ProFunds Group

What are leveraged & inverse ETFs

Up Day
(Index up 1%)
Leveraged inverse fund should fall 2%
Leveraged long fund should gain -2%

Down Day
(Index down 1%)
Leveraged long fund should fall 2%
Leveraged inverse fund should gain -2%

The ETFs seek a daily return that is a multiple of the return of the index (target). Due to compounding of daily returns, results over periods other than a day will likely differ in amount and possibly direction from the target return for the same period.

* Before fees and expenses.
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Leveraged and inverse funds and ETFs: History and Profile

• Leveraged and inverse mutual funds introduced in 1993
  - Grew to over 100 funds with $10 billion in assets
  - ETFs introduced in 2006 in the U.S.

• Today more than 155 funds
  - Assets are now more than $30 billion
  - Daily volume of more than $18 billion per day

<table>
<thead>
<tr>
<th>Ticker</th>
<th>ETF Name</th>
<th>Assets (m$)</th>
<th>$/Assets Daily Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBT</td>
<td>ProShares UltraShort 20+ Year Treasury</td>
<td>$4,601</td>
<td>15.8</td>
</tr>
<tr>
<td>SDS</td>
<td>ProShares UltraShort S&amp;P500</td>
<td>$2,928</td>
<td>1.9</td>
</tr>
<tr>
<td>UYG</td>
<td>ProShares Ultra Financials</td>
<td>$1,929</td>
<td>12.0</td>
</tr>
<tr>
<td>SSO</td>
<td>ProShares Ultra S&amp;P500</td>
<td>$1,725</td>
<td>2.2</td>
</tr>
<tr>
<td>SH</td>
<td>ProShares Short S&amp;P500</td>
<td>$1,545</td>
<td>9.4</td>
</tr>
<tr>
<td>FAS</td>
<td>Direxion Daily Financial Bull 3x Shares</td>
<td>$1,240</td>
<td>0.7</td>
</tr>
<tr>
<td>FAZ</td>
<td>Direxion Daily Financial Bear 3x Shares</td>
<td>$1,074</td>
<td>0.8</td>
</tr>
<tr>
<td>QLD</td>
<td>ProShares Ultra QQQ</td>
<td>$1,002</td>
<td>1.8</td>
</tr>
<tr>
<td>SKF</td>
<td>ProShares UltraShort Financials</td>
<td>$836</td>
<td>1.3</td>
</tr>
<tr>
<td>QID</td>
<td>ProShares UltraShort QQQ</td>
<td>$815</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Why they’re valued

• Efficient tool for investors with a view of the market
• Can trade and follow like a stock
• Can’t lose more than you invest
• Institutional pricing
• Transparent
• Liquid

How they’re used

• Tactical tools like other ETFs
• Component of overall portfolio strategy
  - Target exposure with less cash
  - Overweight or underweight exposure
  - Help manage overall portfolio risk or seek to hedge specific risk exposures
Constructing a 2x S&P 500 Leveraged Fund

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Index daily gain of 2%: Impact on +2x leveraged fund assets and exposure

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Example of compounding on indexes and leveraged funds

<table>
<thead>
<tr>
<th>Index</th>
<th>2x Fund</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INDEX</strong></td>
<td><strong>2x FUND</strong></td>
</tr>
<tr>
<td>Daily Return</td>
<td>Daily Return</td>
</tr>
<tr>
<td><strong>UPWARD TRENDS</strong></td>
<td></td>
</tr>
<tr>
<td>Day 1 Return</td>
<td>+10%</td>
</tr>
<tr>
<td>Day 2 Return</td>
<td>+10%</td>
</tr>
<tr>
<td>Compounded 2-day return</td>
<td>+21%</td>
</tr>
<tr>
<td><strong>DOWNWARD TRENDS</strong></td>
<td></td>
</tr>
<tr>
<td>Day 1 Return</td>
<td>-10%</td>
</tr>
<tr>
<td>Day 2 Return</td>
<td>-10%</td>
</tr>
<tr>
<td>Compounded 2-day return</td>
<td>-19%</td>
</tr>
<tr>
<td><strong>VOLATILE MARKET</strong></td>
<td></td>
</tr>
<tr>
<td>Day 1 Return</td>
<td>+10%</td>
</tr>
<tr>
<td>Day 2 Return</td>
<td>-10%</td>
</tr>
<tr>
<td>Compounded 2-day return</td>
<td>-1%</td>
</tr>
</tbody>
</table>

Universal effects of compounding on investment returns

- **Compounding affects all investments over time**
  - Upward trending periods enhances returns
  - Downward trending periods reduces losses
  - Volatile periods may reduce returns and increase losses

- **Positive and negative effects of compounding may be magnified in leveraged and inverse funds**
Leveraged and inverse funds affected by record volatility

Highest short-term volatility levels for U.S. equities in 80 years affected all investments, including leveraged funds.

<table>
<thead>
<tr>
<th>Year</th>
<th>Volatility (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/31/29</td>
<td>66%</td>
</tr>
<tr>
<td>10/21/32</td>
<td>69%</td>
</tr>
<tr>
<td>1/11/88</td>
<td>60%</td>
</tr>
<tr>
<td>12/16/08</td>
<td>72%</td>
</tr>
</tbody>
</table>

Source: Journal of Indexes, August 2009. Past performance is no guarantee of future results. For illustrative purposes only.

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Historical analysis of leveraged & inverse index returns

- Research study published in *The Journal of Indexes*
- Studied 50 years covering all possible 2, 7, 30, 91, and 183 day holding periods for leveraged and inverse versions (+2x and -2x) of selected indexes
- Compared returns for +2x and -2x Daily Objective Strategies to +2x and -2x index returns for the holding periods studied
  - Results differ for ETFs that are held intraday or for longer periods, that track more narrow or volatile indexes, or that seek a different multiple return for the index

Source: Understanding Returns of Leveraged and Inverse Funds, Journal of Indexes, September/October 2009

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S&P 500 2x & -2x Returns: Compounding effect was close to zero on average

From 1959-2008, average differences between +2x/-2x S&P 500 daily objective strategy and 2 or -2 times the index return:

<table>
<thead>
<tr>
<th>Holding Periods</th>
<th>2 Days</th>
<th>7 Days</th>
<th>30 Days</th>
<th>2 Days</th>
<th>7 Days</th>
<th>30 Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>97.5th Percentile</td>
<td>0.03%</td>
<td>0.14%</td>
<td>0.71%</td>
<td>0.08%</td>
<td>0.42%</td>
<td>2.08%</td>
</tr>
<tr>
<td>75th Percentile</td>
<td>0.00%</td>
<td>0.01%</td>
<td>0.06%</td>
<td>0.00%</td>
<td>0.04%</td>
<td>0.19%</td>
</tr>
<tr>
<td>Median</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.03%</td>
<td>0.00%</td>
<td>-0.01%</td>
<td>-0.09%</td>
</tr>
<tr>
<td>25th Percentile</td>
<td>0.00%</td>
<td>-0.02%</td>
<td>-0.10%</td>
<td>0.00%</td>
<td>-0.05%</td>
<td>-0.29%</td>
</tr>
<tr>
<td>2.5th Percentile</td>
<td>-0.02%</td>
<td>-0.12%</td>
<td>-0.50%</td>
<td>-0.07%</td>
<td>-0.35%</td>
<td>-1.54%</td>
</tr>
<tr>
<td>Average</td>
<td>0.000%</td>
<td>0.000%</td>
<td>-0.002%</td>
<td>0.000%</td>
<td>-0.002%</td>
<td>-0.035%</td>
</tr>
</tbody>
</table>

While individual period returns varied, the effect of compounding on average was about neutral across the holding periods we studied.

Monitoring and rebalancing increased the likelihood of returns being close to multiple

- Some investors want returns close to the fund multiple times the index returns over longer periods
- Must monitor and rebalance to improve chances
- Similar concept as rebalancing asset allocations
- Rebalancing doesn’t always increase returns

Value of fund position vs. index

Index return is greater than fund return

Index return is less than fund return

Note that there are three primary factors that drive the frequency of rebalancing required to achieve the daily target over longer time periods: the volatility of the index, the size of the fund multiple and whether the fund multiple is positive or negative. Higher volatility, higher multiples and inverse objectives each on their own will lead to more frequent rebalancing when trying to achieve the daily target over time.